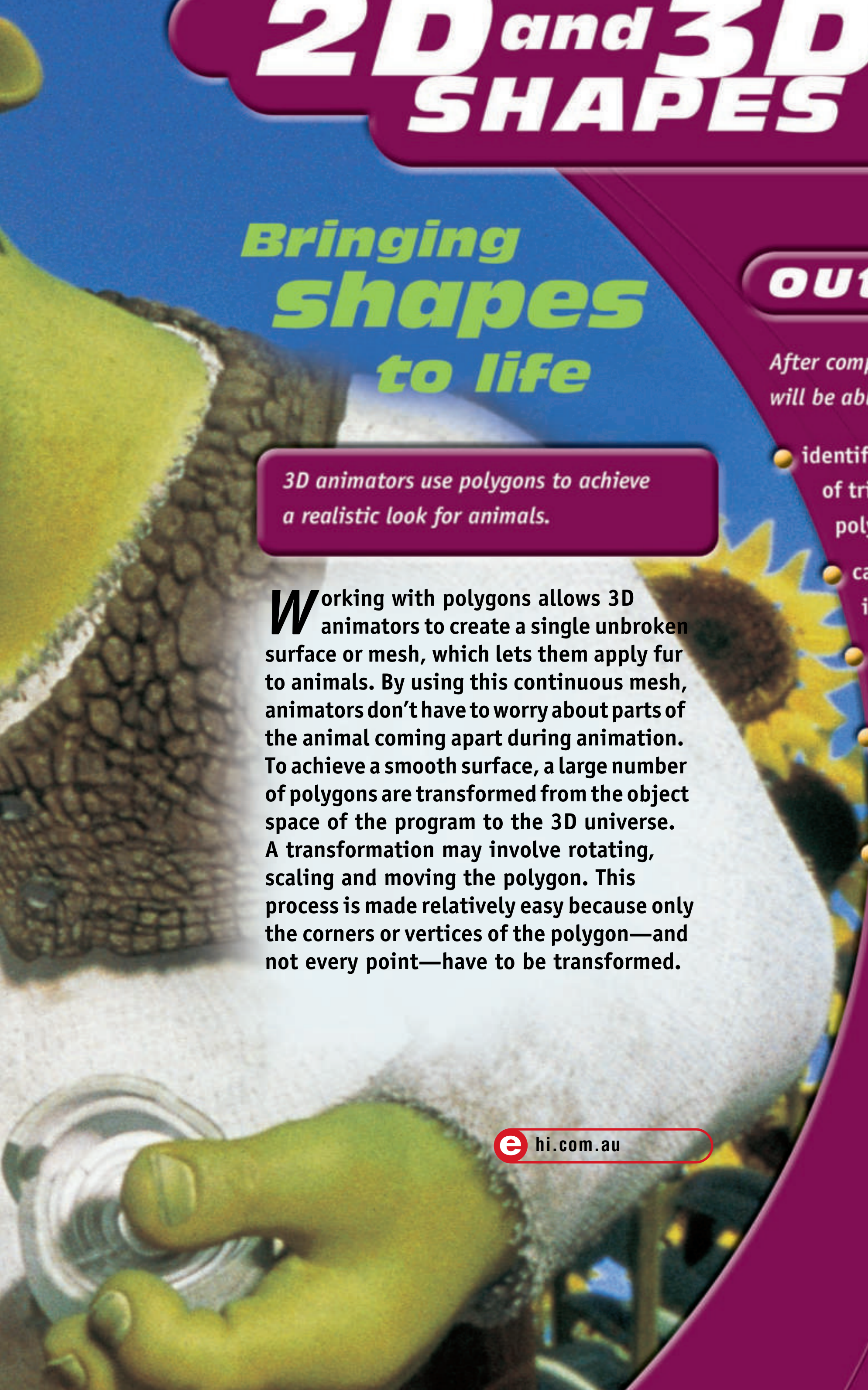
RETURN TO MAIN MENU

8 2D and 3D SHAPES



Bringing shapes to life

3D animators use polygons to achieve a realistic look for animals.

Working with polygons allows 3D animators to create a single unbroken surface or mesh, which lets them apply fur to animals. By using this continuous mesh, animators don't have to worry about parts of the animal coming apart during animation. To achieve a smooth surface, a large number of polygons are transformed from the object space of the program to the 3D universe. A transformation may involve rotating, scaling and moving the polygon. This process is made relatively easy because only the corners or vertices of the polygon—and not every point—have to be transformed.

e hi.com.au

outcomes

After completing this chapter you will be able to:

- identify and determine properties of triangles, quadrilaterals and polygons
- calculate the missing angle in a polygon
- reflect, rotate, dilate and translate simple figures
- draw and name some simple three-dimensional shapes
- determine properties of three-dimensional shapes.

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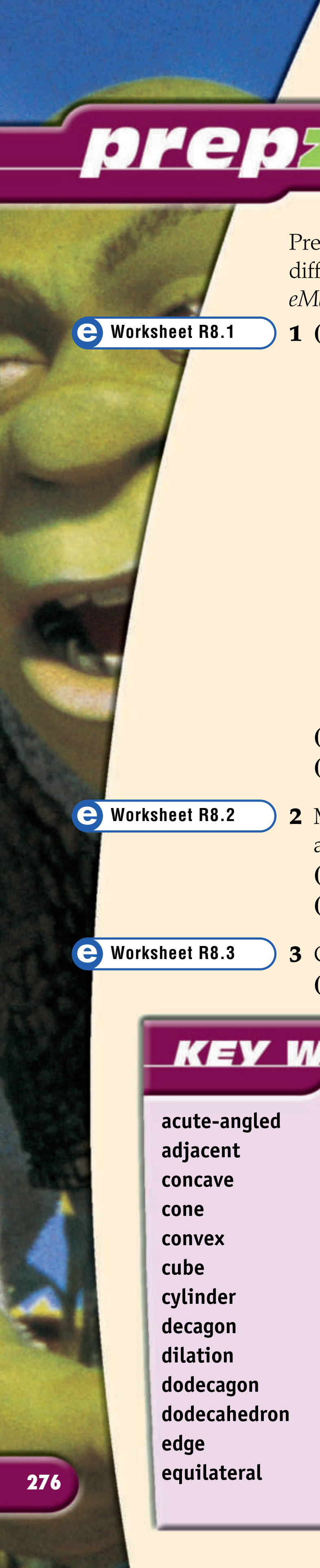
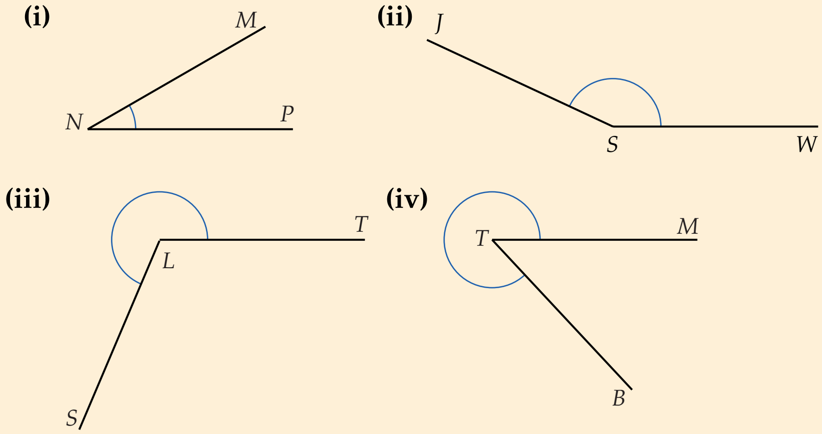
prepzone 8



Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

1 **(a)** Measure the following angles to the nearest degree. — *Worksheet R8.1*



(b) Give each of the angles a letter name.

(c) What type of angle is each?

2 Measure the length of each of the following line segments. Give your answer in centimetres to one decimal place. — *Worksheet R8.2*

(a) ______________________________

(b) ____

3 Calculate: — *Worksheet R8.3*

(a) 56 + 69 – 71 **(b)** 278 – (90 + 23 + 144)

KEY WORDS

acute-angled
adjacent
concave
cone
convex
cube
cylinder
decagon
dilation
dodecagon
dodecahedron
edge
equilateral
Euler's rule
face
heptagon
hexagon
hexahedron
icosahedron
irregularpolygon
isometric
isosceles
kite
line symmetry
net
nonagon
oblique
obtuse-angled
octagon
octahedron
order of rotational symmetry
parallelogram
pentagon
perpendicular bisector
plane shape
polygon
polyhedron
prism
pyramid
quadrilateral
rectangle
reflection
regular
rhombus
right
right-angled
rotation
rotational symmetry
scale factor
scalene
solid
sphere
square
symmetry
tetrahedron
torus
transformations
translation
trapezium
triangle
undecagon
vertex

RETURN TO MAIN MENU

Working mathematically

computer investigation

Cabri

Classifying triangles

Triangles are given special names depending on their angles and side lengths. To answer the following questions, click on the icons to the right to open the Cabri Geometry files on classifying triangles. Observe what happens to the triangles when you move the points.

1 What can you say about the lengths of the sides in each triangle? Which triangle do you think is named:
(a) isosceles **(b)** scalene **(c)** equilateral?

e Interactive

2 What can you say about the sizes of the angles in each triangle? Which triangle do you think is named:
(a) obtuse-angled **(b)** right-angled **(c)** acute-angled?

e Interactive

8.1 Triangles

A **plane shape** is a flat two-dimensional shape that is closed with sides that do not overlap.

The simplest plane shape that can be drawn with straight sides is the **triangle**. There are several different types of triangles. We may name any triangle using either its 'side name' or its 'angle name'.

The side name for a triangle is found by considering the length of all three sides.

An **equilateral** triangle has all sides equal in length.

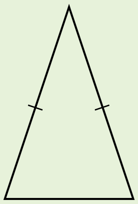
An **isosceles** triangle has only two sides of equal length.

A **scalene** triangle has all sides of different lengths.

To show that sides are of an equal length, we mark them with an equal number of dashes.

worked example 1

Give each of the following triangles its side name.

(a)


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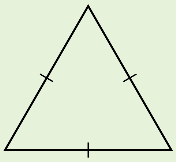

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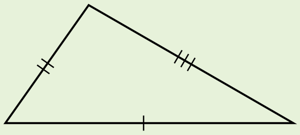


Steps

(a) Determine how many sides are the same length and then use the definitions to name the shape.

(b) Determine how many sides are the same length and then use the definitions to name the shape.

(c) Determine how many sides are the same length and then use the definitions to name the shape.

Solutions

(a) Two sides are the same length, therefore an isosceles triangle.

(b) All sides are the same length, therefore an equilateral triangle.

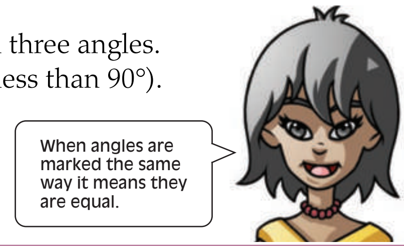
(c) All sides are different in length, therefore a scalene triangle.

The angle name of a triangle is found by finding the size of all three angles.

An **acute-angled** triangle has three angles that are acute (less than 90°).

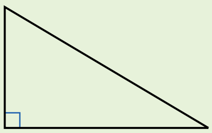
An **obtuse-angled** triangle contains one obtuse angle (greater than 90° but less than 180°).

A **right-angled** triangle contains one right angle (90°).



worked example 2

Give each of the following triangles its angle name.

(a)


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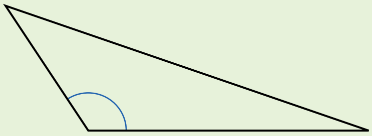
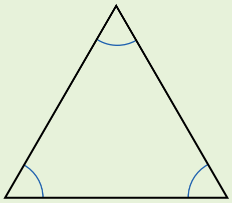


(c)


Steps

(a) Look at all angles and use the definitions above to determine the correct angle name for the triangle.

(b) Look at all angles and use the definitions above to determine the correct angle name for the triangle.

(c) Look at all angles and use the definitions above to determine the correct angle name for the triangle.

Solutions

(a) One angle is 90°, therefore a right-angled triangle.

(b) One angle is greater than 90°, therefore an obtuse-angled triangle.

(c) All angles are less than 90°, therefore an acute-angled triangle.

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exercise 8.1 Triangles

Preparation: Prep Zone Q1 and 2

Core

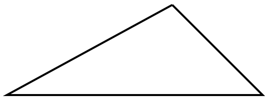
1 Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one.

Hint

Worksheet C8.1

Worksheet C8.2

Worksheet C8.3

(a)


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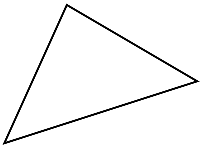

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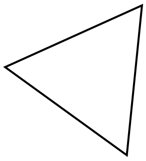

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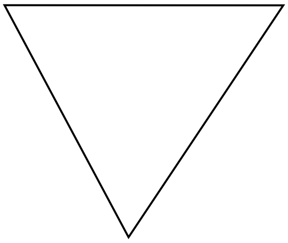

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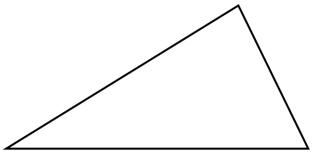

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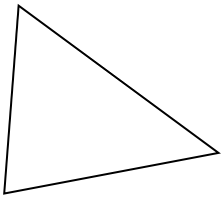
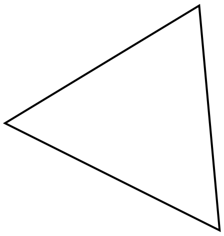


(g)


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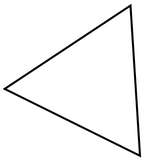

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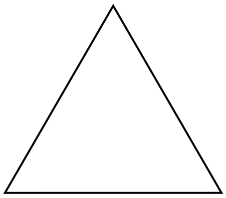

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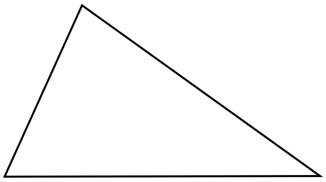

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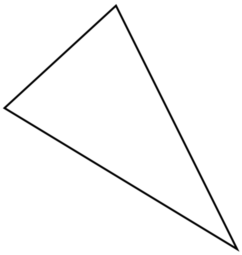

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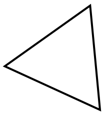
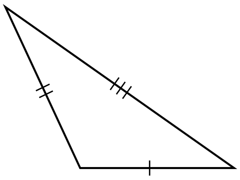


2 Give each of the following triangles its side name.

Hint

(a)


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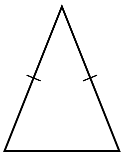

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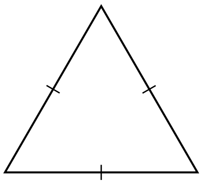

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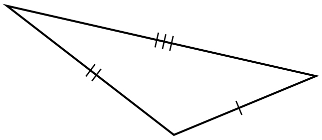
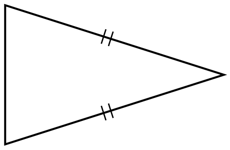
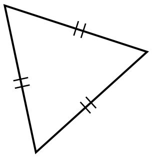
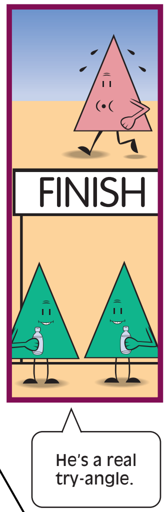
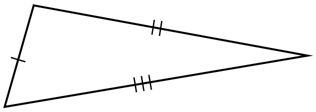
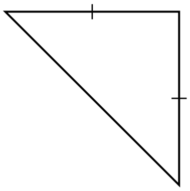
(d) 

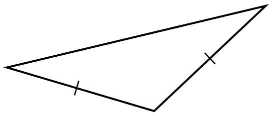
(e) 

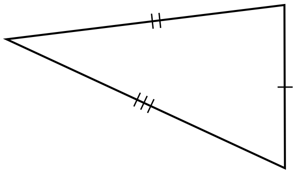
(f) 

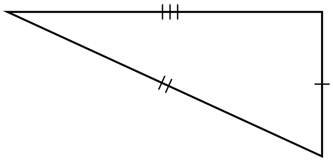


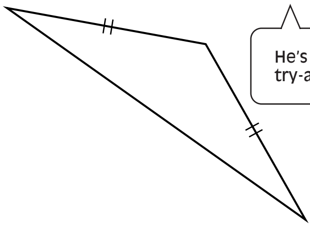
(g) 

(h) 

(i) 

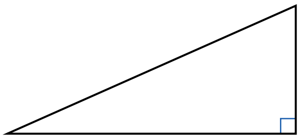
(j) 

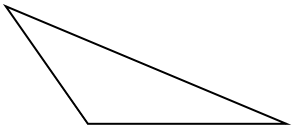
(k) 

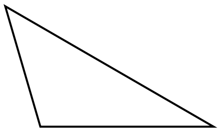
(l) 

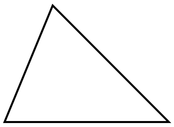
3 Give each of the following triangles its angle name.

