



fractions

9

Fractions give you rhythm

You can improve your understanding of fractions by learning the guitar.

Some studies claim there is a link between musical and mathematical ability. Brain imaging techniques have shown that similar changes occur in the brain when you're playing music and doing maths. This is not that surprising considering music is based on rhythm, and rhythm is based on fractions. The five most commonly used notes are:



whole note



half note



quarter note



eighth note



sixteenth note

Time signatures such as $\frac{4}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{8}$, $\frac{9}{8}$ and $\frac{12}{8}$ tell us how many beats are in a measure and how to count them.

outcomes

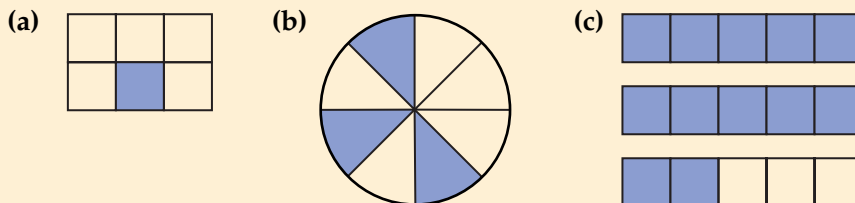
After completing this chapter you will be able to:

- add and subtract fractions and mixed numbers
- multiply and divide fractions and mixed numbers
- use fractions to represent real-life situations
- use your calculator to perform calculations with fractions and mixed numbers.

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

e Worksheet R9.1

1 What fraction could represent the following shaded areas?



- (d) Shane had to divide a birthday cake equally between ten people. What was each person's share?
 (e) Jemima scored three of her team's five goals in last week's hockey match. What was her contribution to the total?
 (f) two-thirds (g) twelve-fifths (h) $5 \div 17$ (i) 7
 (j) A denominator of nine and a numerator of ten.

e Worksheet R9.2

2 State whether each of the following is a proper fraction, an improper fraction, or a mixed number.

- (a) $\frac{8}{5}$ (b) $\frac{2}{9}$ (c) $\frac{200}{300}$
 (d) $8\frac{3}{7}$ (e) $\frac{300}{200}$ (f) $11\frac{1}{2}$

e Worksheet R9.3

3 (a) Write the fractions $\frac{3}{8}$, $\frac{1}{8}$, 0, $\frac{7}{8}$, $\frac{11}{8}$, 1, $\frac{4}{8}$, $\frac{9}{8}$ in ascending order.

(b) Copy and complete by inserting $<$, $>$ or $=$ into each of these expressions.

- (i) $\frac{4}{15}$ _____ $\frac{7}{15}$ (ii) $1\frac{1}{3}$ _____ 1 (iii) $\frac{5}{5}$ _____ 1 (iv) $\frac{8}{1}$ _____ 8

e Worksheet R9.4

4 Find:

- (a) $\frac{4}{7} + \frac{1}{7}$ (b) $\frac{8}{11} - \frac{2}{11}$

e Worksheet R9.5

5 (a) Find the first five multiples of 8. (b) Find the first five multiples of 12.

KEY WORDS

denominator

equivalent

fraction

highest common factor

improper fraction

lowest common

denominator

mixed numeral

numerator

of

proper fraction

recurring decimal

reciprocal

simplest form

simplify

9.1 Introduction to fractions

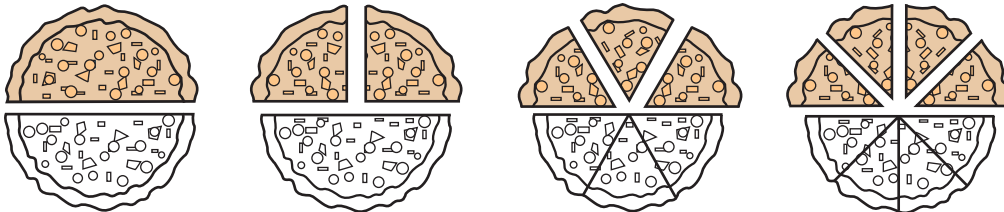
It was the Hindus of 200–1200 AD who began to use **fractions** similar to the ones we use today—they added, subtracted, multiplied and divided fractions in much the way we show in this chapter. The one main difference was that they didn't use the bar between the **numerator** (top number) and **denominator** (bottom number). This meant $\frac{3}{4} = \frac{3}{4}$. The Arabs were the ones who added the bar to the Hindu system.



Equivalent fractions

Often fractions look very different at first glance even though they are in fact equal. Another word for equal is **equivalent**.

Look at the following Mathicana pizzas.



Each picture has the same amount of pizza shaded.

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions. This means $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

Equivalent fractions can be found either by *multiplying* the numerator and the denominator by the *same* number, or by *dividing* the numerator and the denominator by the *same* number.

$$\begin{array}{c} \times 3 \\ \curvearrowright \\ \frac{1}{2} = \frac{3}{6} \\ \curvearrowleft \\ \times 3 \end{array}$$

$$\begin{array}{c} \div 2 \\ \curvearrowright \\ \frac{4}{8} = \frac{2}{4} \\ \curvearrowleft \\ \div 2 \end{array}$$

worked example 1

Find an equivalent fraction for $\frac{7}{10}$ by copying and completing $\frac{7}{10} = \frac{\square}{40}$.

Steps

- Look at the two denominators, and work out what we need to multiply or divide the first one by to get the second.
- Do the same thing to the numerator.

Solution

$$\begin{array}{c} \frac{7}{10} = \frac{\square}{40} \\ \curvearrowright \\ \times 4 \\ \frac{7}{10} = \frac{28}{40} \end{array}$$

worked example 2

Are $\frac{8}{10}$ and $\frac{5}{6}$ equivalent? Complete the following by writing either = (equals) or \neq (does not equal) between the terms.

Steps

1. Write both fractions with the same denominator.
2. Multiply the numerator of each fraction by the number used to multiply the denominator.
3. Look at the two fractions with the same denominators. If they are the same, then the fractions are equivalent. If they are not the same then the fractions are not equivalent.

Solution

$$\frac{8}{10} = \frac{\square}{30} \text{ and } \frac{5}{6} = \frac{\square}{30}$$

$$\frac{8}{10} = \frac{24}{30} \text{ and } \frac{5}{6} = \frac{25}{30}$$

$$\frac{24}{30} \neq \frac{25}{30} \text{ so } \frac{8}{10} \neq \frac{5}{6}$$



Simplifying fractions

There are many ways of writing the same fraction, but one way will usually be the 'best' or most convenient way of writing that fraction.

The best way is usually the **simplest form** of the fraction. We reduce a fraction to its simplest form (**simplify**) by dividing both the numerator and denominator by their **highest common factor (HCF)**.

Answers to questions which involve fractions should be written in simplest form.

Common factors and highest common factors

A common factor is one that appears in the factor lists of two or more given numbers. The HCF is the largest of the common factors. Every pair of numbers has the same lowest common factor. It is 1, as 1 is a factor of every number.



worked example 3

Find the common factors of 12 and 18, and state the HCF.

Steps

1. List all factors of 12.
2. List all factors of 18.
3. Pick out the factors appearing in both lists.
4. Pick out the largest of these. This is the *highest common factor*.

Solution

1, 2, 3, 4, 6, 12

1, 2, 3, 6, 9, 18

The common factors are 1, 2, 3, 6

HCF = 6

worked example 4

Reduce the fraction $\frac{20}{55}$ to its simplest form.

Steps

1. List the factors of the numerator.
2. List the factors of the denominator.
3. Find the highest common factor (HCF).
4. Divide both the numerator and the denominator by the HCF to get the fraction in its simplest form.

Solution

Factors of 20 = 1, 2, 4, 5, 10, 20

Factors of 55 = 1, 5, 11, 55

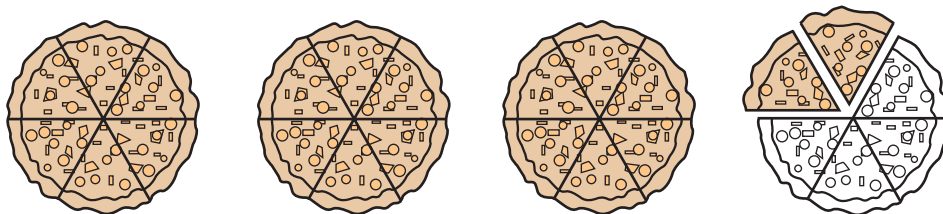
HCF = 5

$$\frac{20}{55} = \frac{4}{11}$$

eTutorial

Mixed numerals and improper fractions

Suppose four pizzas are each cut into six slices. This means each slice is $\frac{1}{6}$ of a pizza.



We can think of 3 whole pizzas as $\frac{18}{6}$ pizzas, i.e. $3 = \frac{18}{6}$

If Minh ate 20 pieces, this could be written as a fraction in two different ways.

As a **mixed numeral**, she has eaten 3 pizzas and 2 slices = $3\frac{2}{6}$ pizzas

As an improper fraction, she has eaten 20 slices = $\frac{20}{6}$ pizzas

This means $3\frac{2}{6} = \frac{20}{6}$

Remember, an **improper fraction** is one where the numerator is greater than the denominator. A **proper fraction** has a smaller numerator than denominator.

eTutorial

worked example 5

Write the mixed numeral $7\frac{3}{5}$ as an improper fraction.

Steps

1. Multiply the whole number by the denominator and add the numerator.
Write down the answer.
2. Put the total over the original denominator.
Write down the answer.

Solution

$$7 \times 5 + 3 = 38$$

$$7\frac{3}{5} = \frac{38}{5}$$

worked example 6

Write the improper fraction $\frac{34}{7}$ as a mixed numeral.

Steps

1. Divide the numerator by the denominator.
Include the remainder with your answer.
2. Write down the whole number part. Put the remainder over the original denominator to form the fractional part.

Solution

$$34 \div 7 = 4 \text{ remainder } 6$$

$$\frac{34}{7} = 4\frac{6}{7}$$

exercise 9.1 Introduction to fractions



Preparation: Prep Zone Q1 and 2

Core

1 Find equivalent fractions by copying and completing the following.

(a) $\frac{3}{4} = \frac{33}{\square}$ (b) $\frac{1}{3} = \frac{\square}{3000}$ (c) $\frac{1}{2} = \frac{\square}{400}$ (d) $\frac{2}{5} = \frac{24}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{36}{20} = \frac{\square}{5}$

(i) $\frac{24}{16} = \frac{\square}{4}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{4}{3} = \frac{\square}{27}$ (l) $\frac{7}{11} = \frac{\square}{99}$

(m) $\frac{5}{7} = \frac{100}{\square}$ (n) $\frac{6}{5} = \frac{42}{\square}$ (o) $\frac{54}{36} = \frac{\square}{6}$ (p) $\frac{72}{36} = \frac{\square}{9}$

(q) $\frac{8}{12} = \frac{96}{\square}$ (r) $\frac{5}{3} = \frac{60}{\square}$ (s) $\frac{11}{9} = \frac{121}{\square}$ (t) $\frac{10}{14} = \frac{50}{\square}$

2 (a) Write down five fractions that are equivalent to $\frac{4}{5}$.

(b) Write down five fractions that are equivalent to $\frac{12}{8}$.

3 Are these pairs of fractions equivalent? Copy the following and complete by inserting = or \neq .

(a) $\frac{1}{5} \text{ — } \frac{3}{15}$ (b) $\frac{1}{4} \text{ — } \frac{2}{5}$ (c) $\frac{2}{3} \text{ — } \frac{6}{10}$ (d) $\frac{3}{5} \text{ — } \frac{9}{15}$

(e) $\frac{2}{11} \text{ — } \frac{8}{33}$ (f) $\frac{35}{100} \text{ — } \frac{7}{20}$ (g) $\frac{12}{100} \text{ — } \frac{3}{50}$ (h) $\frac{6}{11} \text{ — } \frac{60}{100}$

(i) $\frac{10}{25} \text{ — } \frac{4}{10}$ (j) $\frac{15}{20} \text{ — } \frac{9}{12}$ (k) $\frac{6}{24} \text{ — } \frac{5}{20}$ (l) $\frac{30}{50} \text{ — } \frac{9}{15}$

4 Find the HCF for each pair of numbers.

- (a) 10 and 15 (b) 8 and 24 (c) 5 and 12
(d) 26 and 36 (e) 11 and 33 (f) 28 and 70
(g) 44 and 22 (h) 10 and 30 (i) 40 and 70
(j) 32 and 60 (k) 35 and 70 (l) 42 and 48

Worksheet C9.1

Hint

eQuestions

Hint

Hint

eTester

Worksheet C9.2

5 Reduce the following fractions to their simplest form.

- | | | | |
|---------------------|---------------------|----------------------|----------------------|
| (a) $\frac{5}{10}$ | (b) $\frac{3}{12}$ | (c) $\frac{7}{21}$ | (d) $\frac{3}{9}$ |
| (e) $\frac{4}{10}$ | (f) $\frac{8}{22}$ | (g) $\frac{10}{16}$ | (h) $\frac{36}{24}$ |
| (i) $\frac{8}{80}$ | (j) $\frac{12}{36}$ | (k) $\frac{120}{80}$ | (l) $\frac{40}{100}$ |
| (m) $\frac{24}{32}$ | (n) $\frac{33}{18}$ | (o) $\frac{54}{66}$ | (p) $\frac{60}{48}$ |

e Hint

e hi.com.au

e Worksheet C9.3

6 Choose the correct answer.

- (a) Which one of the following is the simplest form of $\frac{3}{15}$?
- A $\frac{1}{3}$ B $\frac{1}{5}$ C $\frac{3}{5}$ D $\frac{6}{30}$
- (b) Which one of the following is the simplest form of $\frac{4}{6}$?
- A $\frac{2}{3}$ B $\frac{6}{4}$ C $\frac{8}{12}$ D $\frac{1}{2}$
- (c) Which one of the following is the simplest form of $\frac{25}{100}$?
- A $\frac{5}{20}$ B $\frac{1}{25}$ C $\frac{5}{25}$ D $\frac{1}{4}$
- (d) Which one of the following is the simplest form of $4\frac{6}{12}$?
- A $4\frac{1}{3}$ B 2 C $4\frac{1}{2}$ D $4\frac{3}{6}$
- (e) Which one of the following is the simplest form of $3\frac{35}{45}$?
- A $3\frac{3}{4}$ B $3\frac{1}{5}$ C $3\frac{7}{9}$ D $3\frac{5}{9}$

Now this is what I call mixed numerals!