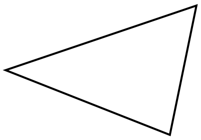
Hint

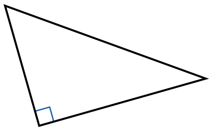
(a) 

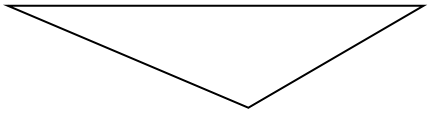
(b) 

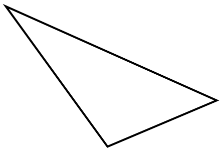
(c) 

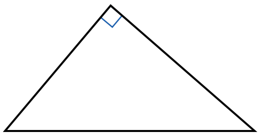
(d) 

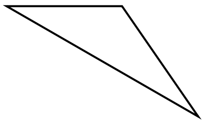
(e) 

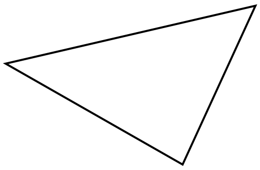
(f) 

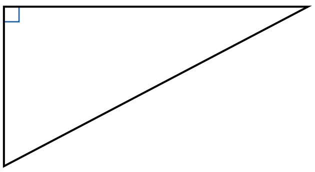
(g) 

(h) 

(i) 

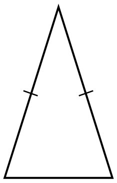
(j) 

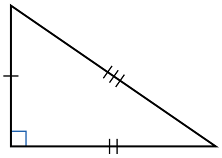
(k) 

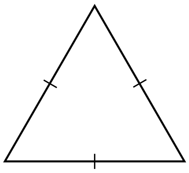
(l) 

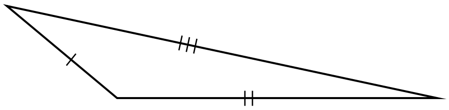
4 Give both the side name and the angle name for each of the following triangles.

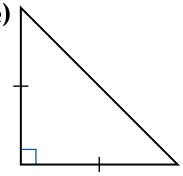
Hint

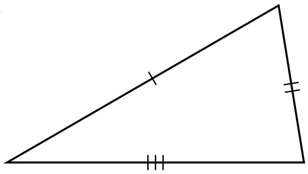
(a) 

(b) 

(c) 

(d) 

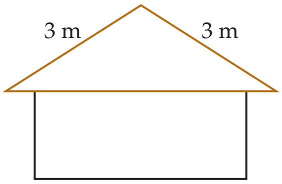
(e) 

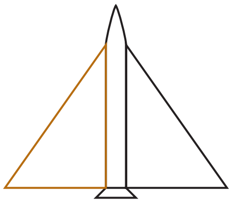
(f) 

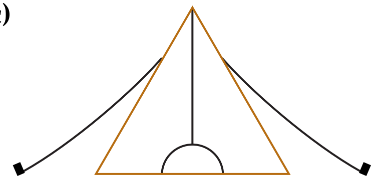
Extension

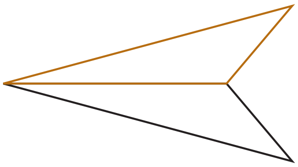
5 Name the type of triangle shown in each diagram below. (Side or angle names may be used.)

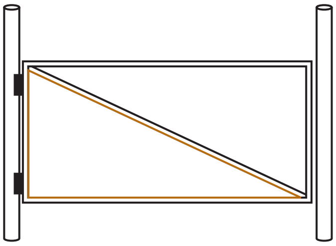
Hint

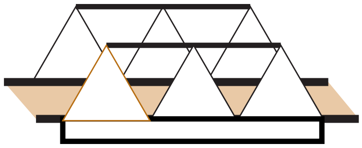
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

6 Look at some of the triangles in this exercise and describe the relationship between the lengths of the sides and their opposite angles.

Worksheet A8.1

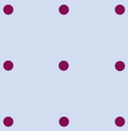
Working mathematically

problem solving

Triangle teasers

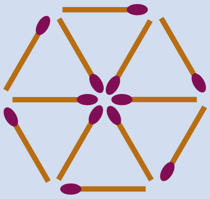
Dotty triangles

How many triangles differing in shape or size can you draw on a dot grid like the one opposite so that each corner lies on a dot?



Moving matchsticks

Re-position 4 matchsticks to form 3 equilateral triangles.



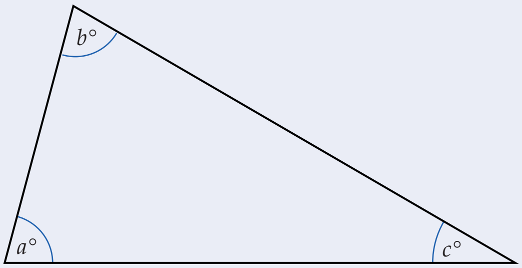
Try guess and check.

Working mathematically

investigation

Angle sum in a triangle

1 On a sheet of blank paper, use a ruler to draw 6 different triangles. Make each triangle about a quarter-page size. Label the angles in each triangle $a°$, $b°$ and $c°$. Number your triangles 1 to 6.



2 Copy the following table. Measure each angle in the triangles you have drawn, and complete the middle three columns of your table.

Triangle	*Angle a°*	*Angle b°*	*Angle c°*	*Angle total a° + b° + c°*
1				
2				
3				
4				
5				
6				

3 Add your angle measurements for each triangle to complete the last column in your table.

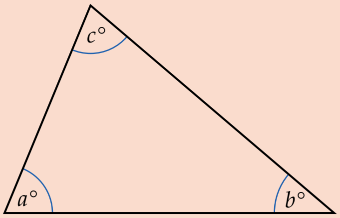
4 Allowing for small measurement errors of a couple of degrees or so, what does your table tell you about the angles in a triangle?

RETURN TO MAIN MENU

8.2 Angle sum in a triangle

If you completed the previous investigation, you would have confirmed the following rule:

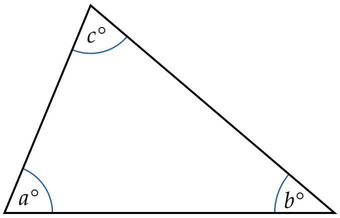
The angles in a triangle add up to 180°.



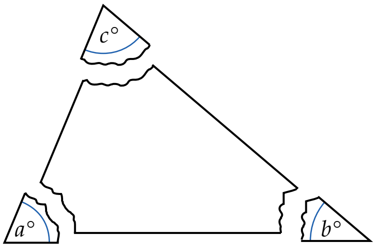
$a° + b° + c° = 180°$

Another way of showing that the angles in a triangle add up to 180° is shown here. You may wish to try it for yourself.

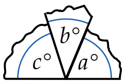
1. Draw any triangle and label its angles.



2. Tear the corners off.



3. Place the cut-off angles together as shown.

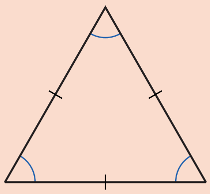


4. The angles form a straight angle, i.e. 180°.
5. Repeat steps 1 to 4 for other triangles if you need further convincing.

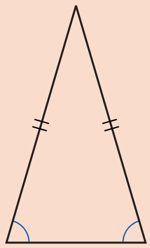
We can use the fact that the angles in a triangle add up to 180° to find a missing angle in a triangle without measuring if we know two other angles.

In Exercise 8.1 we used the number of equal length sides to name equilateral, isosceles and scalene triangles. However it is also possible to use the number of equal angles to work out the name.

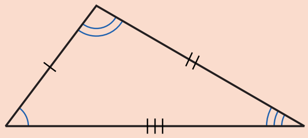
In an equilateral triangle all the angles are equal. (Since they add to 180°, each is 60°.)



An isosceles triangle has two equal angles opposite the equal sides.



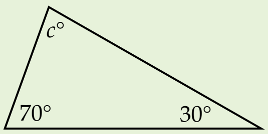
A scalene triangle has no two angles equal.



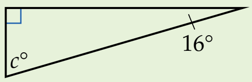
worked example 3

Find the size of the angle labelled $c°$ in each of the following.

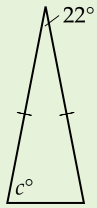
(a)



(b)



(c)

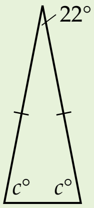


Steps	Solutions
(a) 1. Add the given angles.	**(a)** 70° + 30° = 100°
2. Subtract the sum of the given angles from 180°.	180° − 100° = 80°
3. Write the answer as shown.	$c° = 80°$
(b) 1. Add the given angles. Remember that the small square in the triangle indicates a 90° angle.	**(b)** 90° + 16° = 106°
2. Subtract the sum of the given angles from 180°.	180° − 106° = 74°
3. Write the answer as shown.	$c° = 74°$

(c) 1. Since there is only one given angle, subtract it from 180° to start with.

(c)
$$\begin{array}{r} 180^\circ \\ -\ 22^\circ \\ \hline 158^\circ \\ \hline \end{array}$$

2. The two angles at the bottom of the triangle must be equal since the triangle is isosceles, so divide the answer from step 1 by 2 to find the size of each one.



$$2\overline{)158^\circ} = 79^\circ$$

3. Write the answer as shown.

$c° = 79°$

exercise 8.2 Angle sum in a triangle

Preparation: Prep Zone Q3, Ex 8.1

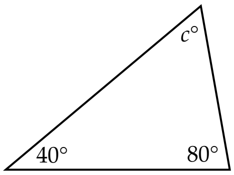
Worksheet C8.4

Hint

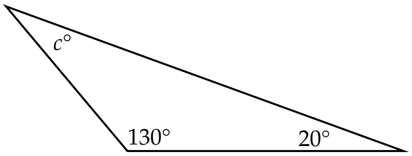
Core

1 Find the value of $c°$ in each of the following triangles.

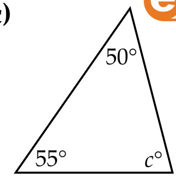
(a)



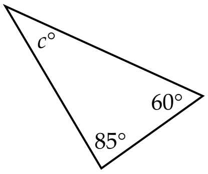
(b)



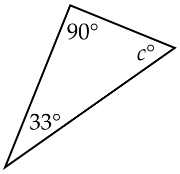
(c)



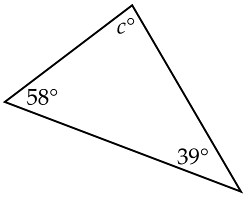
(d)



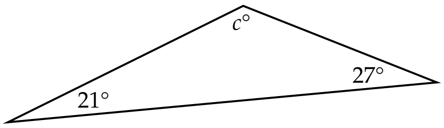
(e)



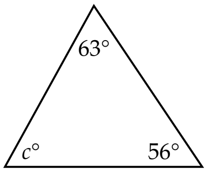
(f)



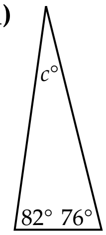
(g)



(h)



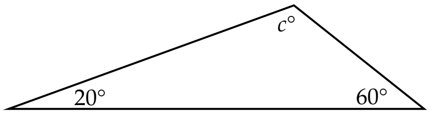
(i)



2 Choose the correct answer.

(a) The size of angle $c°$ is:

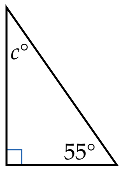
A 20° **B** 60°
C 80° **D** 100°



RETURN TO MAIN MENU

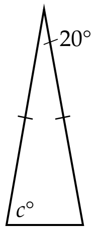
(b) The size of angle $c°$ is:

A 35° **B** 45°
C 55° **D** 145°



(c) The size of angle $c°$ is:

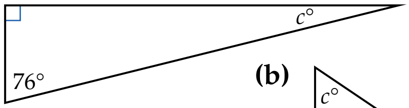
A 20° **B** 80°
C 140° **D** 160°



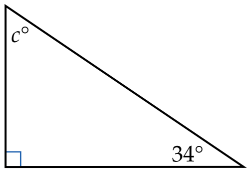
3 Find the value of $c°$ in each triangle.

eTester
Hint

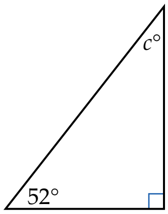
(a)



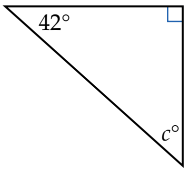
(b)



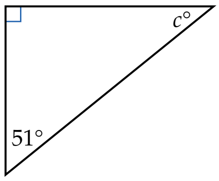
(c)



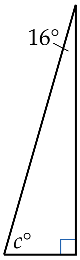
(d)



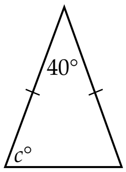
(e)



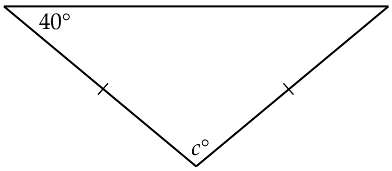
(f)



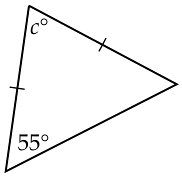
(g)



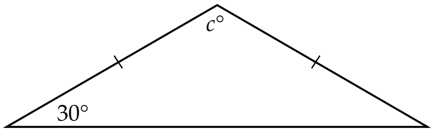
(h)



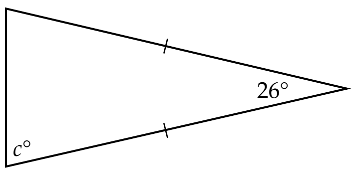
(i)



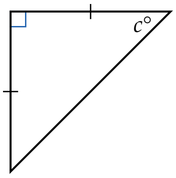
(j)



(k)



(l)



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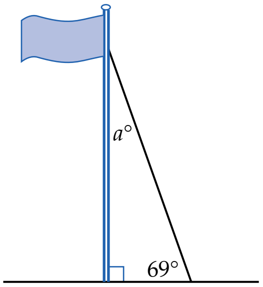
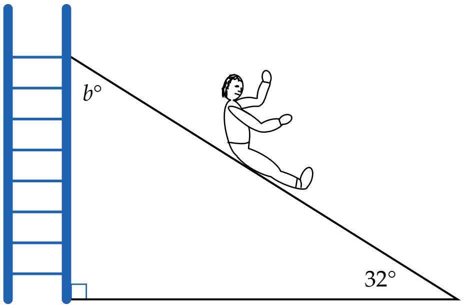
Extension

4 Find the angle represented by the pronumeral in each triangle below.

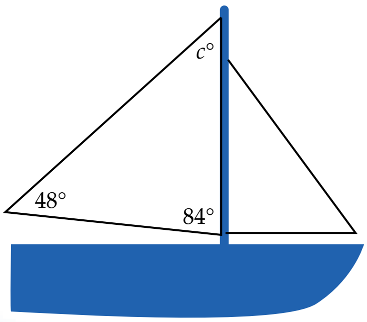
(a)



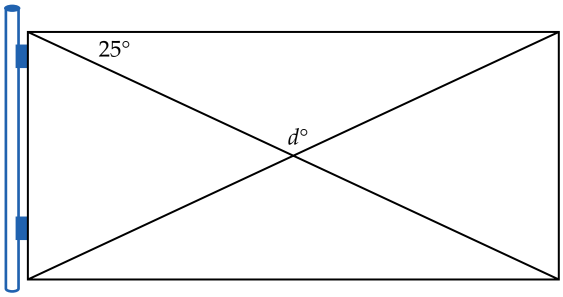
(b)



(c)



(d)



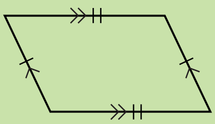
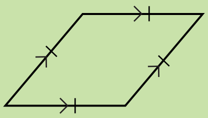
5 Give an example of what the three angles could be in a scalene, right-angled triangle.

Worksheet C8.5
Homework 8.1

8.3 Quadrilaterals

A plane shape with four straight sides is called a **quadrilateral**. We name quadrilaterals by looking at the properties of their sides and angles.

Parallelograms are quadrilaterals with both pairs of opposite sides parallel. Within this group, three smaller groups have extra defining characteristics.

Quadrilateral	*Definition*
Parallelogram 	Two pairs of parallel sides equal and opposite angles of equal size.
Rhombus (or diamond) 	All four sides equal and opposite angles of equal size (a parallelogram with all sides of equal length).

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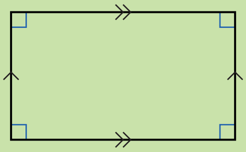
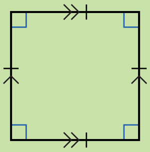
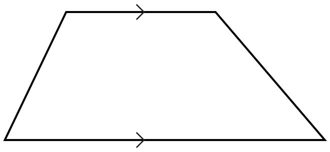
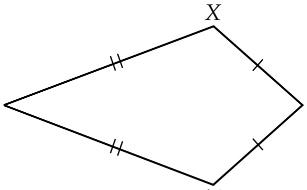
Rectangle		Two pairs of opposite sides equal and all internal angles 90° (a parallelogram with all internal angles 90°).
Square		All sides equal and all internal angles 90° (a rectangle with all sides of equal length).

We will consider three other types of quadrilateral.

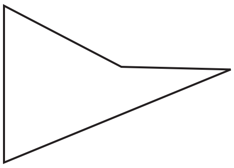
A quadrilateral with only one pair of parallel sides is called a **trapezium**.



A quadrilateral with two pairs of equal **adjacent** sides (sides next to each other) is called a **kite**. The angles marked *X* and *Y* are equal in size.



Sometimes the most accurate name we can give a four-sided shape is just 'quadrilateral'.



Any particular quadrilateral may have more than one correct name. For example a square may also be called a rectangle. However, in this case the name 'square' is the more accurate because all squares are rectangles but not all rectangles are squares.

Some plane shapes

eTutorial

exercise 8.3 Quadrilaterals

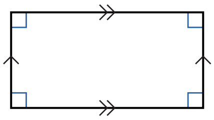
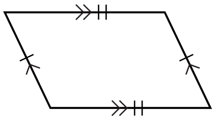
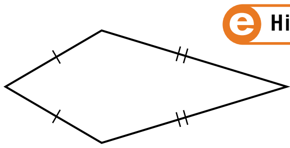
Preparation: Prep Zone Q1 and 2

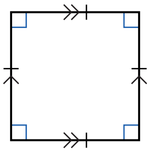
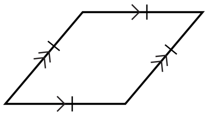
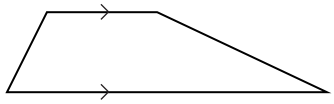
Core

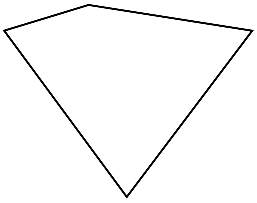
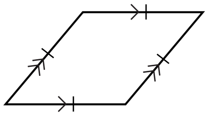
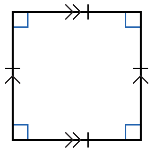
Worksheet C8.6

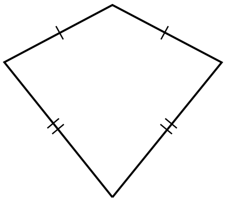
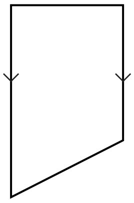
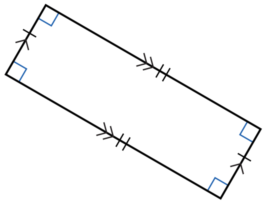
Hint

1 Give the most accurate name for each shape below.

(a)  **(b)**  **(c)** 

(d)  **(e)**  **(f)** 

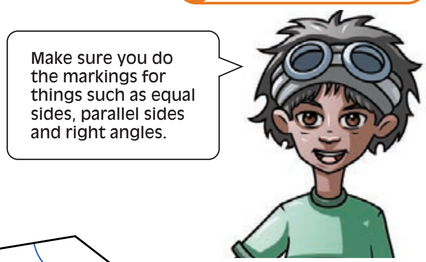
(g)  **(h)**  **(i)** 

(j)  **(k)**  **(l)** 

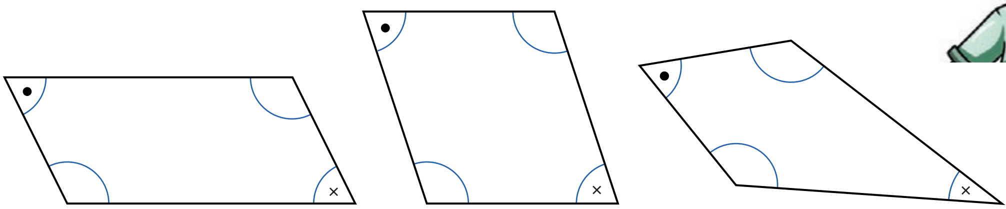
2 Draw two different examples of each of the following quadrilaterals.

(a) a kite **(b)** a parallelogram
(c) a rhombus **(d)** a rectangle
(e) a square **(f)** a trapezium

Hint



3 The opposite angles in three quadrilaterals are marked in the diagrams below.



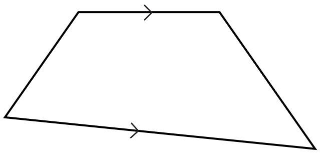
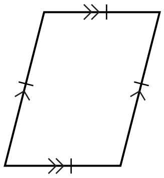
State TRUE or FALSE for each of the following.

(a) The opposite angles in a parallelogram are equal.
(b) All four angles in a parallelogram are always equal.
(c) The opposite angles in a rhombus are equal.
(d) All four angles in a rhombus are equal.
(e) One pair of opposite angles in a kite will always be equal.

Extension

4 The diagrams below were done by a tired author. What type of shape do you think he meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines and angles are correct.

Hint

(a)  **(b)** 

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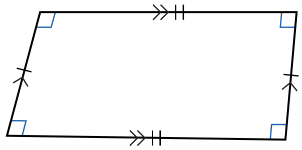
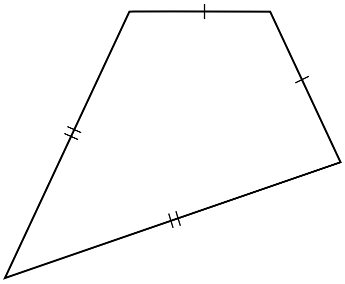
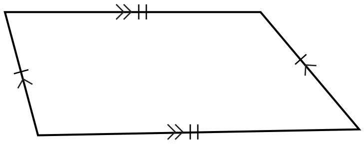
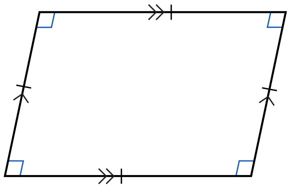
(c) 

(d) 

(e) 

(f) 

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eQuestions
Worksheet A8.2

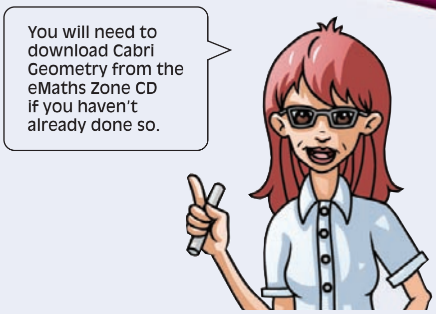
Working mathematically

computer investigation

Cabri

Quadrilaterals

In this investigation you will investigate quadrilaterals further. To answer the following questions, click on the icons to the right to open the Cabri Geometry files on quadrilaterals. Observe what happens to the quadrilaterals when you move the points. For each shape some things will stay the same and others will change. The things that stay the same are called properties of that shape; for example, a property of a square is that all of its angles are 90°.



1 What can you say about the angles in a kite? What can you say about the side and diagonal lengths? Interactive

2 What can you say about the angles of the trapezium? Interactive

3 What can you say about the angles in a parallelogram? Look at opposite angles and adjacent angles (angles next to each other). What can you say about the diagonals of a parallelogram? Interactive

4 What can you say about the diagonals of a rectangle? Interactive

5 List any interesting features of the rhombus you can find. What properties does a rhombus have that a parallelogram does not? Interactive

6 What can you say about the diagonals of a square? Interactive

7 Look at all the quadrilaterals together. Which quadrilaterals are types of parallelograms? Which quadrilateral is a type of rhombus? Interactive

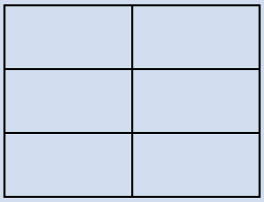
RETURN TO MAIN MENU

Working mathematically

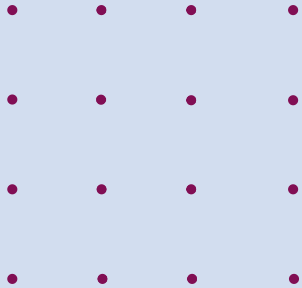
problem solving

Quadrilateral quandaries

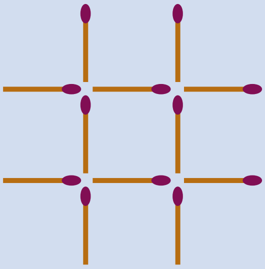
1 How many rectangles can you count here altogether? (Include rectangles that are the same size if they are drawn in different positions.)

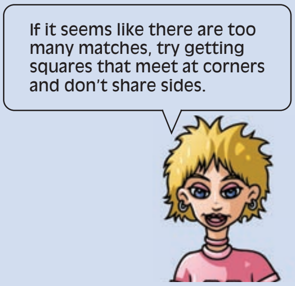


2 Find how many different size squares can be drawn on the grid shown.

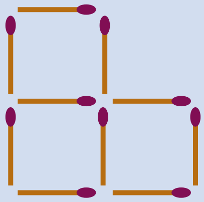


3 Move 3 matchsticks in the arrangement shown to create 3 squares that touch each other.





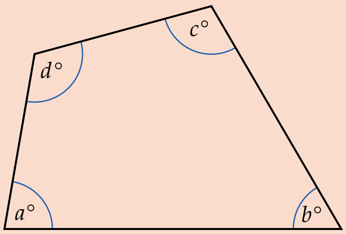
4 Remove 1 match and re-position 3 others to form 1 square and 2 parallelograms.



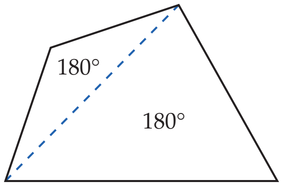
8.4 Angle sum in a quadrilateral

The angles in a quadrilateral add up to 360°.

$$a^\circ + b^\circ + c^\circ + d^\circ = 360^\circ$$

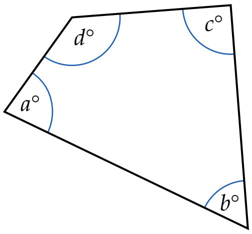


One way of showing this to be true is to draw a line joining two opposite vertices of the quadrilateral. No matter what type of quadrilateral you have drawn, this action will always divide the shape into two triangles. Since we already know that the angles in a triangle add up to 180° then the sum of the angles in a quadrilateral must be 180° + 180° = 360°.

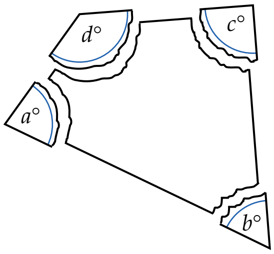


Another way of showing that the angles in a quadrilateral add up to 360° is shown here. You may wish to try it for yourself.

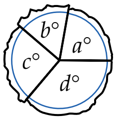
1. Draw any quadrilateral.



2. Cut the corners off.



3. Place the cut-off angles together.



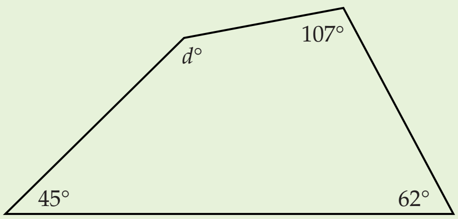
4. Notice that the angles form a complete circle, i.e. 360°.
5. Repeat steps 1 to 4 for other quadrilaterals if you are not convinced.

We can use the fact that the angles in a quadrilateral add up to 360° to find a missing angle in a quadrilateral without measuring if we know three other angles.

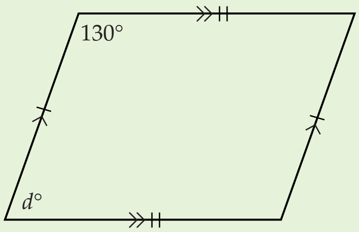
worked example 4

Find the size of the angle labelled $d°$ in each of the following.

(a)



(b)



Steps

(a) 1. Add the given angles.

Solutions

(a)

$$\begin{array}{r} 45° \\ 62° \\ +\ 107° \\ \hline 214° \end{array}$$

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2. Subtract the sum of the given angles from 360°.

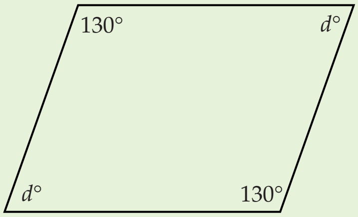
$$360° - 214° = 146°$$

3. Write the answer as shown.

$d° = 146°$

(b) 1. Recall that the opposite angles in a parallelogram are equal.

Add the known angles.