7 Reduce the following mixed numerals to their simplest form.

- | | | | |
|-----------------------|-----------------------|------------------------|------------------------|
| (a) $6\frac{9}{45}$ | (b) $4\frac{6}{42}$ | (c) $2\frac{14}{21}$ | (d) $3\frac{5}{30}$ |
| (e) $1\frac{6}{20}$ | (f) $1\frac{12}{22}$ | (g) $12\frac{24}{30}$ | (h) $6\frac{15}{45}$ |
| (i) $3\frac{25}{100}$ | (j) $2\frac{30}{100}$ | (k) $100\frac{12}{80}$ | (l) $11\frac{42}{48}$ |
| (m) $85\frac{26}{39}$ | (n) $16\frac{16}{40}$ | (o) $32\frac{56}{64}$ | (p) $201\frac{15}{75}$ |

e eQuestions

8 Write these mixed numerals as improper fractions.

- | | | | |
|---------------------|---------------------|-----------------------|-----------------------|
| (a) $3\frac{1}{5}$ | (b) $1\frac{4}{5}$ | (c) $1\frac{3}{4}$ | (d) $5\frac{9}{10}$ |
| (e) $5\frac{3}{10}$ | (f) $6\frac{2}{3}$ | (g) $4\frac{7}{11}$ | (h) $4\frac{3}{8}$ |
| (i) $10\frac{2}{7}$ | (j) $6\frac{8}{11}$ | (k) $6\frac{5}{12}$ | (l) $3\frac{9}{100}$ |
| (m) $8\frac{7}{9}$ | (n) $6\frac{8}{9}$ | (o) $14\frac{11}{20}$ | (p) $21\frac{13}{16}$ |

e Hint

e Worksheet C9.4

e eQuestions

9 Write these improper fractions as mixed numerals.

- | | | | |
|---------------------|-----------------------|---------------------|---------------------|
| (a) $\frac{7}{5}$ | (b) $\frac{10}{7}$ | (c) $\frac{13}{6}$ | (d) $\frac{35}{11}$ |
| (e) $\frac{23}{4}$ | (f) $\frac{15}{7}$ | (g) $\frac{37}{10}$ | (h) $\frac{63}{10}$ |
| (i) $\frac{59}{11}$ | (j) $\frac{48}{5}$ | (k) $\frac{44}{7}$ | (l) $\frac{91}{12}$ |
| (m) $\frac{77}{9}$ | (n) $\frac{107}{100}$ | (o) $\frac{49}{23}$ | (p) $\frac{88}{15}$ |

e Worksheet C9.5

e Hint

Extension

10 (a) Write three improper fractions that simplify to $1\frac{2}{3}$.

(b) Write three improper fractions that simplify to $3\frac{1}{4}$.

11 Are these pairs of fractions equivalent? Copy and complete by inserting = or \neq . (Careful, some of these fractions are written as mixed numerals with improper fractions.)

(a) $\frac{96}{100} \text{ --- } \frac{72}{75}$

(b) $\frac{45}{30} \text{ --- } \frac{63}{45}$

(c) $\frac{56}{12} \text{ --- } \frac{70}{15}$

(d) $\frac{90}{115} \text{ --- } \frac{18}{46}$

(e) $\frac{42}{63} \text{ --- } \frac{24}{10}$

(f) $\frac{84}{27} \text{ --- } 3\frac{7}{63}$

(g) $1\frac{34}{25} \text{ --- } 2\frac{27}{75}$

(h) $\frac{550}{220} \text{ --- } 1\frac{32}{48}$

(i) $\frac{8}{96} \text{ --- } \frac{68}{816}$

(j) $1\frac{88}{22} \text{ --- } 1\frac{36}{12}$

(k) $2\frac{8}{100} \text{ --- } \frac{416}{200}$

(l) $2\frac{5}{15} \text{ --- } 2\frac{3}{12}$

e Hint

e eQuestions

e Worksheet A9.1

9.2 Comparing fractions

Number lines

Fractions can be shown on a number line. This can be done by dividing one unit on the number line into the number of equal segments given in the denominator of the fraction. The position of the fraction can then be found by counting the number of segments given in the numerator of the fraction.

worked example 7

On a number line show the positions of $\frac{11}{6}$ and $1\frac{1}{6}$ and determine which is larger.

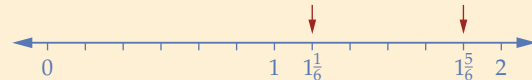
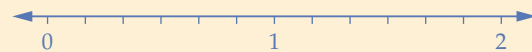
Steps

1. Determine the number of equal segments into which one unit of the number line is to be divided.
2. A number line greater than 1 unit needs to be drawn. Draw each unit with 6 equal segments.
3. Find and label the points. In simplest form, $\frac{11}{6} = 1\frac{5}{6}$.
4. Write the answer.

Solution

$$\frac{11}{6}$$

The denominator is 6. Hence, divide one unit into 6 equal segments.

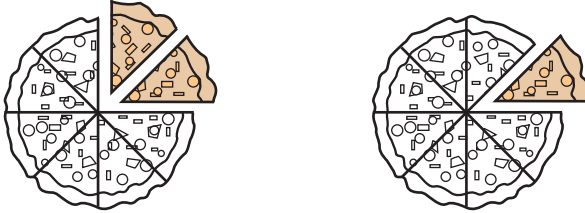


$\frac{11}{6}$ is greater than $1\frac{1}{6}$

Comparing fractions with different denominators

If two fractions have the same denominator, it is clear which fraction is bigger.

For example, $\frac{2}{8}$ of a pizza is obviously more pizza than $\frac{1}{8}$ of a pizza.



To compare fractions with different denominators, we need to find equivalent fractions which have the same denominator. This is called finding the **lowest common denominator** (LCD). The LCD is the lowest common multiple (LCM) of the denominators.

Lowest common multiples

The lowest common multiple (LCM) of two or more numbers is the lowest number that is in the multiples list of all of the given numbers.

worked example 8

Find the LCM of 4 and 6.

Steps

1. List several multiples of 4.
2. List several multiples of 6.
3. Write down the numbers that are in both lists. These are the common multiples.
4. Write the smallest one of these. It is the lowest common multiple.

Solution

4, 8, 12, 16, 20, 24, ...
6, 12, 18, 24, 30, 36, ...
12, 24
LCM = 12



It is not possible to find the highest common multiple. Why not?

worked example 9

Determine which fraction in each of the following pairs is the larger.

(a) $\frac{3}{5}$ or $\frac{4}{7}$

(b) $\frac{7}{10}$ or $\frac{11}{15}$

Steps

- (a) 1. Find multiples of the first denominator.
2. Find multiples of the second denominator.
3. Find the lowest common multiple of these denominators (LCD).
4. Using the LCD as the denominator, find equivalent fractions for both the original fractions.
5. Compare the equivalent fractions to see which of the two original fractions is bigger.
- (b) 1. Find the LCD.
2. Using the LCD as the denominator, find equivalent fractions for both the original fractions.
3. Compare the equivalent fractions to see which of the two original fractions is bigger.

Solutions

(a) $\frac{3}{5}$ or $\frac{4}{7}$

Multiples of 5 are
5, 10, 15, 20, 25, 30, 35, 40, ...

Multiples of 7 are
7, 14, 21, 28, 35, 42, 49, ...

LCD = 35

$$\frac{3}{5} = \frac{21}{35} \quad \frac{4}{7} = \frac{20}{35}$$

$\frac{21}{35} > \frac{20}{35}$

So $\frac{3}{5}$ is bigger than $\frac{4}{7}$.

(b) $\frac{7}{10}$ or $\frac{11}{15}$

LCD = 30

$\frac{7}{10} = \frac{21}{30} \quad \frac{11}{15} = \frac{22}{30}$

$\frac{22}{30} > \frac{21}{30}$

So $\frac{11}{15}$ is bigger than $\frac{7}{10}$.



dangerzone

The LCD is *not* always the two denominators multiplied together. For example, the LCD of $\frac{1}{4}$ and $\frac{5}{6}$ is *not* 24, but 12.

exercise 9.2 Comparing fractions



Preparation: Prep Zone Q1, 3 and 5, Ex 9.1

Core

- 1 Show the position of each of the following fractions on separate number lines.

(a) $\frac{3}{4}$

(b) $\frac{5}{6}$

(c) $\frac{9}{10}$

(d) $\frac{1}{5}$

(e) $\frac{7}{4}$

(f) $\frac{11}{9}$

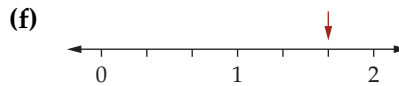
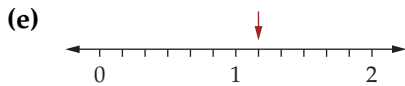
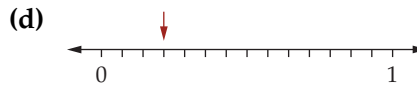
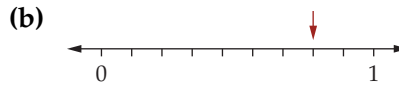
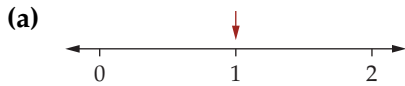
(g) $\frac{9}{2}$

(h) $\frac{29}{12}$

Interactive

Hint

2 Determine the value of the fraction indicated by the arrow on each of the number lines below.



e Hint

3 (a) Draw a number line and show the position of the following fractions.

$$\frac{3}{4}, \frac{1}{4}, \frac{9}{4}, \frac{7}{4}, \frac{4}{4}, \frac{5}{4}$$

(b) From your number line, list these fractions in order from largest to smallest.

4 (a) Draw a number line and show the position of the following fractions.

$$\frac{1}{7}, \frac{4}{7}, \frac{7}{7}, 2\frac{3}{7}, \frac{3}{7}, \frac{13}{7}$$

(b) From your number line, list these fractions in order from largest to smallest.

5 Draw a number line and show the positions of the following fractions.

$$\frac{2}{6}, \frac{4}{12}, \frac{1}{3}, \frac{3}{9}$$

What do you notice about these fractions?

6 Find the LCM of:

(a) 5 and 6

(b) 2 and 5

(c) 8 and 12

(d) 7 and 9

(e) 10 and 12

(f) 3 and 9

(g) 6 and 11

(h) 4 and 7

(i) 5 and 25

(j) 16 and 24

(k) 21 and 28

(l) 20 and 50

7 Which fraction in each pair is larger?

(a) $\frac{2}{5}, \frac{3}{4}$

(b) $\frac{2}{3}, \frac{3}{4}$

(c) $\frac{5}{7}, \frac{5}{8}$

(d) $\frac{3}{7}, \frac{3}{8}$

(e) $\frac{3}{8}, \frac{5}{12}$

(f) $\frac{3}{8}, \frac{1}{3}$

(g) $\frac{5}{6}, \frac{7}{9}$

(h) $\frac{7}{12}, \frac{5}{8}$

(i) $\frac{3}{5}, \frac{7}{15}$

(j) $\frac{3}{4}, \frac{11}{12}$

(k) $\frac{7}{8}, \frac{5}{6}$

(l) $\frac{2}{11}, \frac{1}{10}$

8 Copy the following and complete by inserting $<$, $>$ or $=$.

(a) $\frac{5}{9}$ _____ $\frac{2}{5}$

(b) $\frac{7}{11}$ _____ $\frac{8}{10}$

(c) $\frac{3}{10}$ _____ $\frac{4}{11}$

(d) $\frac{4}{5}$ _____ $\frac{6}{9}$

(e) $\frac{5}{6}$ _____ $\frac{7}{9}$

(f) $\frac{20}{100}$ _____ $\frac{1}{5}$

(g) $\frac{18}{11}$ _____ $\frac{9}{5}$

(h) $\frac{9}{7}$ _____ $\frac{11}{9}$

(i) $\frac{8}{16}$ _____ $\frac{2}{4}$

(j) $\frac{5}{2}$ _____ $\frac{43}{20}$

(k) $\frac{20}{35}$ _____ $\frac{4}{7}$

(l) $\frac{2}{9}$ _____ $\frac{11}{45}$

9 Choose the correct answer in each case.

(a) Which one of the following is bigger than $\frac{2}{3}$?