(b) $130° + 130° = 260°$

2. Subtract the sum found in step 1 from 360° to find the total of the unknown angles.

$$360° - 260° = 100°$$

3. Since the two unknown angles (i.e. the *d*s) are equal, divide the answer in step 2 by 2.

$$100° \div 2 = 50°$$

4. Write the answer as shown.

$d° = 50°$

exercise 8.4 Angle sum in a quadrilateral

Preparation: Prep Zone Q3, Ex 8.3

Interactive

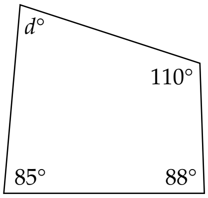
Worksheet C8.7

Hint

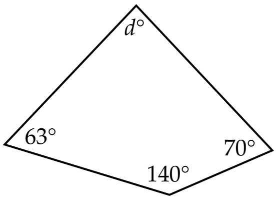
Core

1 Find the value of *d*° in each of the following quadrilaterals.

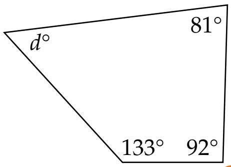
(a)



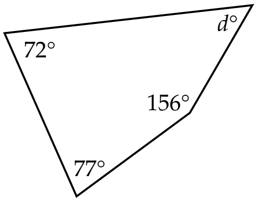
(b)



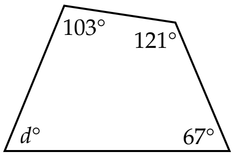
(c)



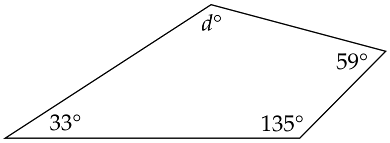
(d)



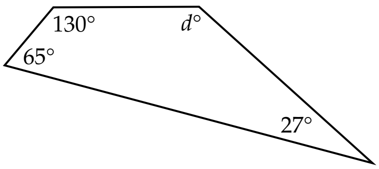
(e)



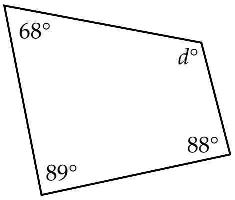
(f)



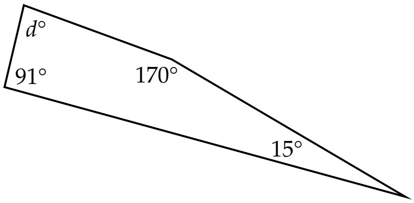
(g)



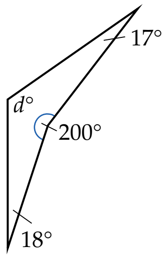
(h)



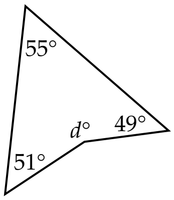
(i)



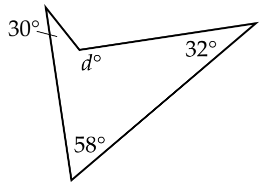
(j)



(k)

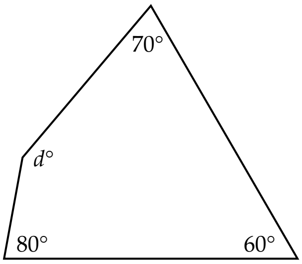


(l)



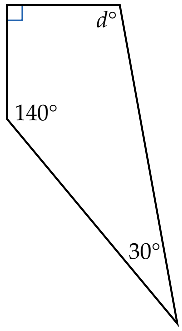
2 Choose the correct answer.

(a) The size of angle $d°$ shown is:

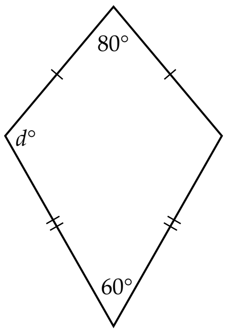


A 70° **B** 90°
C 150° **D** 210°

(b) The size of angle $d°$ shown is:
A 10°
B 100°
C 170°
D 260°



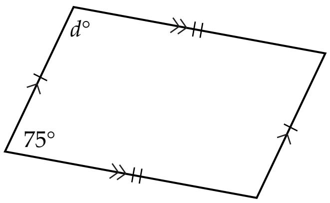
(c) The size of angle $d°$ shown is:
A 55° **B** 110°
C 140° **D** 220°



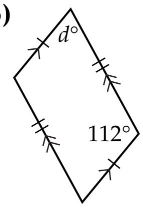
3 Find the value of $d°$ in each quadrilateral.

Hint

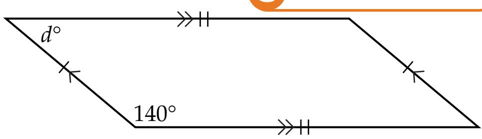
(a)



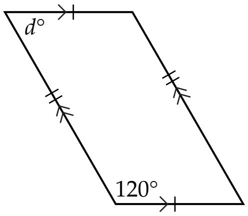
(b)



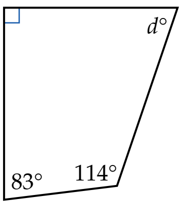
(c)



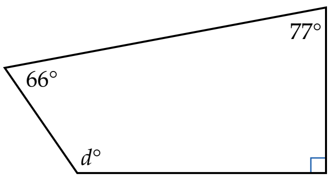
(d)



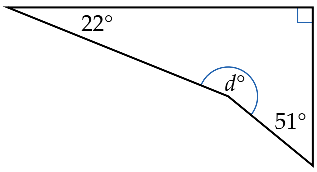
(e)



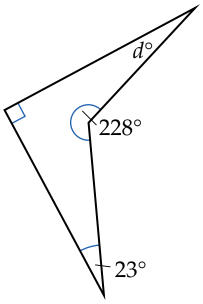
(f)



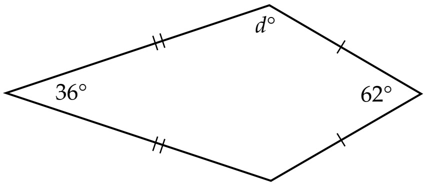
(g)



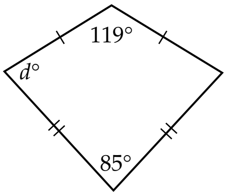
(h)



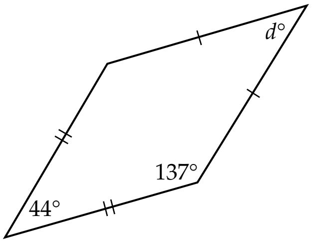
(i)



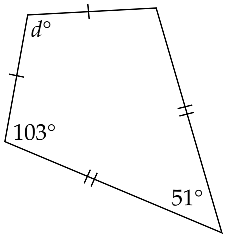
(j)

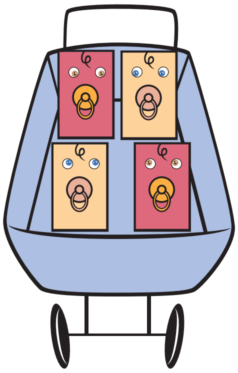


(k)



(l)



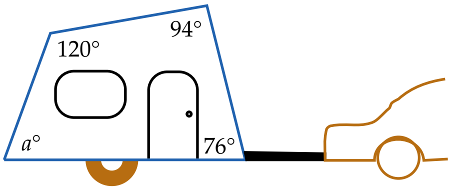


Quads

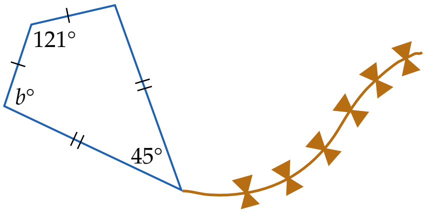
Extension

4 Find the angle represented by the pronumeral in each quadrilateral below.

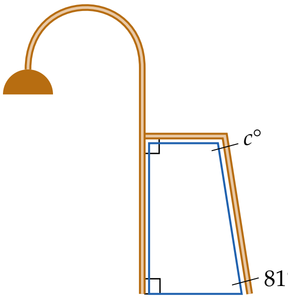
(a)



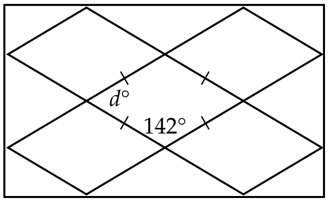
(b)



(c)



(d)



5 A quadrilateral has two right angles. Give possible values for the other two angles.

Worksheet C8.8

Homework 8.2

Worksheet A8.3

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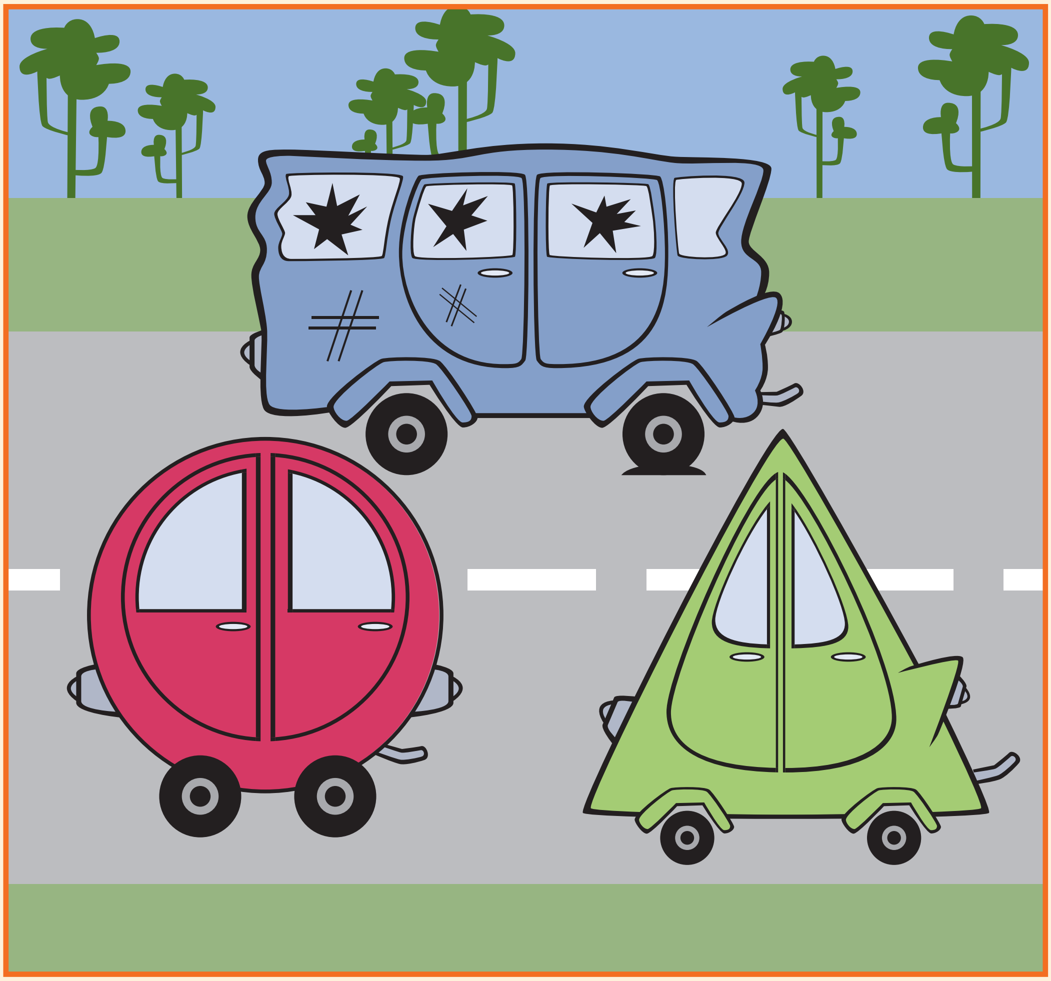
laughzone



Answer the following, showing your working, and then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

Given that each of the following are groups of angles in a triangle, find the value of *x*.

27°, 44°, $x°$	**E**	52°, 72°, $x°$	**N**	90°, $x°$, $x°$	**K**
44°, $x°$, $x°$	**W**	150°, $x°$, $x°$	**A**	$x°$, $x°$, $x°$	**L**

Given that each of the following are groups of angles in a quadrilateral, find the value of *x*.

108°, 95°, 69°, $x°$	**G**	97°, 112°, 68°, $x°$	**R**	90°, 78°, $x°$, $x°$	**C**
68°, 24°, $x°$, $x°$	**D**	54°, $x°$, $x°$, $x°$	**E**	$x°$, $x°$, $x°$, $x°$	**A**

‘

15°	68°	83°	109°	96°	45°	109°	134°	90°	56°	88°	60°	102°

’

RETURN TO MAIN MENU

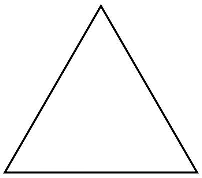
8.5 Polygons

The word **polygon** is made up of two Greek words—*poly* (meaning many) and *gon* (meaning angle), so a polygon is a many-angled shape. As you can see in the following diagrams of polygons, the number of sides in a polygon is equal to its number of angles.

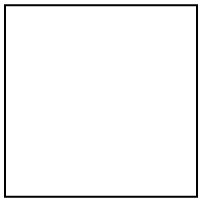
The following table shows the names given to the first eight polygons.

Number of sides	*Polygon name*
3	Triangle
4	Quadrilateral
5	**Pentagon**
6	**Hexagon**
7	**Heptagon**
8	**Octagon**
9	**Nonagon**
10	**Decagon**

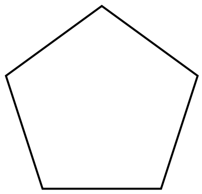
You are probably familiar with several of the **regular** polygons shown below. A regular polygon has all sides of equal length and all angles of equal size.



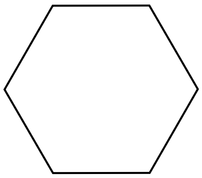
A regular triangle



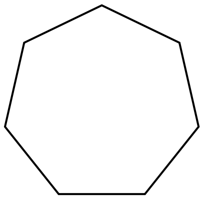
A regular quadrilateral



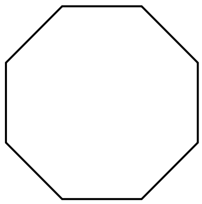
A regular pentagon



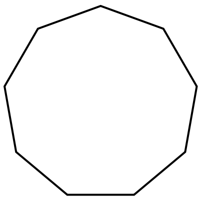
A regular hexagon



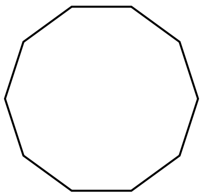
A regular heptagon



A regular octagon

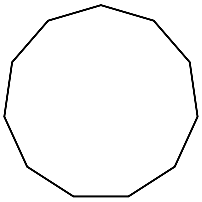


A regular nonagon

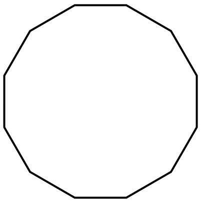


A regular decagon

Regular versions of two less well known polygons are shown below.

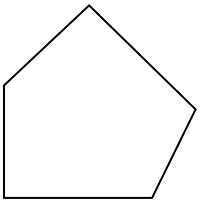


Undecagon—11 sides

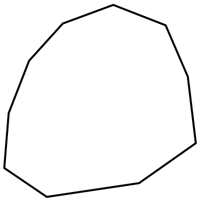


Dodecagon—12 sides

Polygons that have unequal sides are called **irregular polygons**.

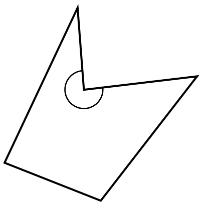


An irregular pentagon

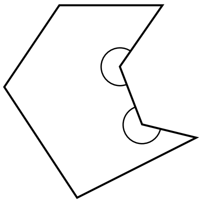


An irregular decagon

All of the polygons shown so far have been **convex** polygons, as they contained no angles greater than 180°. Below are examples of **concave** polygons that contain at least one internal angle greater than 180°.



A concave pentagon



A concave heptagon

eTutorial

exercise 8.5 Polygons

Preparation: Exs 8.1 and 8.3

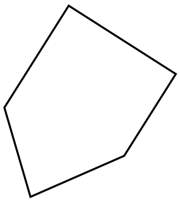
Core

Worksheet C8.9

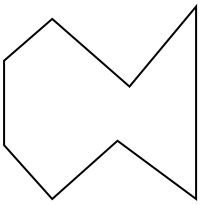
Hint

1 Name each of the polygons below. State whether each one is concave or convex.

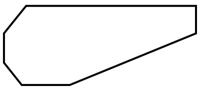
(a)

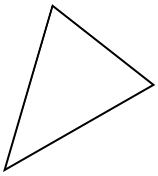
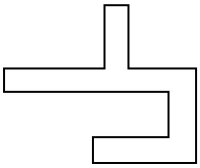
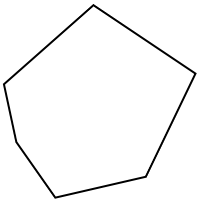


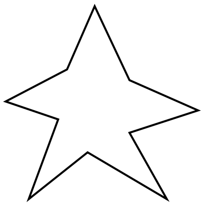
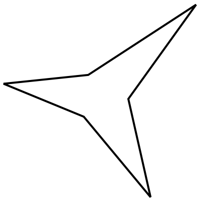
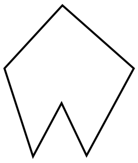
(b)

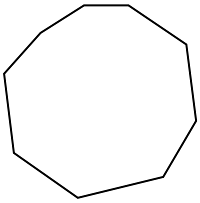
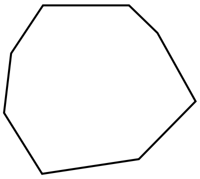
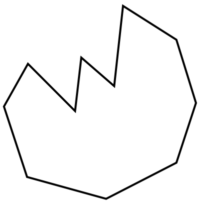


(c)



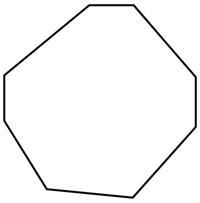
(d)

(e)

(f)


(g)

(h)

(i)


(j)

(k)

(l)


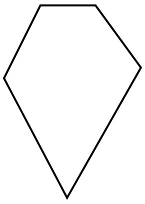
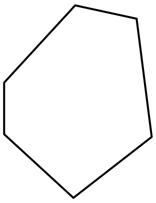
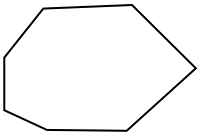
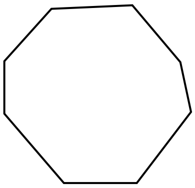
2 Choose the correct answer.

(a) The shape opposite is:



A a hexagon
B a heptagon
C an octagon
D a nonagon

(b) Which of the shapes below is a heptagon?

A

B

C

D


3 Using a ruler, draw any:

Hint

(a) convex hexagon **(b)** concave hexagon **(c)** convex quadrilateral
(d) concave octagon **(e)** convex dodecagon **(f)** concave undecagon

4 Name the type of polygon in each photograph.

(a)


(b)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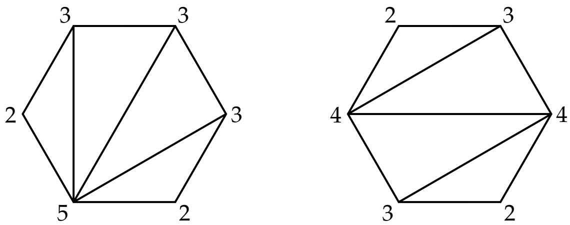
(c) 

(d) 

Extension

5 The diagrams below show how a hexagon may be divided into triangles two different ways, without any lines crossing.



(a) What do the numbers on each corner (vertex) represent?
(b) What do the numbers add up to in each case?
(c) Try dividing an octagon into triangles different ways and number each vertex using the same procedure. What do the numbers add up to?
(d) Write a sentence or two explaining what you have discovered about triangulated polygons.

eQuestions
Worksheet A8.4

8.6 Transformations and symmetry

A **transformation** takes place when a shape is moved or changed in size, according to a set rule. The four major types of transformations are **rotation**, **reflection**, **translation** and **dilation**.

Rotation

When a shape is rotated, the entire shape is moved through an arc of a circle for a given angle. It is necessary to know the location of the point about which the rotation is taking place.

For example:

M. rotated 90° clockwise about the bottom right corner becomes M.
(point of rotation)

eTutorial

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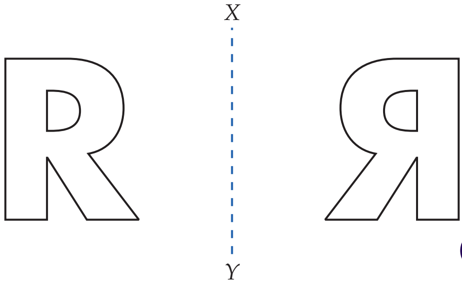
Reflection

When a shape is reflected we see the image that would occur in a mirror. It is necessary to know the line in which the shape is reflected.

For example, this shape is reflected in the line *XY*.

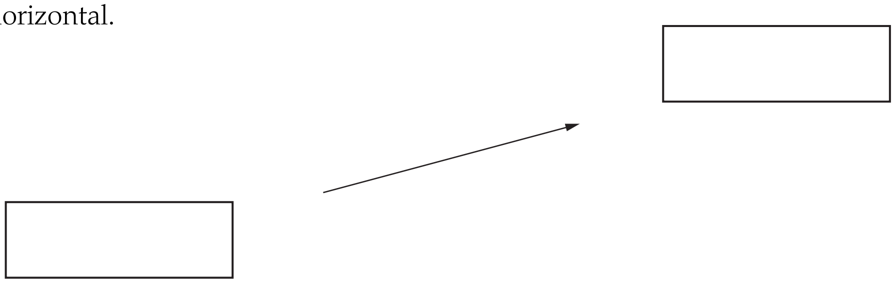


eTutorial
eTutorial

Translation

When a shape is translated the entire shape is moved a specified distance in a straight line in a given direction.

For example, this rectangle is translated 90 mm along a line at 15° to the horizontal.



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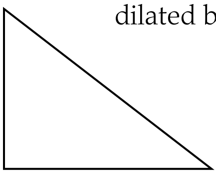
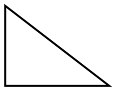
Dilation

When a shape is dilated it is made larger or smaller while retaining its original shape. The **scale factor** tells you how much larger or smaller the image is compared to the original.

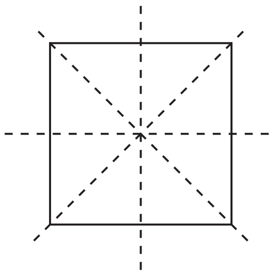
For example  dilated by a scale factor of $\frac{1}{2}$ becomes 

Line and rotational symmetry

Symmetry exists when a shape has corresponding identical parts. There are different kinds of symmetry.

A shape has **rotational symmetry** if it looks the same at least once during a revolution. A square has rotational symmetry because it looks the same four times during one rotation. We say a square has an **order of rotational symmetry** of 4.

Line symmetry exists when you can draw a line through a shape and the part of the shape on one side of the line will be a reflection of the part of the shape on the other side. Sometimes many lines of symmetry exist. A square has four lines of symmetry.



exercise 8.6 Transformations and symmetry

Preparation: Prep Zone Q1 and 2

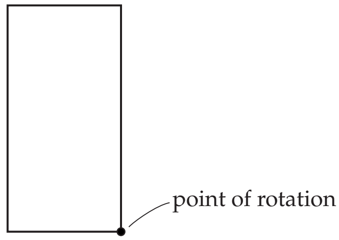
Core

1 Trace each of the following shapes and then rotate each through 90° clockwise about the given point.

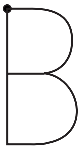
Interactive

Hint

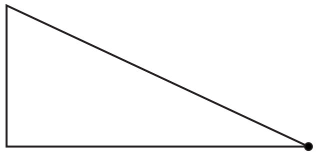
(a)



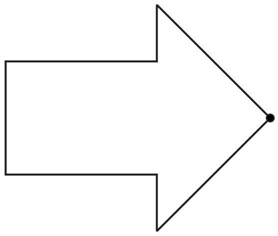
(b)



(c)



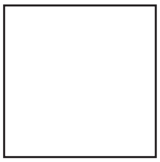
(d)



2 Copy each of the following shapes and then dilate each by a factor of **(i)** 2 **(ii)** $\frac{1}{2}$.

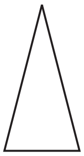
Hint

(a)

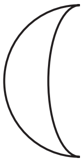


(b)

(c)



(d)



3 Trace the following shape and then translate it by the specified amounts.

(a) 50 mm horizontally left

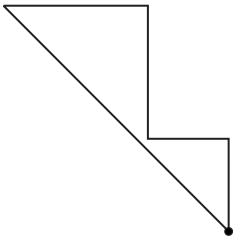
(b) 65 mm vertically down

(c) 45 mm along a line at 20° to the horizontal

(d) 50 mm along a line at 50° to the horizontal

(e) 60 mm along a line at 35° to the vertical

(f) 50 mm along a line at 60° to the vertical



Hint

eQuestions

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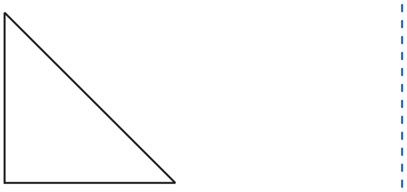
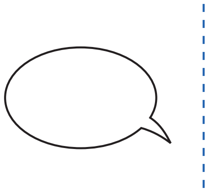
4 In each of the following the dotted line represents a mirror. Trace the diagrams exactly into your book and then draw the shape as it would appear after reflection in the mirror.

Hint

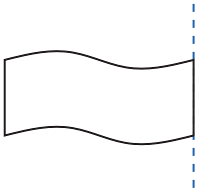
(a)



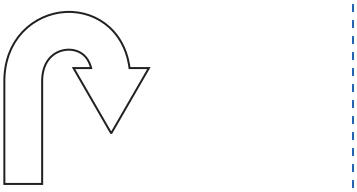
(b)



(c)



(d)



5 For each of the following shapes, write:

(i) the order of rotational symmetry

(ii) the number of lines of symmetry

eQuestions

(a) equilateral triangle **(b)** kite **(c)** rectangle

(d) isosceles triangle **(e)** rhombus **(f)** scalene triangle

(g) circle **(h)** parallelogram **(i)** trapezium

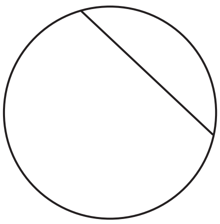
(j) square

6 For each of the following shapes, write:

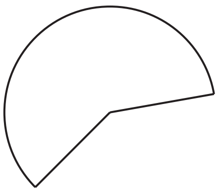
(i) the order of rotational symmetry

(ii) the number of lines of symmetry

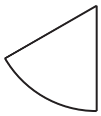
(a)



(b)



(c)



Extension

7 The letter A has one vertical line of symmetry.

(a) Write the other capital letters of the alphabet that have a vertical line of symmetry.

(b) Write the capital letters of the alphabet that have a horizontal line of symmetry.

8 Draw your own shapes (different to Question **5**) that have orders of rotational symmetry and numbers of lines of symmetry of:

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

Worksheet T8.1

eQuestions

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8.7 Constructions

It is not always necessary to have a protractor to be able to draw angles and shapes. A compass, pencil and ruler may be used very accurately. Some compass construction techniques are explained below. When making a construction you should always **(i)** use a grey lead pencil, not a pen and **(ii)** leave in your construction lines—don't erase them.

worked example 5

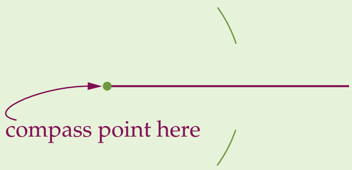
Construct the following angles.

(a) 90°   **(b)** 60°

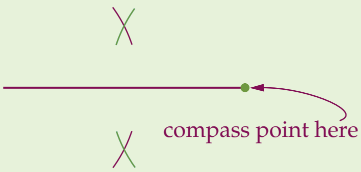
Steps / Solutions

(a) 1. Rule a base line any convenient length.

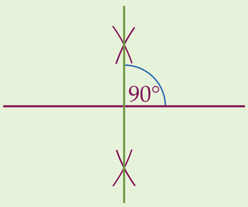
2. Place the compass point on one end of the line and open the compass up to reach a bit more than half the length of the line. Draw a small arc above and below the line as shown.



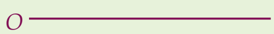
3. Keeping the compass at the same span, place the point on the other end of the line and draw two more small arcs which cut the first two.



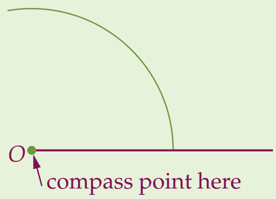
4. Join the crossed arcs as shown. There are now four 90° angles to choose from. (*Note:* The base line has been divided into two equal halves, or 'bisected', by this process. The vertical line is called a **perpendicular bisector**.) We use the notation $\perp$ to show when two lines are perpendicular. For example, $DE \perp LM$ means line *DE* is perpendicular to line *MN*.



(b) 1. Rule a base line any convenient length.



2. Place the compass point on one end of the line (point *O* opposite) and open the compass up to a convenient span. Draw a large arc as shown.



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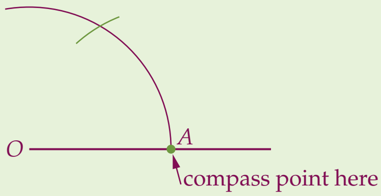
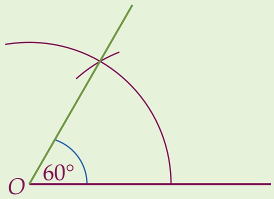
3. Keep the compass at the same span and, with the point on *A*, draw a small arc which cuts the large arc.

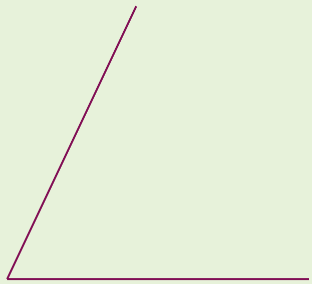


4. Join the intersection of the arcs to *O*.



worked example 6

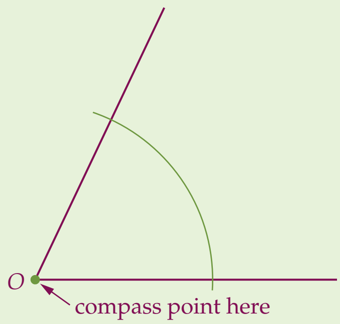
Bisect the angle shown.



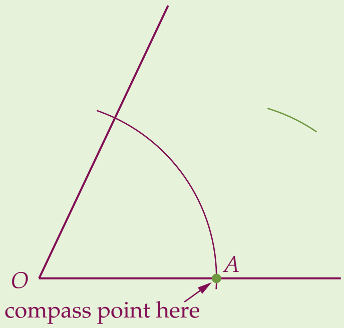
Steps

1. Trace the angle to be halved and draw a large arc centred on *O* as shown.

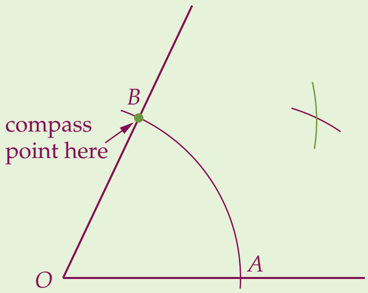
Solution



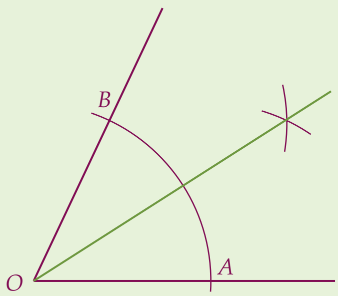
2. Place the compass point on *A* and draw a small arc.



3. Without changing the compass span, place the point on *B* and draw another small arc which cuts the first one.



4. Join the intersection of the arcs to *O* to bisect the angle.



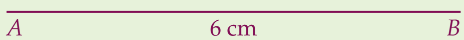
worked example 7

(a) Construct triangle *ABC*, where *AB* = 6 cm, *AC* = 5 cm and *BC* = 4 cm.
(b) Construct triangle *DEF*, where *DE* = 5 cm, *DF* = 4 cm and ∠*EDF* = 40°.
(c) Construct triangle *PQR*, where *PQ* = 4.5 cm, ∠*QPR* = 25° and ∠*RQP* = 30°.

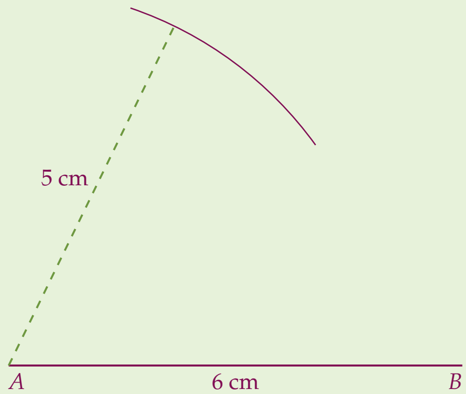
Steps

(a) 1. Draw one side (your base line) and label it.