A $\frac{1}{3}$

B $\frac{2}{9}$

C $\frac{4}{6}$

D $\frac{3}{4}$

(b) Which one of the following is smaller than $\frac{2}{3}$?

A $\frac{4}{5}$

B $\frac{1}{4}$

C $\frac{3}{2}$

D $\frac{20}{30}$

e Hint

This is easy because all the denominators are the same.



e Worksheet C9.6

e Animation

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e Hint

- (c) Which one of the following is bigger than $\frac{5}{8}$?
- A $\frac{2}{3}$ B $\frac{6}{12}$ C $\frac{1}{2}$ D $\frac{5}{9}$
- (d) Which one of the following is smaller than $\frac{4}{11}$?
- A $\frac{1}{2}$ B $\frac{11}{4}$ C $\frac{3}{8}$ D $\frac{1}{3}$
- (e) Which one of the following is bigger than $\frac{7}{9}$?
- A $\frac{3}{5}$ B $\frac{5}{7}$ C $\frac{50}{100}$ D $\frac{11}{12}$

Extension

10 Write each of these lists in ascending order.

- (a) $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{10}{11}$ (b) $\frac{7}{9}, \frac{8}{10}, \frac{4}{6}, \frac{1}{3}, \frac{9}{11}, \frac{10}{12}$ (c) $\frac{4}{7}, 1, \frac{7}{11}, \frac{6}{7}, \frac{7}{8}, 0$

11 Write each of these lists in descending order.

- (a) $\frac{4}{7}, \frac{3}{9}, \frac{2}{3}, 1, \frac{11}{21}, \frac{45}{63}$ (b) $\frac{21}{16}, 1\frac{3}{80}, \frac{5}{4}, 1\frac{7}{20}, \frac{39}{40}, 1\frac{3}{10}$
- (c) $2\frac{6}{7}, \frac{80}{35}, \frac{84}{42}, 2\frac{69}{70}, \frac{60}{28}, 2\frac{14}{28}$

12 Write three fractions that are greater than $\frac{3}{7}$ and less than $\frac{4}{7}$.

e Hint

e eQuestions

e Homework 9.1

e Worksheet A9.2

9.3 Decimals and fractions

As we saw in Chapter 6, a decimal may be expressed in expanded fractional form. For example, 3.72 may be expressed as $3 + \frac{7}{10} + \frac{2}{100}$.

However, 3.72 may also be expressed in single fraction form:

$$3\frac{72}{100} = 3\frac{18}{25}$$

worked example 10

Convert 6.28 to a fraction, simplifying where possible.

Steps

- Write the whole number part first, if necessary.
- Write the digits following the decimal point as the numerator of the fraction.
- Count the number of decimal places and write 1 followed by a number of zeros equal to this number of decimal places.
- Cancel down the fraction if necessary.

Solution

$$6$$

$$6\frac{28}{100}$$

$$6\frac{28}{100}$$

$$6\frac{28}{100} = 6\frac{14}{50} = 6\frac{7}{25}$$

e eTutorial

It can be more difficult to convert fractions to decimals. If the denominator is a multiple of ten, converting to a decimal is quick.

For example, $\frac{7}{100} = 0.07$. But what about $\frac{4}{7}$? It is important to remember that the line between the denominator and numerator means divide. So $\frac{4}{7}$ is the same as $4 \div 7$. To convert $\frac{4}{7}$ to a decimal either do short division or enter $4 \div 7$ on your calculator.

worked example 11

Convert $\frac{4}{7}$ to a decimal, correct to three decimal places, using:

(a) short division

(b) your calculator

Steps

- (a) 1. Set out a short division question. Add some additional zeros because they may be necessary.
2. Perform the short division until it finishes, or one place past the number of decimal places asked for.
3. Write the answer, rounding off to the required number of decimal places.

(b) Press **4** **÷** **7** **=** on your calculator. Write the answer.

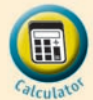
Solutions

(a) $7 \overline{)4.00}$

$$\begin{array}{r} 0.5714 \\ 7 \overline{)4.0501030} \end{array}$$

$$\frac{4}{7} = 0.571$$

(b) $\frac{4}{7} = 0.571$



eTutorial

eTutorial

Some decimals are called **recurring decimals**. This is when the same number or numbers keep being repeated after the decimal point. For example, $\frac{1}{3} = 0.333\ 333\ \dots$ and $\frac{21}{22} = 0.954\ 545\ 45\ \dots$

If a question asked us to round these to three decimal places we would write 0.333 and 0.955 (rounded up).

But another way to write recurring decimals is to put a dot above the recurring digit (when only one digit is recurring) or a line above the recurring digits (if more than one digit is recurring).

So $\frac{1}{3} = 0.\dot{3}$ and $\frac{21}{22} = 0.9\overline{54}$.

exercise 9.3 Decimals and fractions

Preparation: Ex 9.1

Core

1 Convert each of the following to a fraction.

- | | | | |
|-----------|-----------|------------|------------|
| (a) 4.9 | (b) 6.1 | (c) 4.71 | (d) 9.19 |
| (e) 8.13 | (f) 2.37 | (g) 0.29 | (h) 2.47 |
| (i) 3.171 | (j) 0.883 | (k) 6.023 | (l) 5.009 |
| (m) 0.901 | (n) 7.307 | (o) 7.3151 | (p) 8.2243 |

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Hint

- (q) 2.926 47 (r) 1.635 259 (s) 1.008 71 (t) 0.009 21
 (u) 0.008 089 (v) 6.040 03 (w) 2.004 13 (x) 0.000 120 3

2 Convert each of the following to a fraction, and cancel down to simplest form where possible.

- (a) 3.8 (b) 2.2 (c) 4.22 (d) 9.15
 (e) 5.35 (f) 4.26 (g) 6.48 (h) 7.25
 (i) 8.642 (j) 0.546 (k) 0.125 (l) 0.475
 (m) 0.088 (n) 3.072 (o) 7.045 (p) 6.0015
 (q) 1.0075 (r) 8.3314 (s) 4.3484 (t) 5.5025
 (u) 2.000 02 (v) 4.0008 (w) 0.000 375 (x) 1.000 075

e Hint

e eQuestions

3 Choose the correct answer.

5.817 expressed in fraction form is:

- A $5\frac{817}{10}$ B $5\frac{8}{17}$ C $5\frac{817}{100}$ D $5\frac{817}{1000}$

4 Choose the correct answer.

0.55 expressed as a fraction in simplest form is:

- A $\frac{55}{100}$ B $\frac{55}{1000}$ C $\frac{11}{20}$ D $\frac{11}{200}$

5 Choose the correct answer.

2.000 47 is equal to:

- A $2\frac{47}{100}$ B $2\frac{47}{1000}$ C $2\frac{47}{10\,000}$ D $2\frac{47}{100\,000}$

6 Use short division to convert each of the following fractions to a decimal.

- (a) $\frac{1}{4}$ (b) $\frac{7}{8}$ (c) $\frac{2}{5}$ (d) $\frac{1}{8}$
 (e) $\frac{11}{5}$ (f) $\frac{5}{4}$ (g) $\frac{13}{2}$ (h) $\frac{15}{8}$

e Hint

7 Use your calculator to convert each of the following fractions to a decimal.

Write answers correct to three decimal places where necessary.

- (a) $\frac{5}{7}$ (b) $\frac{2}{3}$ (c) $\frac{5}{9}$ (d) $\frac{7}{11}$
 (e) $\frac{17}{25}$ (f) $\frac{15}{68}$ (g) $\frac{13}{200}$ (h) $\frac{82}{83}$
 (i) $\frac{19}{11}$ (j) $\frac{32}{29}$ (k) $\frac{70}{3}$ (l) $\frac{177}{110}$
 (m) $3\frac{4}{15}$ (n) $7\frac{19}{20}$ (o) $12\frac{11}{90}$ (p) $105\frac{7}{900}$

e eQuestions

8 Look back at your answers to Question 7. Write any decimals that are clearly recurring using the correct notation.

e Hint

9 Give two examples of fractions, not from Question 7, that have two digits recurring.

Extension

10 As part of a science experiment, Minh's reaction time was measured to be 0.067 seconds. What fraction of a second is this?

e Hint

11 An inch equals about 2.54 centimetres. What is this when expressed in fraction form?



12 One second equals about $\frac{23}{2000000}$ days. Write this as a decimal.

e Hint

13 A bank offers its customers an interest rate of 5.85 per cent. Write this rate in fraction form. Is this rate higher than the $5\frac{4}{5}$ per cent offered by another bank?

Working mathematically

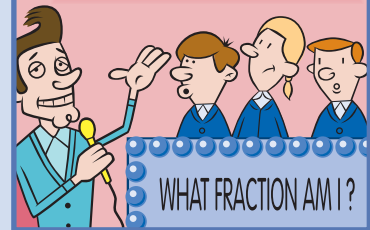
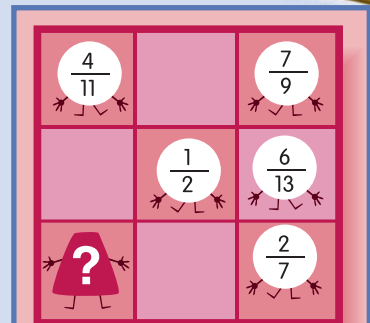
problem solving



What fractions are we?

You may like to work in pairs to solve these.

- 1 My numerator is a single-digit odd number.
My denominator is a two-digit palindrome.
If you add 1 to both my numerator and denominator,
you get a fraction equivalent to $\frac{1}{2}$.
What am I?
- 2 My numerator is a multiple of 5.
My denominator is the LCM of 6 and 9.
I am less than 1.
I am in simplest form.
What am I?
- 3 My numerator is an odd multiple of 3.
My denominator is the HCF of 14 and 21.
I am an improper fraction.
I am greater than 2 but less than 3.
What am I?



9.4 Adding and subtracting fractions

Fractions can be added and subtracted easily if they have the *same* denominator. For example,

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7} \qquad \frac{3}{7} - \frac{2}{7} = \frac{1}{7}$$

If fractions don't have the same denominator they have to be *changed to equivalent fractions* which *do* have the same denominator. This is done by finding the lowest common denominator (LCD).

worked example 12

Find the following. Write your answers as mixed numerals, if appropriate.

(a) $\frac{5}{6} + \frac{3}{4}$

(b) $\frac{7}{10} - \frac{2}{5}$

(c) $\frac{4}{5} + \frac{2}{3} - \frac{1}{15}$

Steps

- (a) 1. Find the lowest common denominator (LCD).
2. Using the LCD as the denominator, find equivalent fractions for both the original fractions.
3. Do the addition.
4. Write your answer as a mixed numeral if appropriate.
- (b) 1. Find the lowest common denominator (LCD).
2. Using the LCD as the denominator, find equivalent fractions for both the original fractions, if necessary.
3. Do the subtraction.
- (c) 1. Find the lowest common denominator (LCD).
2. Using the LCD as the denominator, find equivalent fractions for the three original fractions.
3. Do the addition and subtraction. Remember to do these in the order you come to them from the left.
4. Simplify and write your answer as a mixed numeral.

Solutions

(a) $\frac{5}{6} + \frac{3}{4}$
LCD = 12
$$= \frac{5}{6} + \frac{3}{4}$$
$$= \frac{10}{12} + \frac{9}{12}$$
$$= \frac{19}{12}$$
$$= 1\frac{7}{12}$$

(b) $\frac{7}{10} - \frac{2}{5}$
LCD = 10
$$= \frac{7}{10} - \frac{2}{5}$$
$$= \frac{7}{10} - \frac{4}{10}$$
$$= \frac{3}{10}$$

(c) $\frac{4}{5} + \frac{2}{3} - \frac{1}{15}$
LCD = 15
$$= \frac{4}{5} + \frac{2}{3} - \frac{1}{15}$$
$$= \frac{12}{15} + \frac{10}{15} - \frac{1}{15}$$
$$= \frac{21}{15}$$
$$= \frac{7}{5}$$
$$= 1\frac{2}{5}$$

Don't add or subtract fractions until you've changed them to equivalent fractions.