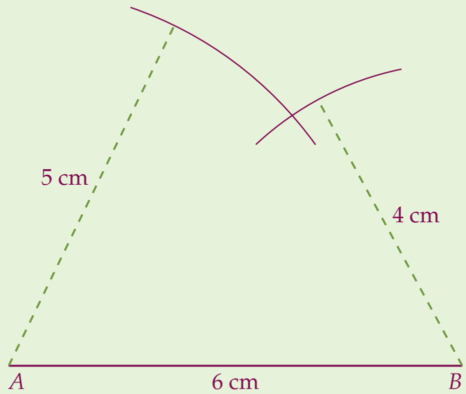
Solutions

(a) 

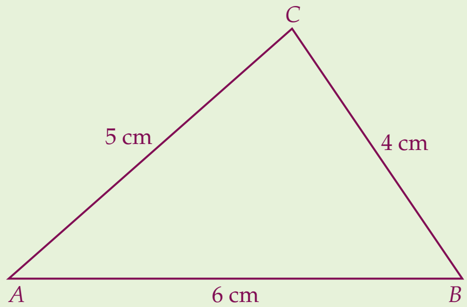
2. Open your compass to the length of another side, place your compass point at one end of the base line and draw an arc above the line.



3. Do the same for the third side with your compass point at the other end of the base line.

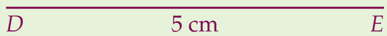


4. The point where the arcs intersect is the third vertex. Label this point and draw lines to each end of the base line.

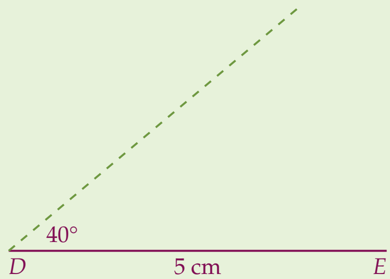


(b) 1. Draw one side and label it.

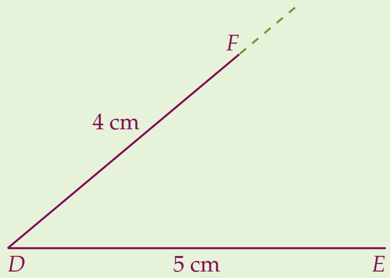
(b)



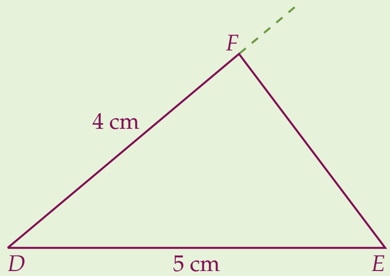
2. With your protractor mark the angle lightly.



3. Rule a line of the specified length in the direction of the angle marked. Label the new vertex.

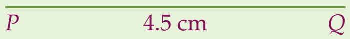


4. Connect the two remaining vertices.

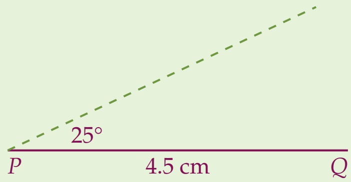


(c) 1. Draw one side and label it.

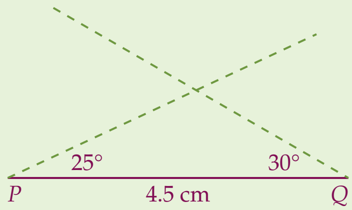
(c)



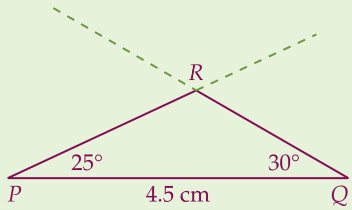
2. With your protractor mark one of the angles and lightly draw a line through it.



3. With your protractor mark the other angle and lightly draw a line through it.



4. Where the lines cross is the third vertex. Label it. Draw in the sides of the triangle.



The steps to follow when constructing quadrilaterals depend on the information given. It is useful to remember the following for constructing both triangles and quadrilaterals:

- When adding a side where the length is known, but the angle isn't, use a compass.
- When adding a side where the angle is known use a protractor.

> The notation $\perp$ means 'is perpendicular to'. So $AB \perp CD$ means line AB is perpendicular to line CD.
> The notation || means 'is parallel to'. So $WX || YZ$ means line WX is parallel to line YZ.

exercise 8.7 Constructions

Preparation: Ex 8.5

In this exercise, use *only* the equipment specified. No protractor!

Core

1 **(a)** Construct a 90° angle on a base line of length 8 cm.

Hint

(b) Construct a right angle on a base line of length 5 cm.

(c) Draw lines AB and DC so that $AB \perp DC$.

2 **(a)** Construct any 60° angle facing right like the one shown here. (The example is not drawn to scale.)

(b) Construct a 60° angle facing left.

3 **(a)** Construct a 90° angle and bisect it to form a 45° angle.

(b) Construct a 30° angle.

(c) Construct a 15° angle.

Hint

4 Accurately construct an angle of:

(a) 150° **(b)** 75°

5 Construct triangles that have the following side lengths.

(a) $AB = 5$ cm, $BC = 2$ cm, $AC = 4$ cm

(b) $XY = 6$ cm, $YZ = 5$ cm, $XZ = 5$ cm

(c) $PQ = 9$ cm, $QR = 6$ cm, $PR = 4$ cm

(d) $JK = 2.7$ cm, $KL = 4.5$ cm, $JL = 3.6$ cm

Hint

Use a protractor for the following questions.

6 Construct triangles with the following angles and side lengths.

(a) $MN = 6$ cm, $MP = 7$ cm, $\angle M = 45°$

(b) $CD = 4.2$ cm, $DE = 5$ cm, $\angle D = 52°$

(c) $JK = 5$ cm, $\angle KJL = 50°$, $\angle LKJ = 30°$

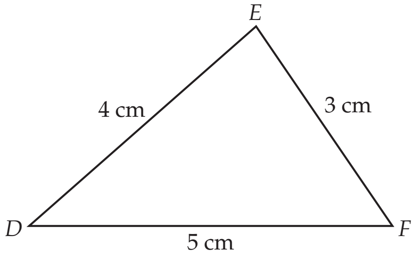
(d) $UV = 6.5$ cm, $\angle WUV = 35°$, $\angle WVU = 55°$

Hint

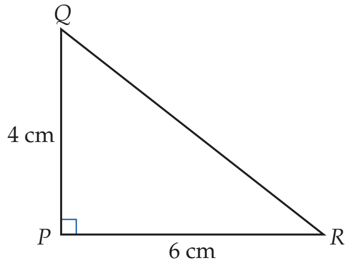
RETURN TO MAIN MENU

7 Make accurate, full-sized drawings of the following triangles.

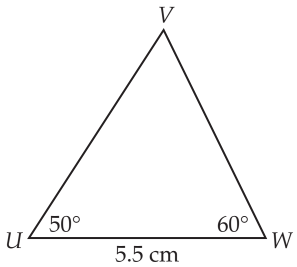
(a)



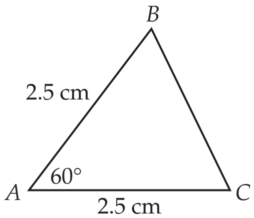
(b)



(c)



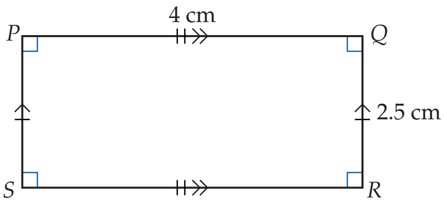
(d)



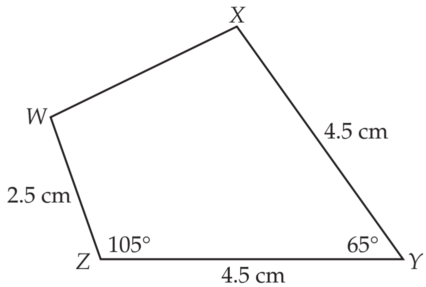
Extension

8 Make accurate, full-sized drawings of the following quadrilaterals.

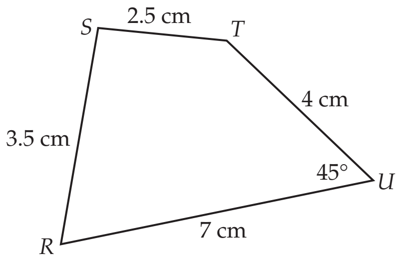
(a)



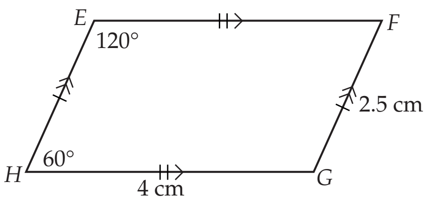
(b)



(c)



(d)



9 Answer TRUE or FALSE for each of the following.

(a) In Question **7(b)** $QR \perp PR$

(b) In Question **8(a)** $QR \perp PS$

(c) In Question **8(a)** $PQ \parallel PR$

(d) In Question **8(b)** $WZ \perp ZY$

(e) In Question **8(d)** $EH \parallel FG$

(f) In Question **8(d)** $HG \perp FG$

e Hint

10 Explain why you can't construct a triangle with side lengths $PQ = 6$ cm, $QR = 5$ cm, $RP = 14$ cm.

e Homework 8.3

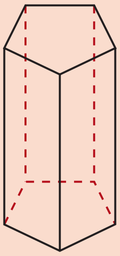
RETURN TO MAIN MENU

8.8 Geometrical solids

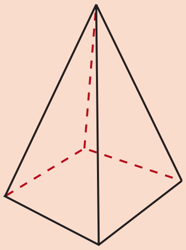
A three-dimensional (3D) shape is called a **solid**.

There are many different types of geometrical solids. Several are defined below.

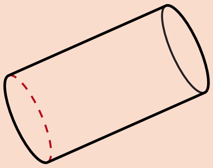
Prism A solid with two ends of the same shape and size joined by straight parallel sides. A prism has a uniform cross-section along its length or height. Prisms are named after the shape of the ends (e.g. a rectangular prism has rectangular ends).



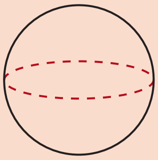
Pyramid A solid with a polygon base and sloping edges which all meet at a point above the base. Pyramids are named after the shape of the base (e.g. a square pyramid has a square base).



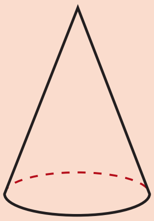
Cylinder A solid with two circular ends joined by a curved surface.



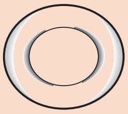
Sphere A ball-shaped solid in which every point on the surface is the same distance from the centre of the shape.



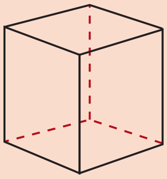
Cone A solid with a circular base joined by a curved surface to a point above the base.



Torus A donut-shaped ring like a cylinder that has been bent so that both ends join.

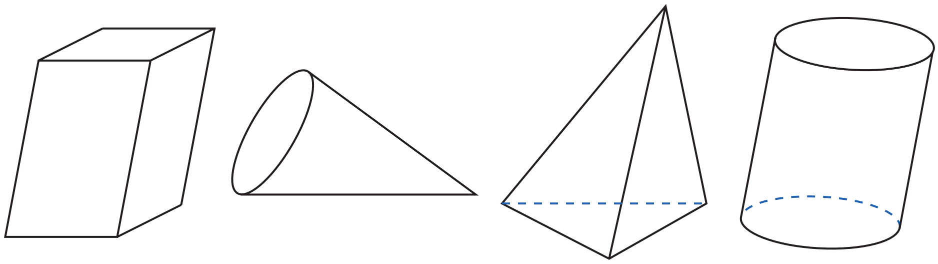


Cube A square prism whose length, breadth and height are equal in size.



Prisms and pyramids, and cylinders and cones, can be classified further as either **right** or **oblique**. Drop a line from the middle of the top face or point to the middle of the base. If the line meets the base at right angles the solid is right; if the line meets the base at an angle then it is oblique. An oblique solid looks like it is leaning over.

Below are some oblique solids.

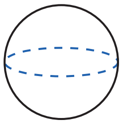


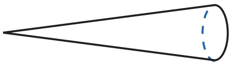
exercise 8.8 Geometrical solids

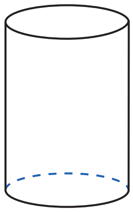
Preparation: Ex 8.5

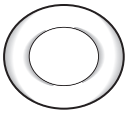
Core

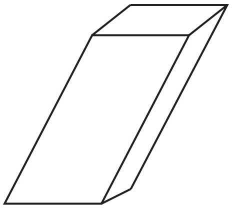
1 Use the definitions above to identify each of the following solids.
For each prism or pyramid, state if it is right or oblique.

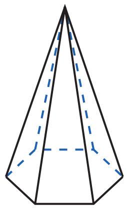
(a) 

(b) 

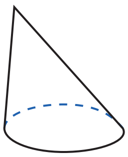
(c) 

(d) 

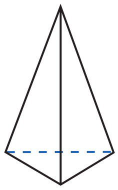
(e) 

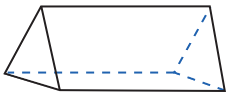
(f) 

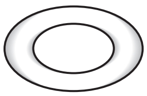
Hint

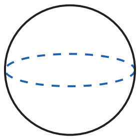
(g) 

(h) 

(i) 

(j) 

(k) 

(l) 

2 Which of the solids in Question **1** have a uniform cross-section?

3 For each of the solids in Question **1**, state if it has any parallel flat faces.
If so describe the shape of these faces.

4 Draw an example of each of the following.

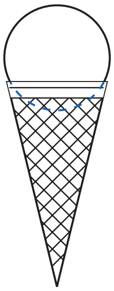
Hint

(a) a pentagonal prism
(b) an oblique hexagonal prism
(c) a hexagonal pyramid
(d) a pentagonal pyramid
(e) a torus
(f) an oblique cone
(g) a sphere
(h) a cylinder

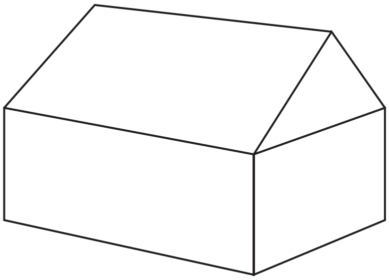
Extension

5 Name the geometrical solids you can find in the objects below.

(a)



(b)



(c)

(d)

(e)

(f)

(g)

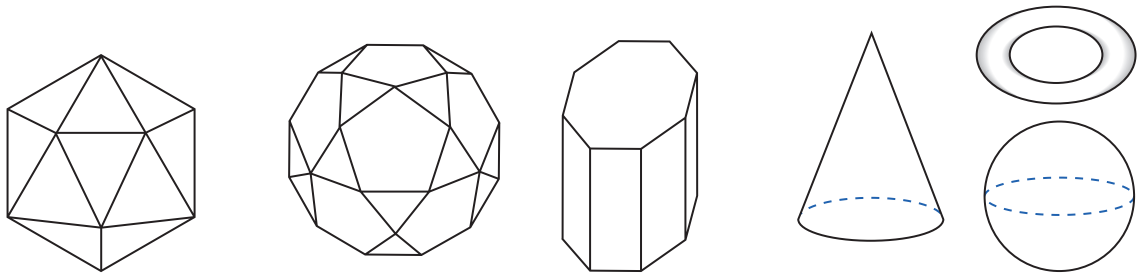
(h)

Extension

6 Give the names of the geometrical solids with the following properties.
- **(a)** a circular base and an apex
- **(b)** a uniform polygonal cross-section
- **(c)** all points on its surface at a fixed distance from its centre
- **(d)** a polygonal base and one further vertex
- **(e)** a uniform circular cross-section.

8.9 Polyhedra

A **polyhedron** is a solid whose faces are polygons. (The plural of polyhedron is polyhedra.) Prisms and pyramids are types of polyhedra.
A regular polyhedron is one whose faces are all the same shape and size (we say its faces are 'congruent').



A regular polyhedron | These polyhedra are not regular | These are not polyhedra

Several types of polyhedra are shown in the following exercise.

eTutorial

exercise 8.9 Polyhedra

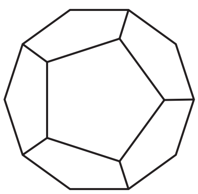
Preparation: Exs 8.5 and 8.8

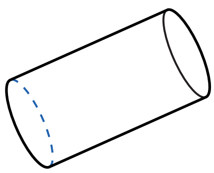
hi.com.au

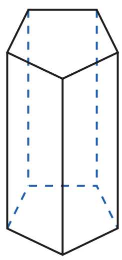
Core

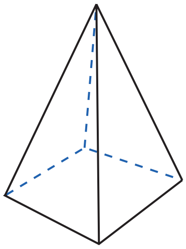
Hint

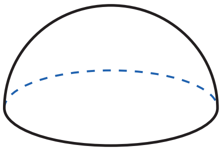
1 State whether each of the following is a polyhedron (P) or not (N).

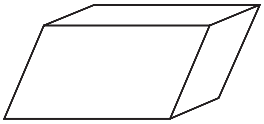
(a)


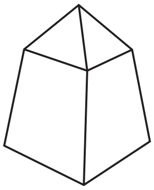
(b)


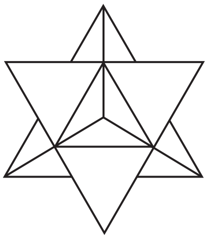
(c)


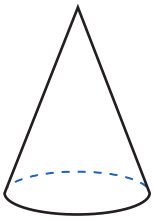
(d)


(e)


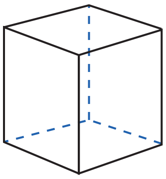
(f)


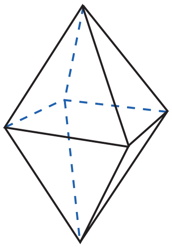
(g)


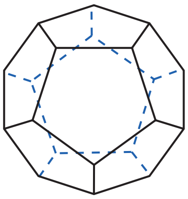
(h)


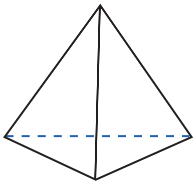
(i)


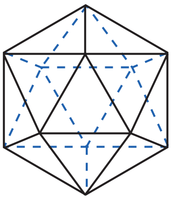
2 There are only five different regular polyhedra. They are known as Platonic solids (see Investigation on page 319). Match each description with one of the diagrams below.

A


B


C


D


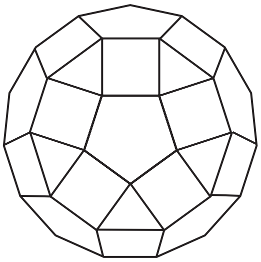
E


(a) **Tetrahedron:** A polyhedron with 4 congruent faces.
(b) **Hexahedron:** A polyhedron with 6 congruent faces.
(c) **Octahedron:** A polyhedron with 8 congruent faces.
(d) **Dodecahedron:** A polyhedron with 12 congruent faces.
(e) **Icosahedron:** A polyhedron with 20 congruent faces.

Extension

3 What is another name for a hexahedron?

4 Name the different shapes that appear as faces of the rhombicosidodecahedron (can you pronounce this?) shown here.

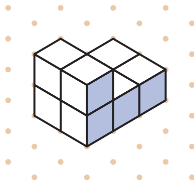


eQuestions

8.10 Drawing and visualising 3D shapes

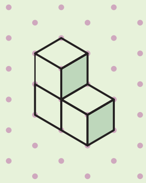
Most people find drawing solids difficult. This is because solids are 3D, i.e. they have height, length and breadth, and we are trying to draw them on paper, which is only 2D.

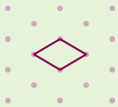
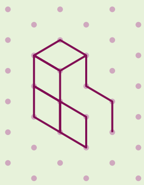
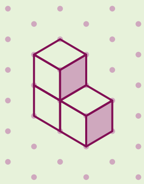
One method of overcoming this difficulty is to use triangular dot paper, sometimes called **isometric** paper.



worked example 8

Use triangular dot paper to copy the following shape.



Steps	Solution
1. Draw one complete horizontal face.	
2. Draw in lines gradually, building from the horizontal face.	
3. To create the illusion of 3D, choose one direction (front, side or top) and shade in all faces you would see from that direction.	

worked example 9

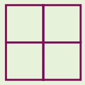
Draw the top, side and front views of the solid at the start of section 8.10. We will take the front as being on the left side of the shape.

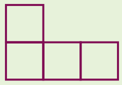
RETURN TO MAIN MENU

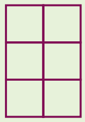
Steps

1. Consider each position separately. Imagine you are standing in front, or to the left side, of the object and just draw this side.
2. Move to the right-hand side of the object and draw what you see.
3. Finally, look at the object from directly above and draw what you see. Make sure the front is at the bottom of the diagram and the back is at the top.

Solution

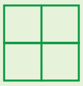

Front

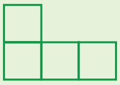

Side

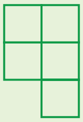

Top

worked example 10

Build the shape represented by these three elevations. Then draw it on isometric dot paper.


Front

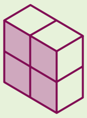

Side

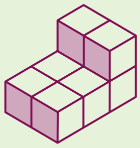

Top

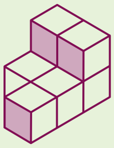
Steps

1. Begin from the front and assume the shape is one layer deep.
2. From the side it is clear that the shape is, in fact, three layers deep with the top blocks at the back. It is okay to use too many blocks and then remove them later.
3. It is only when looking at the top view that we can see that the base must have five cubes. The shape is now complete.

Solution







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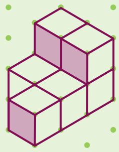
4. Position the shape so that your view of it is similar to the view you will sketch on isometric paper. Follow the steps in worked example 8 to copy the shape onto isometric paper.



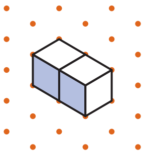
eTutorial

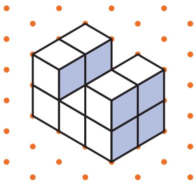
exercise 8.10 Drawing and visualising 3D shapes

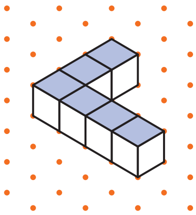
Preparation: Ex 8.9

Core

1 Use triangular dot paper to copy the following shapes.

(a) 

(b) 

(c) 

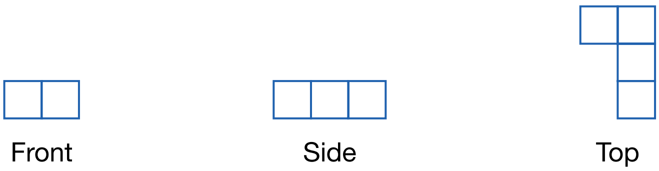
Worksheet A8.5

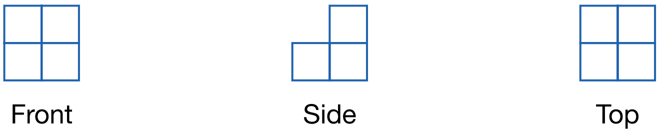
Hint

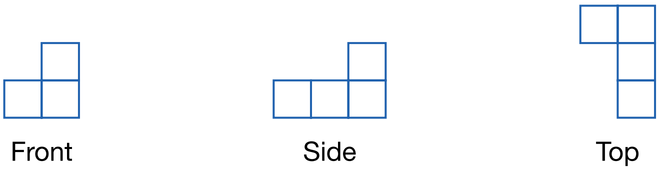
2 Draw the front, side and top views of each of the solids in Question **1**.

3 Use blocks to build each of the solids in Question **1**.

4 Build, then draw on isometric dot paper, the shape represented by each of these elevations.

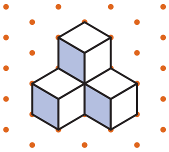
(a) 

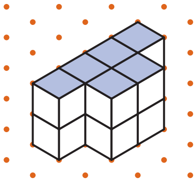
(b) 

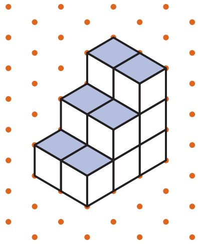
(c) 

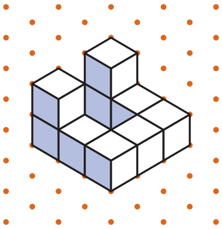
5 How many cubes would be required to build these solids? (Assume that there are no cubes missing at the back of the solids where you cannot see.)

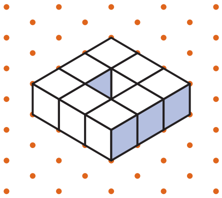
Hint

(a) 

(b) 

(c) 

(d) 

(e) 

Extension

6 Using the triangular dot paper, draw a 3D sketch of a Rubik's cube. (A Rubik's cube is made of 27 cubes stacked three layers high, three layers wide and three layers deep.)

eQuestions

Homework 8.4

Working mathematically

investigation

eTutorial

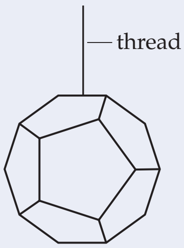
eQuestions

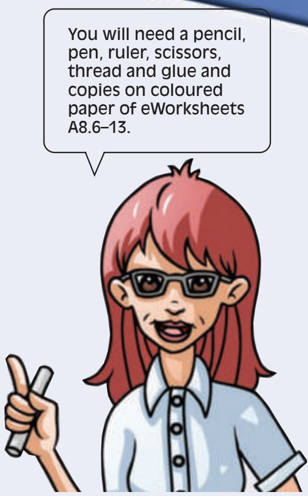
Euler's rule

The **nets** of eight solids are provided on eWorksheets A8.6–13. A net is a 2D plan used to make a 3D shape. You should personally construct the tetrahedron and two other solids. In order to complete the Investigation and derive **Euler's rule**, you will need to be able to see all eight solids. Organise with a group of others in your class so that the group constructs all eight solids.

These hints may be useful:

1. Cut out each net, including the tabs, and then crease along the fold lines so that a sharp line is formed.
2. Fold the nets so the marked fold lines are on the inside of the model.
3. Insert a length of thread in your models before glueing the last tab. The thread may be used for hanging the models in a display later on.



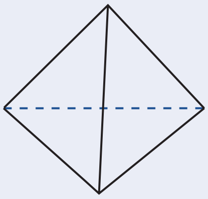


RETURN TO MAIN MENU

Tetrahedron

Worksheet A8.6

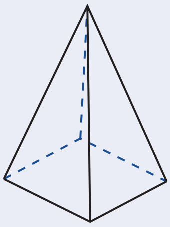
'Tetra' means 4.
A tetrahedron is a solid with 4 faces.
A tetrahedron is a triangular pyramid.



Square pyramid

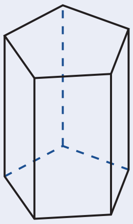
Worksheet A8.7

There are several famous square pyramids in Egypt.



Pentagonal prism

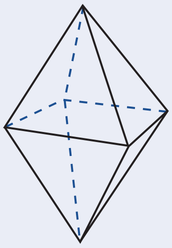
Worksheet A8.8



Octahedron

Worksheet A8.9

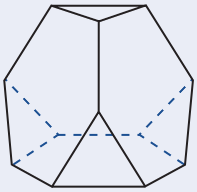
How many faces does this solid have?



Truncated tetrahedron

Worksheet A8.10

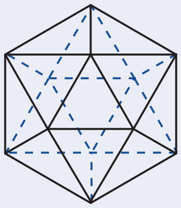
Truncated means 'cut-off'. Can you see how this solid looks like a tetrahedron with pieces cut off?



Icosahedron

Worksheet A8.11

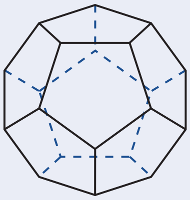
'Icosa' means 20.
An icosahedron has 20 faces.



Dodecahedron

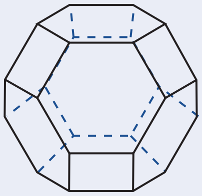
Worksheet A8.12

Can you remember what 'dodeca' means?



Truncated octahedron

Worksheet A8.13



We will now use the solids constructed to derive Euler's rule. Leonhard Euler (pronounced *Oiler*) was a brilliant Swiss mathematician of the 18th century. Before we proceed we need to remind ourselves of some mathematical terminology.

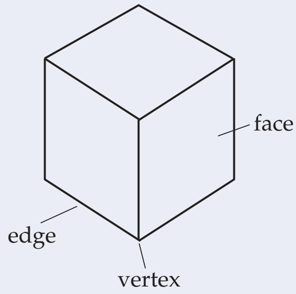
RETURN TO MAIN MENU

Face was the term used earlier to describe a flat part of the surface of a geometrical solid.

Two other terms used when describing such solids are:

Edge The line segment where two faces meet.

Vertex A corner where several edges and faces meet. The plural of vertex is 'vertices'.



The cube shown in the diagram has 6 faces, 8 vertices and 12 edges. Make sure you can count each of these features so that you can fill in the table below correctly.

1 Copy the following table and complete as many rows as you can using your models and the models constructed by other members of your group.

Solid	*Number of faces F*	*Number of vertices V*	*Number of edges E*	*F + V − E*
Tetrahedron				
Square pyramid				
Pentagonal prism				
Octahedron				
Truncated tetrahedron				
Icosahedron				
Dodecahedron				
Truncated octahedron				

2 What can you say about *F*, *V* and *E* for geometric solids?

This relationship is known as Euler's rule and applies to every solid.

3 A geometric solid has 40 faces and 50 vertices. How many edges does it have?

4 William claims he made a mathematical solid that contained 17 faces, 21 vertices and 33 edges. Could such a solid be made?

5 Investigate the history of Platonic solids. Write a paragraph on your findings.

Worksheet A8.14

speedingzone

Do these in your head as quickly as you can and write down the answers.

Time target: 2 minutes

1 8×35

2 $-108 + 48$

3 40^2

4 $3 + 7 \times 50$

5 $81\,000 \div 90$

6 < or >: 1.0972 __ 1.2007

7 52c + 68c + 75c − $1

8 $\frac{7}{8} + 4\frac{2}{8}$

9 Reece types at a speed of 58 words per minute. How many words can he type in 30 seconds?

10 What change is left over after 6 pens at 75 cents each are purchased with a $5 note?

RETURN TO MAIN MENU

maths in action

Putting painting in perspective

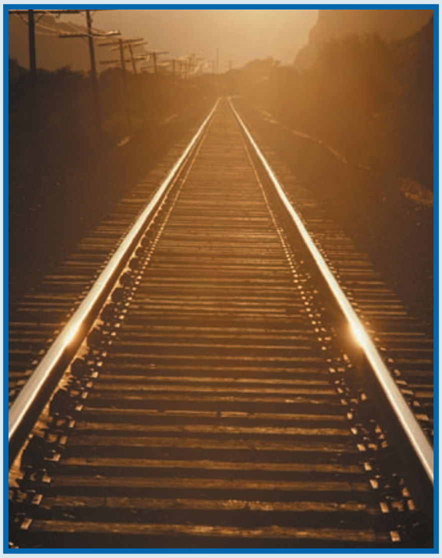
Tommaso Masaccio, *St. Peter Distributing the Common Goods of the Church and the Death of Ananias*, c. 1427

A period of history known as the Renaissance began in Italy during the fourteenth century.