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exercise 9.4 Adding and subtracting fractions

P Preparation: Prep Zone Q1 and 5, Ex 9.1

Core

1 Find the following. Write your answers as mixed numerals if appropriate.

(a) $\frac{3}{10} + \frac{1}{10}$

(b) $\frac{6}{13} + \frac{8}{13}$

(c) $\frac{10}{21} - \frac{2}{21}$

(d) $\frac{7}{8} + \frac{1}{8}$

(e) $\frac{7}{20} + \frac{2}{5}$

(f) $\frac{5}{14} + \frac{1}{7}$

(g) $\frac{5}{6} + \frac{1}{3}$

(h) $\frac{4}{9} + \frac{2}{27}$

(i) $\frac{4}{21} - \frac{1}{7}$

(j) $\frac{5}{8} - \frac{7}{40}$

(k) $\frac{7}{33} - \frac{2}{11}$

(l) $\frac{29}{35} - \frac{5}{7}$

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e Hint

(m) $\frac{4}{55} + \frac{2}{11}$

(n) $\frac{17}{30} + \frac{5}{6}$

(o) $\frac{15}{49} + \frac{5}{7}$

(p) $\frac{43}{60} + \frac{2}{3}$

(q) $\frac{1}{2} - \frac{13}{40}$

(r) $\frac{1}{3} - \frac{17}{66}$

(s) $\frac{16}{81} - \frac{1}{9}$

(t) $\frac{19}{90} - \frac{1}{30}$

2 Find the following. Write your answers as mixed numerals if appropriate.

(a) $\frac{3}{4} + \frac{1}{5}$

(b) $\frac{1}{5} - \frac{1}{6}$

(c) $\frac{3}{4} - \frac{1}{5}$

(d) $\frac{1}{5} + \frac{1}{6}$

(e) $\frac{7}{10} + \frac{1}{4}$

(f) $\frac{1}{6} + \frac{1}{8}$

(g) $\frac{7}{10} - \frac{1}{4}$

(h) $\frac{1}{6} - \frac{1}{8}$

(i) $\frac{5}{9} - \frac{1}{6}$

(j) $\frac{6}{11} - \frac{2}{5}$

(k) $\frac{5}{8} + \frac{3}{4}$

(l) $\frac{3}{10} + \frac{2}{3}$

(m) $\frac{2}{3} - \frac{2}{9}$

(n) $\frac{6}{7} + \frac{3}{14}$

(o) $\frac{1}{2} + \frac{9}{25}$

(p) $\frac{2}{3} - \frac{7}{22}$

(q) $\frac{7}{12} + \frac{5}{8}$

(r) $\frac{9}{10} - \frac{5}{12}$

(s) $\frac{11}{15} - \frac{7}{20}$

(t) $\frac{13}{20} + \frac{22}{25}$

3 Work out the following additions and subtractions, and choose the correct answer.

(a) $\frac{4}{7} + \frac{1}{3}$

A $\frac{19}{21}$

B $\frac{1}{2}$

C $\frac{5}{7}$

D $\frac{5}{21}$

(b) $\frac{2}{3} + \frac{2}{5}$

A $\frac{4}{8}$

B $\frac{4}{15}$

C $\frac{16}{15}$

D $\frac{4}{5}$

(c) $\frac{5}{8} - \frac{1}{3}$

A $\frac{23}{24}$

B $\frac{7}{24}$

C $\frac{4}{24}$

D $\frac{4}{5}$

(d) $\frac{7}{8} - \frac{1}{6}$

A $\frac{17}{24}$

B $\frac{25}{24}$

C $\frac{6}{48}$

D $\frac{8}{48}$

(e) $\frac{7}{8} + \frac{1}{6}$

A $\frac{24}{25}$

B $\frac{8}{14}$

C $1\frac{1}{24}$

D $\frac{8}{24}$

4 Find the following. Write your answers as mixed numerals if appropriate.

(a) $\frac{3}{8} + \frac{1}{6} + \frac{1}{3}$

(b) $\frac{1}{2} + \frac{3}{4} - \frac{1}{3}$

(c) $\frac{7}{10} + \frac{4}{5} - \frac{1}{2}$

(d) $\frac{5}{8} - \frac{1}{12} + \frac{2}{3}$

(e) $\frac{2}{3} - \frac{1}{2} + \frac{1}{6}$

(f) $\frac{3}{4} - \frac{1}{3} + \frac{1}{2}$

(g) $\frac{2}{5} + \frac{1}{10} - \frac{3}{20}$

(h) $\frac{2}{3} + \frac{1}{4} + \frac{5}{6}$

(i) $\frac{5}{12} - \frac{1}{3} - \frac{1}{24}$

(j) $\frac{11}{12} - \frac{1}{3} - \frac{1}{4}$

(k) $\frac{7}{15} - \frac{1}{3} + \frac{2}{5}$

(l) $\frac{11}{30} - \frac{7}{20} + \frac{1}{6}$

Extension

5 Find the following. Write your answers as mixed numerals if appropriate.

(a) $-\frac{1}{5} + \frac{2}{5}$

(b) $\frac{4}{7} - \frac{6}{7}$

(c) $-\frac{1}{9} - \frac{4}{9}$

(d) $-\frac{10}{11} + \frac{4}{11}$

(e) $\frac{1}{4} - 2$

(f) $5 - 7\frac{2}{9}$

(g) $-\frac{3}{7} - 8$

(h) $3\frac{1}{8} - 6$

(i) $\frac{5}{11} - \frac{1}{2}$

(j) $-\frac{5}{24} + \frac{5}{8}$

(k) $\frac{1}{20} - \frac{3}{8}$

(l) $-\frac{5}{9} - \frac{1}{15}$

(m) $\frac{1}{2} - \frac{3}{4} - \frac{1}{6}$

(n) $-\frac{17}{30} + \frac{5}{6} - \frac{3}{10}$

(o) $-\frac{2}{5} + \frac{3}{10} + \frac{1}{4}$

(p) $\frac{3}{7} + \frac{1}{2} - \frac{20}{21}$

6 (a) Find two fractions that add to $\frac{3}{4}$ and do not have a denominator of 4.

(b) Find two fractions that have a difference of $\frac{5}{6}$ and do not have a denominator of 6.

e Worksheet C9.7

e Hint

e eQuestions

e eQuestions

e Hint

e Hint

e Homework 9.2

e Worksheet E9.1

9.5 Adding and subtracting mixed numerals

worked example 13

Find $6\frac{5}{8} - 2\frac{3}{4}$. Write your answer as a mixed numeral.

Steps

1. Write the mixed numerals as improper fractions.
2. Find the equivalent fractions which have the LCD as their denominator.
3. Do the subtraction.
4. Simplify if possible and write your answer as a mixed numeral.

Solution

$$\begin{aligned} 6\frac{5}{8} - 2\frac{3}{4} &= \frac{53}{8} - \frac{11}{4} \\ \text{LCD} &= 8 \\ &= \frac{53}{8} - \frac{22}{8} \\ &= \frac{31}{8} \\ &= 3\frac{7}{8} \end{aligned}$$

Remember, always give answers in simplest form.



e eTutorial

exercise 9.5 Adding and subtracting mixed numerals

P Preparation: Prep Zone Q1 and 5, Exs 9.1 and 9.4

Core

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1 Find the following. Write your answers as mixed numerals if appropriate.

- | | | | |
|-------------------------------------|---------------------------------------|---------------------------------------|-------------------------------------|
| (a) $3\frac{1}{7} + 4\frac{2}{7}$ | (b) $2\frac{3}{8} + 7\frac{1}{8}$ | (c) $6\frac{3}{10} - 1\frac{1}{10}$ | (d) $3\frac{6}{7} - 2\frac{4}{7}$ |
| (e) $2\frac{1}{2} + 3\frac{1}{6}$ | (f) $2\frac{1}{8} - 1\frac{1}{6}$ | (g) $2\frac{1}{4} - 1\frac{2}{5}$ | (h) $3\frac{4}{9} + 1\frac{1}{6}$ |
| (i) $10\frac{7}{8} - 2\frac{3}{4}$ | (j) $4\frac{1}{2} - 2\frac{11}{12}$ | (k) $7\frac{1}{2} - 3\frac{9}{10}$ | (l) $3\frac{3}{10} + 7\frac{2}{3}$ |
| (m) $1\frac{4}{11} + 3\frac{2}{5}$ | (n) $5\frac{5}{8} - 1\frac{1}{16}$ | (o) $4\frac{2}{3} + 3\frac{2}{7}$ | (p) $2\frac{1}{13} - 1\frac{5}{26}$ |
| (q) $5\frac{1}{4} - 2\frac{5}{6}$ | (r) $4\frac{83}{100} + 5\frac{1}{20}$ | (s) $2\frac{4}{25} + 7\frac{59}{100}$ | (t) $10\frac{4}{5} + 11\frac{1}{2}$ |
| (u) $2\frac{7}{12} + 2\frac{1}{10}$ | (v) $1\frac{4}{25} + 2\frac{3}{20}$ | (w) $2\frac{13}{20} - 1\frac{3}{100}$ | (x) $3\frac{9}{10} - 2\frac{8}{25}$ |

e Hint

2 Find the following, writing your answers as mixed numerals. Can you find some shortcuts?

- | | | | |
|------------------------|-------------------------|--------------------------|-------------------------|
| (a) $4 + 2\frac{4}{5}$ | (b) $1 + 3\frac{1}{5}$ | (c) $2 + 3\frac{1}{4}$ | (d) $3 + 1\frac{6}{11}$ |
| (e) $4\frac{3}{8} + 5$ | (f) $2\frac{5}{7} + 8$ | (g) $6\frac{10}{11} + 3$ | (h) $6\frac{3}{13} + 7$ |
| (i) $3\frac{5}{7} - 2$ | (j) $12\frac{3}{4} - 9$ | (k) $5\frac{2}{9} - 2$ | (l) $10\frac{7}{8} - 6$ |
| (m) $6 - 1\frac{2}{3}$ | (n) $8 - 1\frac{3}{4}$ | (o) $10 - 3\frac{4}{5}$ | (p) $6 - 3\frac{2}{9}$ |

e Hint

3 Find the following. Write your answers as mixed numerals.

(a) $1\frac{1}{2} - \frac{2}{5} + 2\frac{3}{10}$

(b) $2\frac{1}{3} + 5\frac{3}{4} - 1\frac{2}{5}$

(c) $3\frac{1}{4} + 2\frac{1}{5} - 3\frac{1}{3}$

(d) $5\frac{2}{3} - 1\frac{1}{10} + \frac{3}{4}$

(e) $3\frac{2}{5} + 11 - 2\frac{2}{3}$

(f) $5\frac{1}{6} + 6 - 1\frac{2}{3}$

(g) $12\frac{2}{25} - 4 - 3\frac{51}{100}$

(h) $8\frac{13}{20} - 2 - 3\frac{98}{100}$

(i) $12\frac{7}{16} - 4 - 2\frac{1}{8}$

e Hint

4 Copy and complete the following magic squares. Remember all the rows, columns and diagonals should add up to the same total.

(a)

		$1\frac{1}{6}$
	$1\frac{2}{3}$	
$2\frac{1}{6}$	1	

(b)

$4\frac{1}{5}$		
$1\frac{7}{10}$		$3\frac{7}{10}$
$2\frac{1}{5}$		

Extension

5 Find the following. Write your answers as mixed numerals if appropriate.

(a) $-2\frac{1}{6} + \frac{5}{6}$

(b) $\frac{3}{8} - 1\frac{1}{8}$

(c) $-3\frac{3}{5} - 1\frac{4}{5}$

(d) $-\frac{4}{9} + 5\frac{1}{9}$

(e) $\frac{5}{7} - 6$

(f) $-\frac{4}{9} + 4$

(g) $-3\frac{1}{20} + 2$

(h) $-\frac{7}{16} - 5$

(i) $3\frac{1}{2} - 5\frac{3}{4}$

(j) $-2\frac{5}{6} - 1\frac{3}{4}$

(k) $-1\frac{2}{3} + \frac{3}{20}$

(l) $-1\frac{3}{4} + 1\frac{1}{5}$

e Hint

6 Find two fractions that add to give $2\frac{1}{4}$ and do not have a denominator of 4.

7 Explain why the following is incorrect.

$$\frac{4}{7} + \frac{3}{5} = \frac{7}{12}$$

What should the answer be?

e Worksheet C9.8

e eQuestions



speeding zone

Do these in your head as quickly as you can and write down the answers.