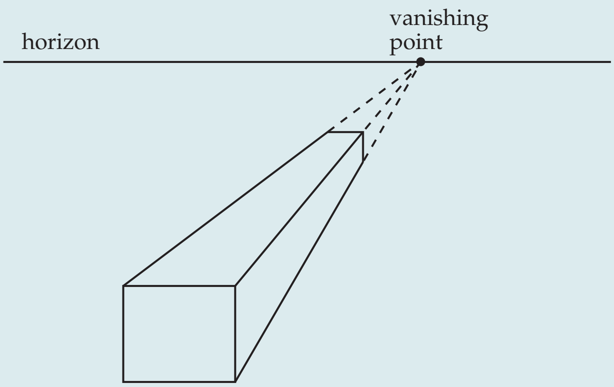
In art, the biggest distinction between the Renaissance and preceding periods was the use of linear perspective. This made it possible to represent 3D objects in a convincing fashion. Prior to this, 3D objects had appeared flat and the most important figure in the work was made more prominent than any other figure.

Perspective is an illusion. A painting is done on a flat 2D canvas, but usually represents a 3D scene.

Maths can directly influence how we see things. We know that a straight stretch of railway track has parallel rails, otherwise the train would fall off the track. But we also know that if we look down such a stretch of track it will appear as if the rails get closer and closer to each other. The point at which they appear to meet is called the 'vanishing point'. This is simply linear perspective in action.



In using linear perspective, the artist draws a horizontal line to represent the horizon and chooses a point on that line to represent the vanishing point. However, the lines do not necessarily go all the way to the vanishing point, as shown in the diagram below.



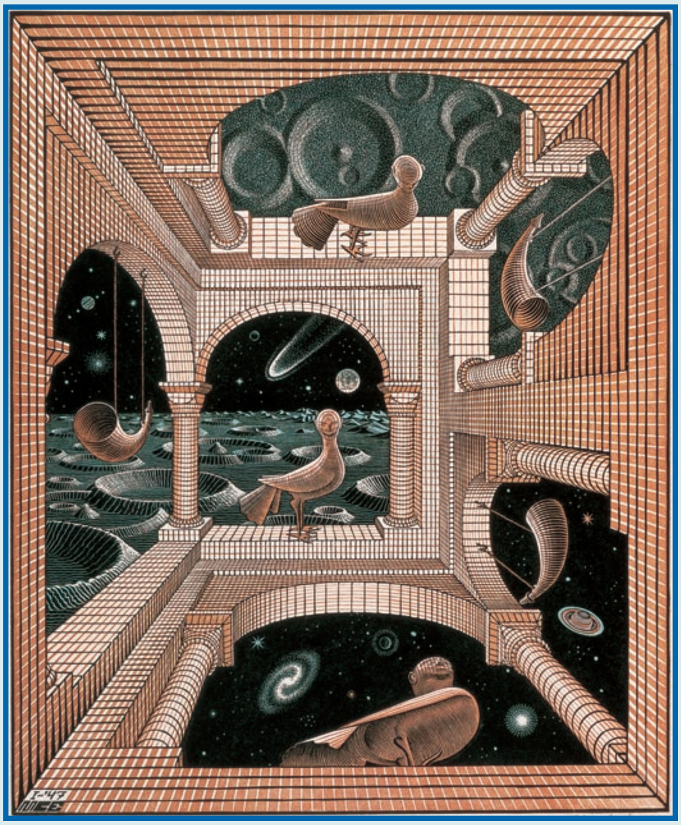
Remember, this drawing is still 2D, but it appears 3D. The mind has been tricked by the use of perspective.

Questions

1 Look at the painting by Masaccio. Describe how perspective has been used.

2 Draw the Rubik's cube with three different vanishing points: one to the left of the cube, one to the right of the cube, and one directly behind the cube.

3 The Dutch artist Maurits Escher played with the ideas of perspective. Describe what you see in *Another World*, the picture shown.



Research

e hi.com.au

Tommaso Masaccio was only one of the many Renaissance artists. Prepare a report on Renaissance art from a mathematical point of view. You should mention the following individuals: Donatello, Ghiberti, Brunelleschi, Alberti, da Vinci, Dürer.

RETURN TO MAIN MENU

languagezone

Summary

Copy and complete the following summary of this chapter using the words and phrases from the list. A word or phrase may be used more than once.

1 A t_____ with all three sides and angles equal is called ______.

2 A q_____ with only opposite sides equal and parallel is either a ______ or a ______.

3 A cube is a prism with a ______ for each face.

4 A quadrilateral with two pairs of a_____ sides equal is called a ______.

5 A donut-shaped solid is called a ______.

6 A dilation is a kind of transformation where it is necessary to know the ______ ______.

7 A cube has 6 f_____, 8 v______ and 12 e______.

Key words

acute-angled
adjacent
concave
cone
convex
cube
cylinder
decagon
dilation
dodecagon
dodecahedron
edges
equilateral
Euler's rule
faces
heptagon
hexagon
hexahedron
icosahedron
irregular polygon
isometric
isosceles
kite
line symmetry
nets
nonagon
oblique
obtuse-angled
octagon
octahedron
order of rotational symmetry
parallelogram
pentagon
perpendicular bisector
plane shape
polygon
polyhedron
prism
pyramid
quadrilateral
rectangle
reflection
regular
rhombus
right
right-angled
rotation
rotational symmetry
scale factor
scalene
solid
sphere
square
symmetry
tetrahedron
torus
transformations
translation
trapezium
triangle
undecagon
vertices

Questions

1 Is it possible to have a reflex-angled triangle? Explain.

2 'Quad' means four. Write two other words that start with 'quad' and their meanings.

3 List the plane shapes from the list in order from the 12-sided shape down to the three-sided shape.

4 Explain, in words, the difference between concave and convex polygons.

5 Are all parallelograms rectangles? Are all rectangles parallelograms? Explain.

6 Suggest why most tinned foods are packed into cylinders, and why carboard boxes are rectangular prisms. What are the advantages and disadvantages of these shapes?

7 List all the words you can find in 'parallelogram' without rearranging the letters. For example 'log' is one word.

8 Write the plurals of each of the following words:
vertex polyhedron prism

e Worksheet L8.1

e Worksheet L8.2

RETURN TO MAIN MENU

chapter 8 REVIEW

FAQs

Is a cylinder a prism?
No. A cylinder is similar to a prism because its size doesn't change all the way along. However, prisms have polygons for ends, and flat sides.

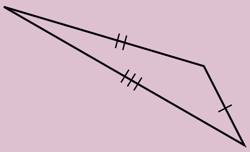
In a polygon, if a side is marked with one dash and another side marked with two dashes, does this mean the side with two dashes is longer than the side with one dash?
No. The dashes are only to show which sides are of *equal* length. They say nothing about the actual length of the side.

Is a square also a rectangle?
Yes. To be a rectangle, a quadrilateral must have opposite sides equal and all angles must be right angles. All squares have opposite sides equal and all angles are right angles, so all squares are rectangles. Not all rectangles are squares because not all rectangles have all sides of equal length.

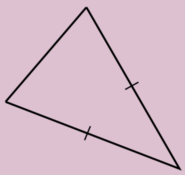
Core

1 Give the side and angle name for each of the following triangles. Use a ruler or protractor to make measurements if necessary. **8.1**

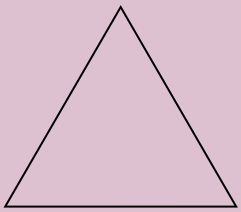
(a)



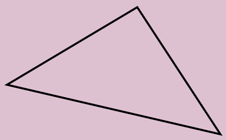
(b)



(c)

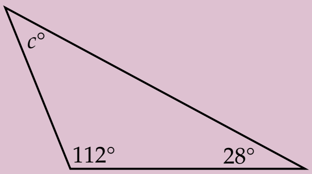


(d)

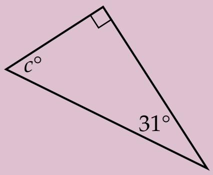


2 Find the value of angle $c°$ in each case. **8.2**

(a)



(b)



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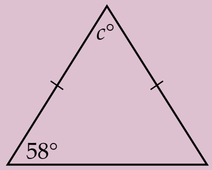
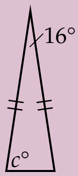
(c)

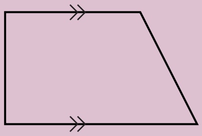


(d)

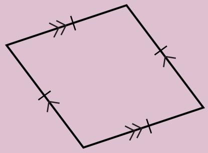


3 Name the following quadrilaterals. **8.3**

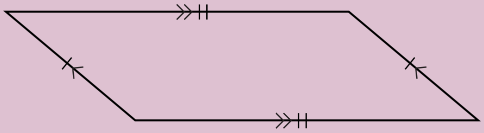
(a)



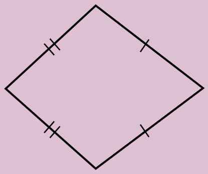
(b)



(c)

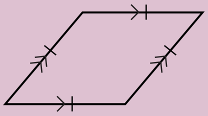


(d)



4 Choose the correct answer. **8.3**

This shape is most precisely named:

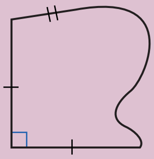


A a quadrilateral **B** a square

C a parallelogram **D** a rhombus

5 Choose the correct answer. **8.3**

The best definition for this shape would be:



A three straight and one wavy side

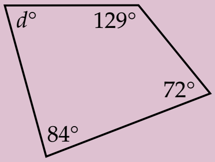
B a quadrilateral

C a four-sided figure with a right angle

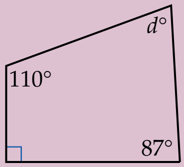
D three straight sides, two of which are the same length, one right angle between the equal sides, and a wavy side

6 Find the size of angle $d°$ in each of the following. **8.4**

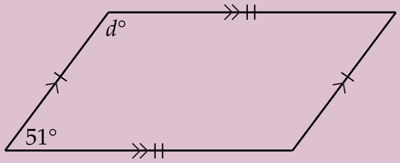
(a)



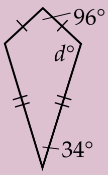
(b)



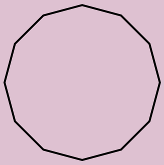
(c)

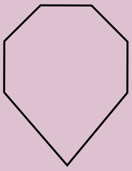


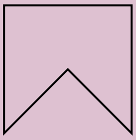
(d)

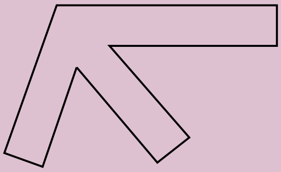


7 Name each polygon, and state whether each is convex or concave. **8.5**

(a) 

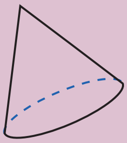
(b) 

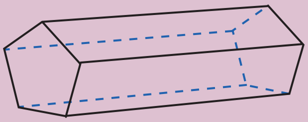
(c) 

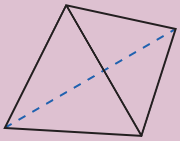
(d) 

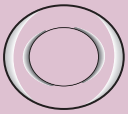
8 Without using a protractor, construct an angle of 30°. **8.7**

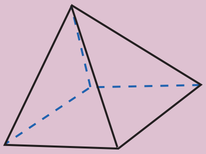
9 Name the following solids. **8.8**

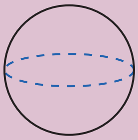
(a) 

(b) 

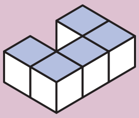
(c) 

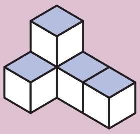
(d) 

(e) 

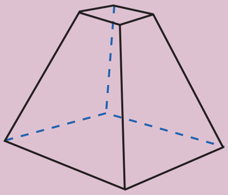
(f) 

10 Use triangular dot paper to copy the following solids. **8.10**

(a) 

(b) 

11 A **frustum** is a pyramid with the top cut off. Choose the correct answer. Investigation, p. 319



(a) The number of faces is:

A 5   **B** 6

C 4   **D** 12

(b) The number of vertices is:

A 12   **B** 6

C 4   **D** 8

12 A particular polyhedron has 8 faces and 12 vertices. How many edges does it have? Investigation, p. 319

Extension

13 Draw an example of each of the following, complete with correct side markings. 8.3

(a) parallelogram **(b)** kite

(c) square **(d)** irregular quadrilateral

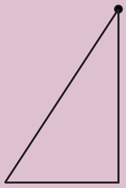
14 Draw an example of each of the following. 8.5

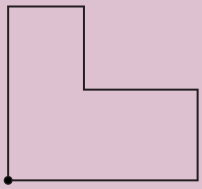
(a) a convex quadrilateral **(b)** a concave pentagon

(c) a convex hexagon **(d)** a convex heptagon

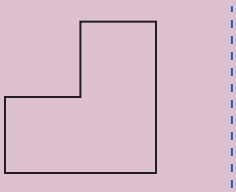
15 Using a compass, protractor and ruler, redraw the following shapes after a clockwise rotation of 90° about the point shown. 8.6

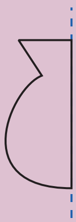
(a)

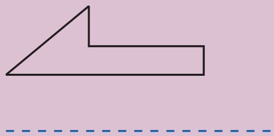

(b)


(c)


16 Redraw the following shapes after a reflection in the dotted line. 8.6

(a)


(b)


(c)


17 Construct the following shapes using only a protractor, pencil and ruler. 8.7

(a) a square **(b)** a regular pentagon

REPLAY

1 Simplify: 1.6

(a) $44 - 8 \times 4$ **(b)** $8 \times 6 \div 3 \times 4$ **(c)** $28 \div 4 + 3 \times 6$

2 Calculate: 2.5

(a) $19 - 28$ **(b)** $^-78 - 34$ **(c)** $^-108 + 100 - 22$

3 Calculate: 2.7

(a) $56 \div {}^-7$ **(b)** $\frac{^-108}{^-12}$ **(c)** $^-99 \div 9$

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4 List the first four multiples of each of the following numbers. 3.1
(a) 13 **(b)** 21 **(c)** 25

5 Simplify: 3.8
(a) $2^4 - 3^2$ **(b)** $3^3 \times 10^2$ **(c)** $10^5 - 10^3$

6 Use the following rules to complete the tables. 4.3
(a) $n = 3m$ **(b)** $t = s - 12$ **(c)** $q = 2p + 11$

m	n
4	
0	
-11	
25	
15	

s	t
8	
-100	
12	
2	
312	

p	q
15	
50	
-11	
0	
-20	

7 Draw angles of the following sizes. 5.3
(a) 22° **(b)** 195° **(c)** 320°

8 Find the supplementary angles for each of the following. 5.5
(a) 35° **(b)** 127° **(c)** 90°

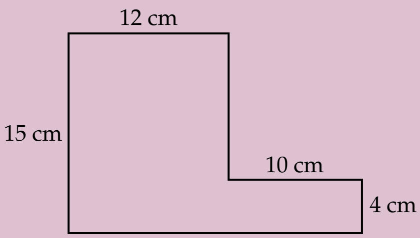
9 Set out the following and add. 6.4
(a) 2.6 + 3.05 **(b)** 0.0954 + 1.294 + 3 **(c)** 15.006 + 2.45 + 0.059

10 Calculate: 6.8
(a) 0.2×0.036 **(b)** 0.29×0.8 **(c)** 0.597×0.3

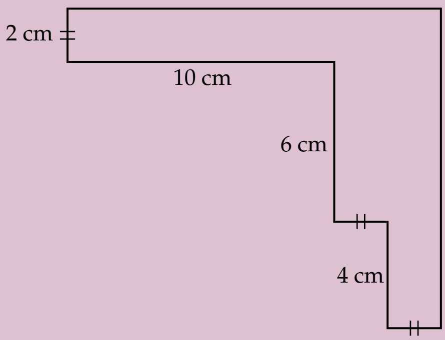
11 Copy and complete the following conversions. 7.2
(a) 7.2 cm = ___ mm **(b)** 320 cm = ___ m **(c)** 57 m = ___ km

12 Find the area of each of the following shapes. (All angles are right angles.) 7.6

(a)



(b)



Assignment 8