Time target: 2 minutes

1 8×15

2 $2\frac{3}{4} + \frac{3}{4}$

3 $36\,000 \div 120$

4 $0.89 + 2.0073$

5 20^3

6 $36 - 106$

7 $\$4.50 + \10.60

8 $1.8 - 0.9$

9 0.5×11

10 $\$10.05 - \5.50

Egyptian fractions



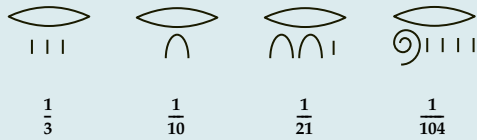
The three pyramids at Giza, featuring the largest one, the Great Pyramid of Khufu (Cheops)

The Great Pyramid of Khufu is the largest of the 80 pyramids that exist in Egypt. It is the only one of the Seven Wonders of the Ancient World still standing and is still among the most amazing buildings ever constructed. For more than 43 centuries it was the tallest building on Earth. Originally standing around 160 m high, the Great Pyramid consists of approximately 2 500 000 blocks of stone, and took 20 years to build. The four sides are 2390 m long and are aligned exactly with the compass directions north, south, east and west. It has been estimated that there is enough stone contained within the three pyramids at Giza to build a wall 3 m high and 30 cm thick all the way around New South Wales.

There have been many theories about how such a massive construction could be built more than 4500 years ago. Recent theories have shown evidence that Egyptian fractions were used in the building of the pyramids. What we know for certain is that it was mathematics that made these great monuments possible.

Much of what we know about Egyptian fractions comes from the Ahmes Papyrus and the Akhmin Papyrus. Egyptians wrote fractions like the ones shown below for over 2000 years.

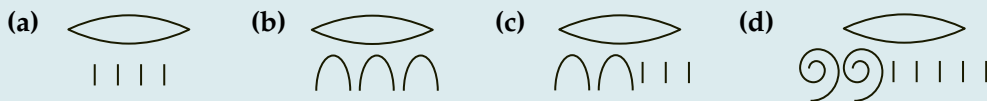
One of the unusual things about Egyptian fractions is that they almost always have a numerator of 1 (the fraction $\frac{2}{3}$ is the only exception)—these are called unit fractions. There was no direct way of writing a fraction with a numerator that wasn't 1.



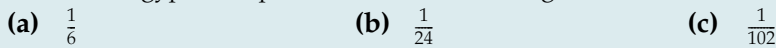
Questions

1 What do we think the original height of the Great Pyramid of Khufu was? Why do you think the pyramid is no longer its original height?

2 How would we write the following Egyptian fractions?



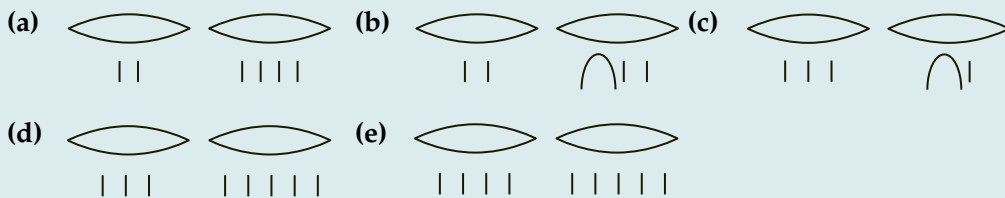
3 Draw the Egyptian equivalent of the following fractions.



4 The Egyptians used the sum of unit fractions to show fractions where the numerator isn't 1. For example, $\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$. The Egyptians would have written $\frac{3}{8}$ as

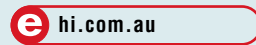


What fractions do these show?



- 5** (a) How would the Egyptians have written $\frac{5}{8}$? (Hint: one of the fractions is $\frac{1}{2}$.)
 (b) How would the Egyptians have written $\frac{3}{5}$? (Hint: one of the fractions is $\frac{1}{2}$.)
 (c) Using the way we write fractions, find out which of the fractions $\frac{5}{8}$ and $\frac{3}{5}$ is bigger.
 (d) Look at the way the Egyptians would have written the two fractions in part (c). Why is it easier to tell which one is bigger when you write them like this?

Research



Do a PowerPoint or other electronic presentation about the history of fractions. Mention the following: the ancient Babylonians, the ancient Greeks, Fibonacci, the Hindus and the Arabs. What practical problems were fractions used for throughout history?

9.6 Multiplying fractions

When we have two of something it means we have that item times two, or multiplied by two. So, for example, we could say that 2×3 is the same as 2 of 3.

So, 3 of 4 is 12 5 of 2 is 10 6 of 3 = 18

In fraction problems the multiplication sign means 'of':

So $\frac{1}{2} \times 6 = \frac{1}{2}$ of 6 = 3

What about multiplying fractions by fractions?

We will display $\frac{1}{2} \times \frac{1}{3}$ using the bar below.

$\frac{1}{2} \times \frac{1}{3}$ can be considered as $\frac{1}{2}$ of $\frac{1}{3}$. So first shade one-third.



Then we need to find $\frac{1}{2}$ of $\frac{1}{3}$, so cut $\frac{1}{3}$ into half.



You should be able to see that $\frac{1}{2}$ of $\frac{1}{3}$ is actually $\frac{1}{6}$.

Can you see that $\frac{1}{2}$ of $\frac{2}{3}$ gives $\frac{1}{3}$? This leads us to the written process we will use for multiplying fractions.

$$\frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{6}$$

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{1}{3}$$

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$\frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

$$\frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$$

worked example 14

Find $\frac{5}{6} \times \frac{9}{20}$.

Steps

1. Cancel factors between numerators and denominators where possible.

$$\left. \begin{array}{l} 5 \div 5 = 1 \\ 20 \div 5 = 4 \end{array} \right\} \div 5$$

$$\left. \begin{array}{l} 9 \div 3 = 3 \\ 6 \div 3 = 2 \end{array} \right\} \div 3$$

2. Multiply out the numerator and the denominator.

Solution

$$\frac{\cancel{5}^1 \times \cancel{9}^3}{\cancel{6}^2 \times \cancel{20}^4}$$

$$= \frac{3}{4}$$

worked example 15

Find $4 \times 2\frac{3}{20}$.

Steps

1. Write the terms as improper fractions (if appropriate).
Remember, whole numbers can be written as fractions with a denominator of 1.
2. Cancel factors between numerators and denominators where possible.
 $4 \div 4 = 1$
 $20 \div 4 = 5$
3. Multiply out the numerator and the denominator.
4. Write as a mixed numeral.

Solution

$$4 \times 2\frac{3}{20} = \frac{4}{1} \times \frac{43}{20}$$

$$\frac{\cancel{4}^1}{1} \times \frac{43}{\cancel{20}_5}$$

$$= \frac{43}{5}$$

$$= 8\frac{3}{5}$$

e eTutorial

worked example 16

Find $\frac{3}{4}$ of $\frac{2}{7}$.

Steps

Change the 'of' to \times and multiply the fractions in the usual way.

Solution

$$\begin{aligned} & \frac{3}{4} \text{ of } \frac{2}{7} \\ &= \frac{3 \times \cancel{2}^1}{\cancel{4}_2 \times 7} \\ &= \frac{3 \times 1}{2 \times 7} \\ &= \frac{3}{14} \end{aligned}$$

e eTutorial

exercise 9.6 Multiplying fractions

p Preparation: Ex 9.1

Core

1 Find the following. Write your answers as mixed numerals if appropriate.

(a) $\frac{5}{7} \times \frac{1}{10}$

(b) $\frac{3}{4} \times \frac{8}{11}$

(c) $\frac{9}{11} \times \frac{1}{12}$

(d) $\frac{3}{7} \times \frac{7}{13}$

(e) $\frac{4}{5} \times \frac{5}{9}$

(f) $\frac{2}{9} \times \frac{1}{2}$

(g) $\frac{3}{20} \times \frac{5}{8}$

(h) $\frac{5}{6} \times \frac{8}{9}$

(i) $\frac{2}{9} \times \frac{15}{16}$

(j) $\frac{33}{40} \times \frac{4}{11}$

(k) $\frac{2}{3} \times \frac{9}{10}$

(l) $\frac{4}{15} \times \frac{25}{16}$

(m) $\frac{5}{2} \times \frac{2}{15}$

(n) $\frac{3}{14} \times \frac{7}{3}$

(o) $\frac{14}{15} \times \frac{25}{7}$

(p) $\frac{14}{3} \times \frac{3}{7}$

(q) $\frac{25}{6} \times \frac{12}{5}$

(r) $\frac{5}{12} \times \frac{9}{35}$

(s) $\frac{9}{20} \times \frac{16}{21}$

(t) $\frac{8}{5} \times \frac{20}{12}$

(u) $\frac{30}{6} \times \frac{4}{20}$

(v) $\frac{12}{7} \times \frac{14}{21}$

(w) $\frac{27}{12} \times \frac{21}{18}$

(x) $\frac{33}{18} \times \frac{24}{44}$

e Worksheet C9.9

e Hint

Always write the answer in simplest form.



2 Find the following. Write your answers as mixed numerals if appropriate.

e Hint

- | | | | |
|---|--|--|---|
| (a) $3 \times \frac{2}{9}$ | (b) $\frac{3}{4} \times 8$ | (c) $\frac{5}{8} \times 2$ | (d) $3 \times \frac{5}{12}$ |
| (e) $5 \times 2\frac{3}{10}$ | (f) $8 \times 1\frac{3}{4}$ | (g) $1\frac{4}{5} \times 15$ | (h) $5\frac{1}{12} \times 8$ |
| (i) $2\frac{5}{8} \times 4$ | (j) $2\frac{3}{25} \times 10$ | (k) $6\frac{2}{3} \times \frac{3}{4}$ | (l) $5\frac{3}{4} \times \frac{2}{23}$ |
| (m) $\frac{7}{8} \times 2\frac{6}{21}$ | (n) $\frac{5}{7} \times 2\frac{4}{5}$ | (o) $2\frac{3}{4} \times 2\frac{2}{11}$ | (p) $3\frac{1}{3} \times 2\frac{4}{5}$ |
| (q) $3\frac{4}{5} \times 2\frac{2}{19}$ | (r) $4\frac{2}{9} \times 1\frac{1}{2}$ | (s) $5\frac{1}{3} \times 1\frac{17}{28}$ | (t) $2\frac{7}{7} \times 1\frac{6}{15}$ |

Be careful not to cancel out two things on the same line. You have to cancel something on the bottom with something on the top of the fraction.



e Hint

3 Find the following. Write your answers as mixed numerals if appropriate.

- | | | |
|--|--|---|
| (a) $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ | (b) $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ | (c) $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$ |
| (d) $\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ | (e) $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ | (f) $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$ |
| (g) $2\frac{1}{7} \times 1\frac{1}{15} \times \frac{3}{4}$ | (h) $1\frac{7}{12} \times \frac{13}{19} \times \frac{1}{26}$ | (i) $\frac{5}{11} \times \frac{5}{6} \times 1\frac{1}{10}$ |
| (j) $2 \times 4\frac{4}{5} \times 1\frac{5}{12}$ | (k) $3\frac{5}{9} \times \frac{3}{19} \times 1\frac{3}{16}$ | (l) $2\frac{6}{17} \times 1\frac{3}{4} \times 3\frac{2}{5}$ |

e eQuestions

e Worksheet C9.10

e Hint

4 Work out the following.

- | | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| (a) $\frac{5}{6}$ of $\frac{1}{11}$ | (b) $\frac{4}{7}$ of $\frac{2}{3}$ | (c) $\frac{7}{12}$ of $\frac{5}{6}$ | (d) $\frac{1}{3}$ of $\frac{7}{10}$ |
| (e) $\frac{7}{9}$ of $\frac{5}{6}$ | (f) $\frac{4}{9}$ of $\frac{3}{14}$ | (g) $\frac{1}{2}$ of $\frac{2}{13}$ | (h) $\frac{5}{7}$ of $\frac{7}{25}$ |
| (i) $\frac{2}{3}$ of 18 | (j) $\frac{3}{4}$ of 12 | (k) $\frac{2}{9}$ of 36 | (l) $\frac{5}{6}$ of 18 |
| (m) $\frac{3}{4}$ of 28 | (n) $\frac{4}{5}$ of 30 | (o) $\frac{6}{7}$ of 56 | (p) $\frac{3}{8}$ of 72 |

Extension

5 Work out the following.

- | | | |
|---|---|---------------------------|
| (a) $\frac{3}{4}$ of \$28 | (b) $\frac{3}{5}$ of \$35 | (c) $\frac{2}{5}$ of \$40 |
| (d) $\frac{2}{3}$ of \$27 | (e) $\frac{5}{6}$ of \$24 | (f) $\frac{3}{4}$ of \$60 |
| (g) $\frac{1}{2}$ of 3 cups of flour | (h) $\frac{1}{2}$ of 5 cups of flour | |
| (i) $\frac{1}{2}$ of 6 cups of flour | (j) $\frac{3}{4}$ of $2\frac{1}{3}$ cups of sugar | |
| (k) $\frac{2}{3}$ of $1\frac{1}{2}$ cups of sugar | (l) $\frac{5}{6}$ of $3\frac{1}{2}$ cups of sugar | |

e Hint

6 Work out the following. Convert fractional hours into minutes where appropriate.

- | | | |
|---|---|---|
| (a) $\frac{2}{3}$ of 6 hours | (b) $\frac{1}{3}$ of $4\frac{1}{2}$ hours | (c) $\frac{3}{4}$ of 6 hours |
| (d) $\frac{5}{6}$ of 4 hours | (e) $\frac{1}{2}$ of $2\frac{1}{2}$ hours | (f) $\frac{1}{3}$ of $1\frac{1}{2}$ hours |
| (g) $\frac{1}{3}$ of 4 hours | (h) $\frac{3}{5}$ of $1\frac{1}{4}$ hours | (i) $\frac{2}{5}$ of 6 hours |
| (j) $\frac{2}{3}$ of $3\frac{1}{4}$ hours | (k) $\frac{2}{3}$ of $2\frac{3}{4}$ hours | (l) $\frac{1}{5}$ of a $\frac{3}{4}$ hour |

e Hint

7 Write two fractions that multiply to give $\frac{3}{4}$.

e eQuestions

e Homework 9.3

problem solving

What fractions are we?

We are two fractions.

Altogether, we are made up of four different digits.

Neither of us is an improper fraction.

We are both in simplest form.

If you add 1 to each of our numerators, we are equal.

Each of us has a digit which is a multiple of four.

Neither of us has a digit which is a multiple of three.

What are we?



9.7 Dividing fractions

To understand dividing fractions we need to think back to primary school when we thought of division as 'how many'. So $\frac{1}{2} \div \frac{1}{4}$ means $\frac{1}{2}$ how many $\frac{1}{4}$ s.

Look at the divided bar below to find the answer.



We can see that there are 2 lots of $\frac{1}{4}$ in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{4} = 2$

Look at the divided bars below.



Can you see that 3 how many $\frac{1}{5}$ s gives 15? So $3 \div \frac{1}{5} = 15$.

Can you see that finding how many $\frac{1}{5}$ s there are in a number is the same as multiplying that number by 5? This leads us to the written process we will use for dividing fractions.



worked example 17

Find $9 \div \frac{3}{8}$.

Steps

1. Write as improper fractions (if appropriate).
2. Turn the second fraction upside down and change the \div to \times .
3. Do the multiplication as usual.

Solution

$$\begin{aligned} & \frac{9}{1} \div \frac{3}{8} \\ &= \frac{9}{1} \times \frac{8}{3} \\ &= \frac{\cancel{3}^9 \times \cancel{8}_2}{\cancel{1}^1 \times \cancel{3}_1} = \frac{24}{1} = 24 \end{aligned}$$

Turning a fraction upside down is also called inverting the fraction or finding the reciprocal.



worked example 18

Find $9\frac{1}{3} \div 4\frac{4}{7}$.

Steps

1. Write as improper fractions.
2. Turn the second fraction upside down and change the \div to \times (i.e. multiply by the reciprocal of the second fraction).
3. Do the multiplication as usual.
4. Write your answer as a mixed numeral.

Solution

$$\begin{aligned} & \frac{28}{3} \div \frac{32}{7} \\ &= \frac{28}{3} \times \frac{7}{32} \\ &= \frac{\cancel{7}^1 \times 7}{3 \times \cancel{32}_8} = \frac{49}{24} \\ &= 2\frac{1}{24} \end{aligned}$$

e eTutorial

exercise 9.7 Dividing fractions

p Preparation: Exs 9.1 and 9.6

Core

1 Copy and complete the following. Do all working in your head.

- (a) (i) There are ____ halves in one, so $1 \div \frac{1}{2} = \underline{\hspace{2cm}}$. (ii) $1 \times 2 = \underline{\hspace{2cm}}$
 (b) (i) There are ____ halves in three, so $3 \div \frac{1}{2} = \underline{\hspace{2cm}}$. (ii) $3 \times 2 = \underline{\hspace{2cm}}$
 (c) (i) There are ____ quarters in one, so $1 \div \frac{1}{4} = \underline{\hspace{2cm}}$. (ii) $1 \times 4 = \underline{\hspace{2cm}}$
 (d) (i) There are ____ quarters in two, so $2 \div \frac{1}{4} = \underline{\hspace{2cm}}$. (ii) $2 \times 4 = \underline{\hspace{2cm}}$

e Hint

2 Find the reciprocals of these fractions.

- | | | | |
|--------------------|--------------------|---------------------|---------------------|
| (a) $\frac{2}{11}$ | (b) $\frac{6}{7}$ | (c) $\frac{5}{6}$ | (d) $\frac{11}{15}$ |
| (e) $\frac{17}{8}$ | (f) $\frac{10}{9}$ | (g) $\frac{20}{19}$ | (h) $\frac{26}{21}$ |
| (i) $\frac{1}{4}$ | (j) $\frac{1}{8}$ | (k) $\frac{1}{71}$ | (l) $\frac{1}{108}$ |
| (m) 12 | (n) 101 | (o) 156 | (p) 80 |



3 Find the following, writing your answers as mixed numerals if appropriate.

- | | | | |
|----------------------------|----------------------------|----------------------------|---------------------------|
| (a) $5 \div \frac{3}{7}$ | (b) $7 \div \frac{4}{9}$ | (c) $6 \div \frac{3}{11}$ | (d) $8 \div \frac{2}{3}$ |
| (e) $8 \div \frac{4}{9}$ | (f) $10 \div \frac{5}{11}$ | (g) $\frac{6}{7} \div 5$ | (h) $\frac{9}{10} \div 7$ |
| (i) $\frac{3}{10} \div 12$ | (j) $\frac{7}{8} \div 28$ | (k) $\frac{12}{13} \div 8$ | (l) $\frac{8}{13} \div 6$ |

e Hint

4 Find the following, writing your answers as mixed numerals if appropriate.

- | | | | |
|---------------------------------------|---------------------------------------|--|--|
| (a) $\frac{4}{5} \div \frac{8}{9}$ | (b) $\frac{2}{9} \div \frac{18}{19}$ | (c) $\frac{6}{11} \div \frac{10}{55}$ | (d) $\frac{2}{33} \div \frac{8}{11}$ |
| (e) $\frac{5}{8} \div \frac{55}{12}$ | (f) $\frac{10}{9} \div \frac{20}{27}$ | (g) $\frac{21}{12} \div \frac{14}{9}$ | (h) $\frac{48}{9} \div \frac{24}{9}$ |
| (i) $\frac{16}{5} \div \frac{32}{15}$ | (j) $\frac{14}{9} \div \frac{49}{15}$ | (k) $\frac{33}{16} \div \frac{77}{40}$ | (l) $\frac{21}{10} \div \frac{39}{20}$ |

5 Find the following, writing your answers as mixed numerals if appropriate.

- (a) $6 \div 3\frac{1}{5}$ (b) $3 \div 2\frac{1}{4}$ (c) $5\frac{1}{4} \div 7$ (d) $9 \div 4\frac{1}{2}$
 (e) $2\frac{3}{7} \div 17$ (f) $5\frac{6}{11} \div 61$ (g) $5\frac{1}{2} \div 5\frac{3}{8}$ (h) $6\frac{4}{7} \div 12$
 (i) $3\frac{3}{4} \div 2\frac{7}{10}$ (j) $3\frac{3}{4} \div 3\frac{1}{3}$ (k) $10\frac{2}{3} \div 6\frac{2}{5}$ (l) $1\frac{3}{7} \div 2\frac{2}{9}$

e Hint

Extension

6 Find the following, writing your answers as mixed numerals if appropriate.

- (a) $-8 \div \frac{16}{19}$ (b) $-\frac{8}{11} \div -2$ (c) $-5 \div -\frac{15}{8}$ (d) $\frac{40}{49} \div -10$
 (e) $-\frac{11}{4} \div \frac{33}{40}$ (f) $\frac{16}{7} \div -\frac{20}{49}$ (g) $-\frac{13}{5} \div -\frac{39}{20}$ (h) $-\frac{9}{10} \div \frac{63}{100}$
 (i) $5 \div -3\frac{4}{7}$ (j) $-4\frac{6}{11} \div 25$ (k) $-3\frac{3}{8} \div 2\frac{1}{16}$ (l) $-5\frac{5}{7} \div 10\frac{10}{21}$

e Hint

7 Find the following, writing your answers as mixed numerals if appropriate.

- (a) $-3 \div \frac{9}{10} \div -\frac{20}{21}$ (b) $-\frac{4}{9} \div -2 \div -\frac{8}{27}$ (c) $-\frac{5}{8} \div -\frac{15}{16} \div 10$
 (d) $-\frac{6}{5} \div \frac{18}{25} \div \frac{10}{21}$ (e) $\frac{16}{7} \div -\frac{20}{49} \div -\frac{16}{25}$ (f) $\frac{14}{27} \div -\frac{7}{9} \div \frac{8}{15}$
 (g) $2\frac{1}{4} \div -3 \div -\frac{9}{16}$ (h) $-7 \div -\frac{14}{15} \div 4\frac{2}{7}$ (i) $-\frac{15}{16} \div -2\frac{1}{4} \div -10$

8 Write a fraction that when divided by 2 will become a fraction with a denominator of 12.

e eQuestions

Working mathematically

Investigation

Ideal fractions

The fractions $\frac{9}{4}$ and $\frac{9}{5}$ have a special property.

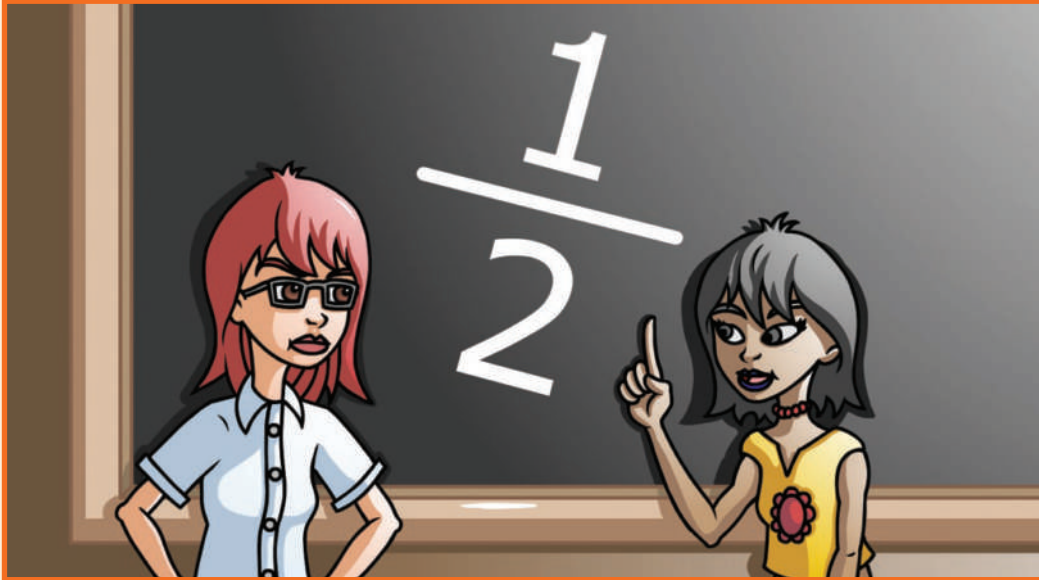
$$\left. \begin{array}{l} \frac{9}{4} + \frac{9}{5} \\ = \frac{45}{20} + \frac{36}{20} \\ = \frac{81}{20} \end{array} \right\} \begin{array}{l} \frac{9}{4} \times \frac{9}{5} \\ = \frac{9 \times 9}{4 \times 5} \\ = \frac{81}{20} \end{array} \quad \text{So} \quad \frac{9}{4} + \frac{9}{5} = \frac{9}{4} \times \frac{9}{5}$$

Fractions with this property are called ideal fractions.

- (a) Show that $\frac{7}{3}$ and $\frac{7}{4}$ are ideal fractions. (b) Show that $\frac{5}{3}$ and $\frac{5}{2}$ are ideal fractions.
 (c) Try to find a pattern in the pairs of ideal fractions we have looked at so far. Copy and complete the following.
 (i) $\frac{7}{5}$ and ____ are ideal fractions. (ii) $\frac{11}{9}$ and ____ are ideal fractions.
 (iii) $\frac{13}{6}$ and ____ are ideal fractions.
 (d) Write down three more pairs of ideal fractions.
 (e) Can you find any pairs of ideal whole numbers?

I deal, Minh deals, then Polly deals.





Answer the following, showing your working, then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

Evaluate the following.

- | | | | | | |
|---|----------|---|----------|--|----------|
| $\frac{1}{6} \times \frac{2}{7}$ | C | $\frac{3}{4} \times \frac{2}{9}$ | R | $\frac{4}{9}$ of $\frac{15}{8}$ | A |
| $\frac{5}{11}$ of $\frac{33}{40}$ | L | $4 \times 2\frac{1}{8}$ | N | $2\frac{3}{5} \times \frac{10}{27}$ | O |
| $2\frac{2}{5} \times 3\frac{1}{4} \times \frac{1}{6}$ | T | $\frac{1}{5} \times 1\frac{1}{4} \times 4\frac{2}{3}$ | F | $5 \div \frac{3}{7}$ | G |
| $\frac{5}{8} \div \frac{15}{8}$ | E | $2 \div 4\frac{4}{9}$ | S | $3\frac{3}{4} \div \frac{1}{2}$ | V |
| $3\frac{3}{4} \div 13\frac{1}{2}$ | W | $8 \div 4\frac{4}{9} \div \frac{1}{2}$ | I | $3\frac{1}{8} \div \frac{5}{10} \div 1\frac{1}{4}$ | H |

$3\frac{3}{5}$	$9\frac{1}{2}$	$\frac{5}{6}$	$7\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{1}{6}$	$3\frac{3}{5}$	$1\frac{3}{10}$	$1\frac{3}{10}$	$\frac{1}{3}$	$8\frac{1}{2}$								

$1\frac{3}{10}$	$9\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{5}{6}$	$\frac{1}{6}$	$11\frac{2}{3}$	$\frac{1}{3}$	$\frac{9}{20}$	$1\frac{3}{10}$										

$1\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{21}$	$1\frac{3}{10}$	$3\frac{3}{5}$	$\frac{26}{27}$	$8\frac{1}{2}$	$3\frac{3}{5}$	$\frac{1}{21}$	$\frac{5}{6}$	$8\frac{1}{2}$								

9.8 Mixed operations with fractions

Order of operations

You should recall from Chapter 1 that when we use the order of operations the first step is to work out the value within the brackets. The next step is to work out any multiplication or division as you come to it, working from left to right. Then work out any addition or subtraction as you come to it, working from left to right.

worked example 19

Simplify the following using the correct order of operations.

(a) $3 + \frac{2}{5} \times \frac{1}{2}$

(b) $\frac{5}{7} \times (\frac{2}{3} + 4 \div \frac{2}{3}) + 5 \times \frac{3}{7}$

Steps

- (a) 1. As there are no brackets do the multiplication first.
2. Do the addition and state your answer in simplest form.
- (b) 1. Simplify the brackets first. Do the division first then add.
2. If necessary, express any mixed numbers as improper fractions.
3. Multiply the fractions, moving from left to right.
4. Add the fractions, moving from left to right.
5. Simplify, if necessary.

Solutions

(a) $3 + \frac{2}{5} \times \frac{1}{2}$

$$= 3 + \frac{\cancel{2} \times 1}{5 \times \cancel{2}}$$

$$= 3 + \frac{1}{5}$$

$$= 3\frac{1}{5}$$

(b) $\frac{5}{7} \times (\frac{2}{3} + 4 \div \frac{2}{3}) + 5 \times \frac{3}{7}$

$$= \frac{5}{7} \times (\frac{2}{3} + \frac{4}{1} \times \frac{3}{2}) + 5 \times \frac{3}{7}$$

$$= \frac{5}{7} \times (\frac{2}{3} + \frac{\cancel{4} \times 3}{1 \times \cancel{2}}) + 5 \times \frac{3}{7}$$

$$= \frac{5}{7} \times (\frac{2}{3} + 6) + 5 \times \frac{3}{7}$$

$$= \frac{5}{7} \times 6\frac{2}{3} + 5 \times \frac{3}{7}$$

$$= \frac{5}{7} \times \frac{20}{3} + \frac{5}{1} \times \frac{3}{7}$$

$$= \frac{100}{21} + \frac{15}{7}$$

$$= \frac{100}{21} + \frac{45}{21}$$

$$= \frac{145}{21}$$

$$= 6\frac{19}{21}$$

Dealing with fractions using a calculator



When dealing with fractions, calculators can be used efficiently if they have the special fraction button a^b/c . The screen will show symbols such as $\frac{\quad}{\quad}$, $\frac{\quad}{\quad}$ or $\frac{\quad}{\quad}$ to separate the whole numbers, numerator and denominator.

worked example 20

By using your calculator, work out the following, giving your answers as fractions.

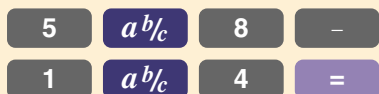
(a) $\frac{5}{8} - \frac{1}{4}$

(b) $4\frac{3}{8} \times 3\frac{9}{10}$

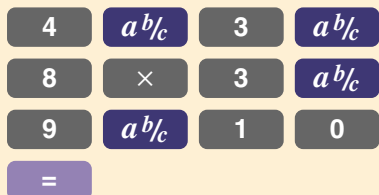
(c) $\frac{3}{8} \div (\frac{5}{6} + \frac{1}{4}) \times \frac{8}{15}$

Steps

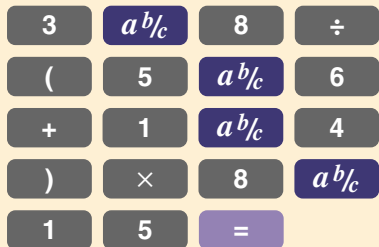
- (a) Press the buttons on your calculator in the following sequence.



- (b) Press the buttons on your calculator in the following sequence.



- (c) Press the buttons on your calculator in the following sequence.



Solutions

(a) $\frac{5}{8} - \frac{1}{4}$

Screen shows $3\frac{-}{8} = \frac{3}{8}$

(b) $4\frac{3}{8} \times 3\frac{9}{10}$

Screen shows $17\frac{-}{16} = 17\frac{1}{16}$

(c) $\frac{3}{8} \div (\frac{5}{6} + \frac{1}{4}) \times \frac{8}{15}$

Screen shows $12\frac{-}{65} = \frac{12}{65}$

exercise 9.8

Mixed operations with fractions



Preparation: Exs 9.1, 9.4–9.7

Core

1 Simplify the following using the correct order of operations.

(a) $2 + \frac{3}{4} \times \frac{1}{3}$

(b) $5 - \frac{7}{8} \times \frac{1}{7}$

(c) $\frac{2}{9} \times \frac{1}{2} + 7$

(d) $\frac{5}{11} \div 5 + 6$

(e) $\frac{6}{7} \div \frac{9}{14} + 2$

(f) $7 - \frac{3}{8} \div \frac{9}{10}$

(g) $2\frac{3}{4} - (4\frac{1}{2} - 3\frac{1}{4})$

(h) $7\frac{3}{5} - (3\frac{3}{10} - 1\frac{1}{5})$

(i) $13\frac{1}{3} - (2\frac{5}{6} - 1\frac{2}{3})$

(j) $1\frac{1}{6} + (3\frac{1}{3} - 2\frac{5}{6})$

(k) $4\frac{7}{8} + (8\frac{1}{4} - 6\frac{3}{8})$

(l) $7\frac{2}{7} + (2\frac{6}{7} - 1\frac{3}{14})$

2 Simplify the following using the correct order of operations.

(a) $\frac{3}{4} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{2}$

(b) $\frac{1}{6} \times \frac{1}{4} + \frac{5}{8} \times \frac{1}{3}$

(c) $\frac{5}{6} \times (\frac{3}{5} + \frac{1}{3})$

(d) $(\frac{1}{3} - \frac{1}{4}) \div \frac{3}{8}$

(e) $\frac{8}{9} \div (\frac{3}{4} - \frac{1}{8})$

(f) $(\frac{7}{10} + \frac{1}{5}) \times \frac{9}{11}$

(g) $\frac{3}{7} \times (\frac{1}{4} + 7 \div \frac{7}{9})$

(h) $(5 \div \frac{5}{11} - \frac{7}{12}) \times \frac{1}{6}$

(i) $\frac{8}{9} \times (\frac{3}{4} + 2 \times \frac{1}{5}) + 2 \times \frac{1}{6}$

(j) $14 \times \frac{5}{8} - (\frac{2}{9} + 4 \times \frac{5}{12}) \times 2\frac{1}{4}$

3 Use your calculator to find the following, writing your answers as mixed numerals if appropriate.

(a) (i) $\frac{4}{5} - \frac{3}{7}$

(ii) $\frac{4}{5} \times \frac{3}{7}$

(iii) $\frac{4}{5} \div \frac{3}{7}$

(b) (i) $\frac{7}{15} + \frac{3}{10}$

(ii) $\frac{7}{15} \div \frac{3}{10}$

(iii) $\frac{7}{15} \times \frac{3}{10}$

(c) (i) $\frac{10}{9} \times \frac{5}{12}$

(ii) $\frac{10}{9} - \frac{5}{12}$

(iii) $\frac{10}{9} \div \frac{5}{12}$

(d) (i) $\frac{11}{30} \div \frac{1}{2}$

(ii) $\frac{11}{30} \times \frac{1}{2}$

(iii) $\frac{11}{30} + \frac{1}{2}$

(e) (i) $2\frac{4}{9} \times 5$

(ii) $2\frac{4}{9} + 5$

(iii) $2\frac{4}{9} \div 5$

(f) (i) $4\frac{3}{8} - 3\frac{9}{10}$

(ii) $4\frac{3}{8} \div 3\frac{9}{10}$

(iii) $4\frac{3}{8} \times 3\frac{9}{10}$

Extension

4 Work out the following using a calculator, and write your answers as mixed numerals if appropriate.

(a) $\frac{2}{3} \times (\frac{2}{5} + \frac{5}{6})$

(b) $(\frac{1}{4} - \frac{1}{5}) \div \frac{4}{7}$

(c) $\frac{7}{9} \div (\frac{5}{6} - \frac{2}{9})$

(d) $(\frac{3}{10} + \frac{2}{5}) \times \frac{5}{14}$

(e) $(\frac{1}{3} + \frac{2}{5}) \div (\frac{2}{5} - \frac{1}{10})$

(f) $(\frac{5}{4} - \frac{6}{7}) \times (\frac{1}{5} + \frac{2}{3})$

(g) $(\frac{5}{2} - \frac{3}{7}) \times (\frac{1}{6} + \frac{8}{9})$

(h) $(\frac{9}{4} + \frac{7}{2}) \div (\frac{3}{7} - \frac{2}{14})$

(i) $\frac{1}{5} \times \frac{5}{7} + \frac{7}{10} \div \frac{14}{15}$

(j) $\frac{5}{8} \div \frac{15}{4} - \frac{1}{5} \times \frac{5}{6}$

(k) $(2\frac{2}{3} \div 4 - \frac{1}{10}) \times \frac{15}{17}$

(l) $3\frac{2}{3} \times (6 \div \frac{4}{7} + \frac{7}{8})$

Don't use your calculator for Questions 1 and 2.



e Hint

e Hint



Do not write the calculator symbol as part of your answer. Just write a normal fraction answer.



e Hint



9.9 Problems involving fractions

Fractions are used in many real-life situations.

exercise 9.9 Problems involving fractions

P Preparation: Exs 9.1, 9.2, 9.4–9.7

Core

Answer the following in simplest form. Write your answers as mixed numbers where appropriate.

- 1 Be A Sport sporting goods store advertises a giant end-of-year sale in which prices are slashed by $\frac{1}{4}$. What are the sale prices of the following items if these are their normal prices?
 - (a) a football for \$20
 - (b) a soccer ball for \$16
 - (c) a T-shirt for \$28
 - (d) a pair of tennis shoes for \$83
- 2 A survey was taken of 120 people as they left Wally's Sandwich Bar.
 - (a) If 80 were male, what fraction was male?
 - (b) What fraction was female?
 - (c) If 45 were under 21 years of age, what fraction was under 21?
 - (d) What fraction was 21 or over?
 - (e) If $\frac{3}{4}$ had bought a drink, how many had bought a drink?
 - (f) If $\frac{3}{10}$ had bought a salad roll, how many had bought a salad roll?
- 3 Valerie is running laps of her school oval to train for Rigby High's Mini Marathon. She ran $4\frac{1}{2}$ laps before having to rest. After her rest, she ran another $3\frac{1}{4}$ laps before stopping again. Then she struggled through another $\frac{1}{3}$ of a lap. How many laps did she complete all together?
- 4 Grant is training in his backyard pool for the 1500 m freestyle at the next Olympics. Unfortunately, the pool is only 20 m in length.
 - (a) What fraction of the total 1500 m race would he have swum after completing just one lap?
 - (b) How many laps would he have to complete to swim 1500 m?
 - (c) What fraction of the total 1500 m race would he have swum after completing six laps?

e Hint

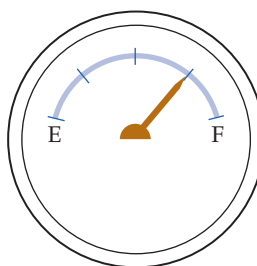
e Worksheet C9.11

e Animation



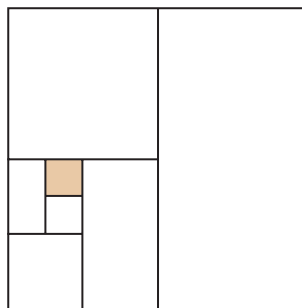
- (d) How far would he have swum if he had completed $\frac{1}{10}$ of the race distance?
- (e) How far would he have swum if he had completed $\frac{3}{4}$ of the race distance?
- 5 (a) What fraction of an hour has passed from 3.49 p.m. to 3.51 p.m.?
 (b) What fraction of an hour has passed from 11.03 a.m. to 11.23 a.m.?
 (c) What fraction of an hour has passed from 8.13 p.m. to 8.25 p.m.?
 (d) What fraction of an hour has passed from 2.36 a.m. to 2.45 a.m.?
- 6 A packet of Munchies weighs 150 grams. If the packet itself weighs 10 grams and there are 85 Munchies in the packet, what fraction of the full packet does one Munchie weigh?
- 7 A packet of SML cheese weighs $\frac{3}{4}$ kg. Sithivani cuts it up into 15 equal cubes. What fraction of a kilogram does each cube weigh?
- 8 A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.
- (a) How much petrol has been used?
 (b) How much petrol is still in the tank?

e Hint



e Hint

- 9 The Happy Valley Farming Cooperative has divided its square block of land in the following way. Each section of land is exactly half of a section next to it. The May family has been given the shaded region to farm. If the total block of land is 1, what fraction of the total *don't* the May family farm?



e Animation

- 10 Despina's department store advertises a mammoth winter sale in which they claim everything is between $\frac{1}{3}$ and $\frac{1}{2}$ off. For which of the following items is this incorrect?

Item	Normal price	Sale price
Bath towels	\$21	\$12
Video camera	\$942	\$612
Cutlery set	\$235	\$120
Microwave oven	\$584	\$392
Jeans	\$53	\$34



- 11** One weekend Louisa and Andrew walk from Ferndale to Greenhill and back, a total of $2\frac{3}{4}$ km. The next weekend they walk from Ferndale through Greenhill to Highvale, which is $5\frac{1}{3}$ km.

How far (in km) is it from:

- Ferndale to Greenhill
- Greenhill to Highvale
- Ferndale to Highvale and back again?

e Hint

- 12** Theresa, Isabelle and Bryce worked on a mathematical problem and came up with the answers $8\frac{1}{2}$, $8\frac{2}{3}$ and $8\frac{3}{5}$ respectively. The correct answer was $8\frac{7}{12}$. Which of the three students was closest to the correct answer?

e Hint

- 13** Tung is planning his European holiday. He has five months and has worked out the following itinerary. He will be in England for $1\frac{1}{2}$ months, in Germany for $1\frac{2}{3}$ months, and in Italy for $\frac{3}{4}$ of a month. The other country he will be visiting is France. How much time (in months) will he spend in France?



Extension

- 14** In 7F at Richwood High, $\frac{2}{3}$ of the students are boys, $\frac{1}{4}$ of the boys have fair hair, twice as many boys as girls have fair hair, and Julie and Daniella are the only girls with fair hair.

- How many boys are there in the class?
- How many students are there in the class?

- 15** At Julio's party, which was due to start at 8.00 p.m., $\frac{3}{5}$ of the guests arrived at least one hour late, $\frac{1}{8}$ of the guests arrived at least half an hour early, and 22 guests arrived between 7.30 p.m. and 9.00 p.m.

- What fraction of the guests arrived between 7.30 p.m. and 9.00 p.m.?
- How many guests were there at the party altogether?
- How many guests arrived at least one hour late?
- How many guests arrived at least half an hour early?

e Hint

- 16** It takes Felicity $\frac{3}{4}$ of an hour to lay a row of 50 bricks.

- How many rows will she lay in $4\frac{1}{2}$ hours?
- How many bricks will she lay in that time?
- How long will it take her to build a wall made up of 22 rows?
- How long will it take her to build a section made up of 462 bricks?

e Homework 9.4



Summary

Copy and complete the following summary of this chapter using the words and phrases from the list. A word or phrase may be used more than once.

- 1 In a fraction the number above the line is called the _____ and the bottom number is called the _____.
- 2 A m _____ _____ can be re-written as an _____ fraction.
- 3 The first step when adding or subtracting fractions with different denominators is to find the _____.
- 4 It is best to write the answer to a question involving fractions in _____ . This is done by dividing the numerator and denominator by their _____ to find an _____ fraction.
- 5 2 is the _____ of $\frac{1}{2}$.
- 6 If a pattern of digits keep repeating somewhere after the decimal point, the number is called a _____ decimal.


Key words

denominator
equivalent
fraction
highest common factor
improper
lowest common denominator
mixed numeral
numerator
of
proper
recurring
reciprocal
simplest form
simplify

Questions

- 1 Explain the difference between an improper and a proper fraction.
- 2 'Equivalent' means equal in value, this is why it begins with the letters 'equ'. Use a dictionary to help you explain what the following terms mean and why they begin with 'equ'.
equilateral triangle equator equilibrium equinox
- 3 Explain why you think we call a fraction such as $4\frac{1}{2}$ a mixed number.
- 4 Find each of the following: $\frac{1}{4}$ of \$40 and $\frac{1}{4}$ off \$40.
- 5 What do the letters HCF and LCD stand for?
- 6 Turning a fraction upside down gives the reciprocal fraction. There is another way to define the reciprocal. What does a fraction multiplied by its reciprocal always result in? Use this to write a different definition of a reciprocal number.
- 7 Which individual word in the above list has:
(a) the most letters (b) the fewest letters?
- 8 Write at least 15 words of four letters or more from the letters in 'reciprocal'.

 Worksheet L9.1

 Worksheet L9.2

chapter REVIEW

9

FAQS

Is multiplying the denominators the best way to get the LCD?

This method will always give you a common denominator but it may not be the lowest common denominator, so you may have to work with bigger numbers and do more cancelling to get the answer. It is better to go through the multiples of each denominator and find the smallest common one.

When I am dividing two fractions, does it matter which of the fractions I turn upside down?

Yes, it does matter. You must turn the second fraction only, otherwise you will get an incorrect answer.



Core

- Find equivalent fractions by copying and completing the following.
 - $\frac{2}{7} = \frac{12}{\square}$
 - $\frac{12}{18} = \frac{\square}{6}$
 - $\frac{15}{30} = \frac{45}{\square}$
- Reduce the following fractions to their simplest form.
 - $\frac{55}{99}$
 - $\frac{48}{20}$
 - $3\frac{60}{72}$
- Write $3\frac{2}{7}$ as an improper fraction.
 - Write $\frac{48}{33}$ as a mixed numeral.
- Position each of the following on separate number lines.
 - $\frac{1}{4}$
 - $\frac{4}{9}$
 - $\frac{17}{10}$
- Copy and complete the following by inserting either $<$, $>$ or $=$.
 - $\frac{5}{13} \text{ — } \frac{1}{2}$
 - $\frac{6}{16} \text{ — } \frac{30}{80}$
 - $\frac{5}{12} \text{ — } \frac{3}{8}$
- Write each set of numbers in order from smallest to largest.
 - $\frac{3}{4}, \frac{1}{2}, 1, \frac{1}{3}, \frac{3}{5}$
 - $2, \frac{13}{5}, \frac{5}{4}, \frac{30}{50}, 1\frac{2}{5}$
- Convert each of the following decimals to fractions in their simplest form.
 - 3.087
 - 0.45
 - 2.000 75
- Use short division to convert each of the following fractions to a decimal.
 - $\frac{3}{8}$
 - $\frac{3}{5}$
 - $\frac{11}{4}$
- Use your calculator to convert each fraction to decimal form. Write answers correct to three decimal places where necessary.
 - $\frac{9}{11}$
 - $\frac{18}{7}$
 - $3\frac{12}{35}$
- Evaluate the following. Write your answers as mixed numerals if appropriate.
 - $\frac{5}{12} + \frac{7}{8}$
 - $\frac{5}{18} - \frac{2}{9}$
 - $\frac{2}{7} + \frac{2}{3}$
- Evaluate the following. Write your answers as mixed numerals.
 - $3\frac{2}{5} - 1\frac{3}{4}$
 - $2\frac{1}{4} + 5\frac{11}{12}$
 - $8 - 2\frac{5}{7}$

hi.com.au

9.1

9.1

9.1

9.2

9.2

9.2

9.3

9.3

9.3

9.4

9.5



12 Evaluate the following. Write your answers as mixed numerals if appropriate.

(a) $\frac{6}{11} \times \frac{55}{18}$

(b) $2\frac{4}{9} \times 5$

(c) $\frac{2}{3} \times \frac{9}{16} \times 1\frac{1}{7}$

9.6

13 Work out the following.

(a) $\frac{3}{7}$ of \$28

(b) $\frac{2}{9}$ of $\frac{3}{4}$

9.6

14 Work out the following divisions, and choose the correct answer.

(a) $\frac{15}{24} \div \frac{5}{12}$

A $\frac{10}{12}$

B $1\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{3}{19}$

(b) $3\frac{2}{7} \div 23$

A $20\frac{2}{7}$

B 7

C $\frac{1}{7}$

D $\frac{7}{529}$

9.7

15 Simplify, without the use of a calculator.

(a) $4 + \frac{5}{6} \times 2\frac{1}{3}$

(b) $7\frac{2}{5} - (3\frac{1}{10} - \frac{3}{5})$

(c) $\frac{7}{8} \times (\frac{3}{4} + 4 \times \frac{3}{4})$

9.8

16 Use your calculator to work out the following, writing your answer as a mixed numeral if appropriate.

(a) $\frac{7}{8} - \frac{17}{20}$

(b) $\frac{72}{600} \div \frac{3}{400}$

(c) $2\frac{2}{9} \times 4\frac{1}{2}$

(d) $5\frac{2}{3} + (3\frac{1}{6} - \frac{7}{12})$



9.8

Extension

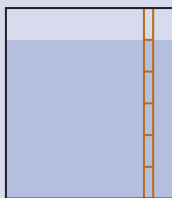
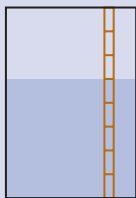
17 Jarrod owns a petrol station which has three main fuel tanks. There is a dip stick in each tank which shows the level of fuel remaining in each tank. How many litres are left in each tank?

9.9

(a) Full tank = 48 000 L

(b) Full tank = 60 000 L

(c) Full tank = 30 000 L



18 In a class of 24 students, three-eighths are girls.

9.4, 9.9

(a) How many girls are in the class?

(b) What fraction of the class is boys?

19 Jenny earns \$768 a week. Bills, food and expenses use up seven-twelfths of her weekly pay. How much money does Jenny spend on these items?

9.6, 9.9

20 Elise and Reece each have identical blocks of chocolate. Elise eats $\frac{2}{5}$ of her block and Reece eats $\frac{2}{3}$ of his block.

9.4, 9.9

(a) What is the smallest number of pieces that each block of chocolate can have?

(b) What fraction does Elise and Reece each have left?

(c) How much more has Reece eaten as a fraction of a block?

(d) Write as a fraction the total amount they have eaten.



REPLAY

- 1** Write the following Roman numbers as Hindu–Arabic numerals.
(a) XXIV (b) CDLXXVIII (c) MMDCCXLIX
- 2** Calculate:
(a) $27 - 40$ (b) $-12 - 8$ (c) $-16 + 28$
- 3** Calculate:
(a) -3×8 (b) $-35 \div -7$ (c) $\frac{81}{-9}$
- 4** List all the factors of the following numbers.
(a) 15 (b) 50 (c) 4
- 5** Evaluate:
(a) 7^2 (b) $\sqrt{36}$ (c) $\sqrt[3]{27}$
- 6** Substitute the given values into $h = 2g + 3$ to find the value of h .
(a) $g = 10$ (b) $g = 2$ (c) $g = 0$ (d) $g = -5$
- 7** What type of angle is each of the following?
(a) 137° (b) 24° (c) 360°
- 8** A straight angle is divided into three equal angles. What is the size of each angle?
- 9** Calculate:
(a) 69.8×4 (b) 4.68×1000 (c) 3.68×0.007
- 10** Find the perimeter of each of the following shapes.
(a) a square with side length 4.2 cm
(b) a rectangle of length 12 cm and breadth 2.5 cm
- 11** Find the area of each of the following shapes.
(a) a rectangle of length 8 cm and breadth 4 cm
(b) a triangle with base length 4 cm and height 6 cm
- 12** (a) What is the sum of the angles in a quadrilateral?
(b) What is the size of each angle in an equilateral triangle?

1.1

2.5

2.6, 2.7

3.3

3.6, 3.7

4.3

5.4

5.5

6.6, 6.7, 6.8

7.4

7.6, 7.7

8.2, 8.4

e Assignment 9