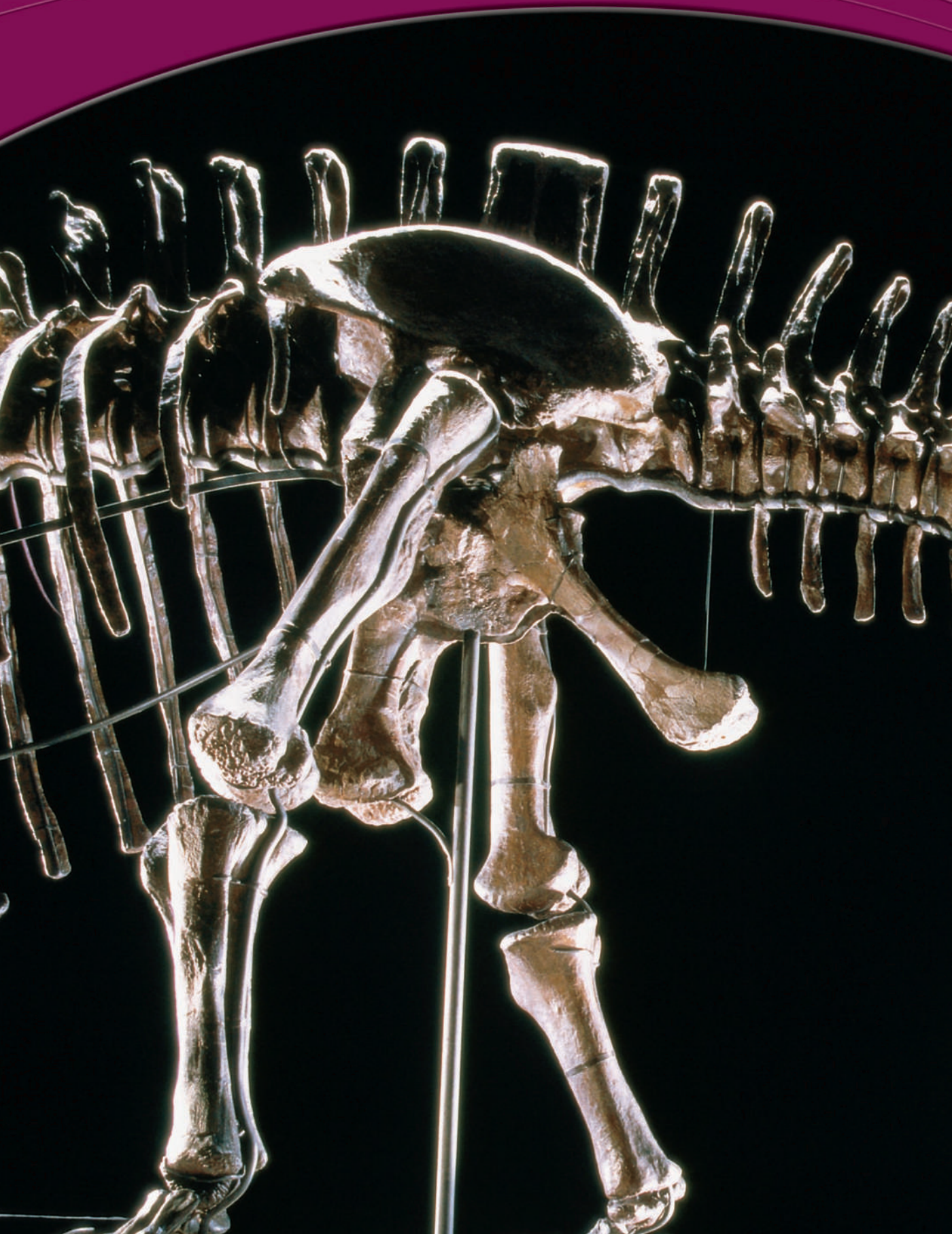
RETURN TO MAIN MENU



# angles 5

## In the steps of the dinosaur

*Scientists are now finding the right angle on dinosaurs by studying their tracks.*

In recent years there has been an explosion in research based on dinosaur tracks. Using trackways we can tell whether a dinosaur was walking, trotting, running or wading. We can estimate its speed by looking at the length of the stride and the step angle. Dinosaurs, like humans, are diagonal walkers, which means a line drawn between the heels of opposite footprints is at an angle to the direction of travel. Other important measurements include the angle made between the innermost and outermost toes, and track rotation, which measures the degree to which tracks are turned inward (pigeon-toed) or outward (duck-footed).

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## outcomes

*After completing this chapter you will be able to:*

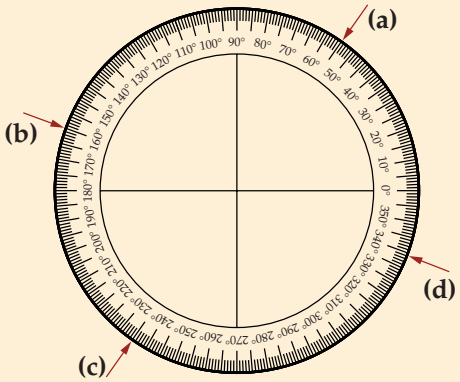
- measure angles accurately
- name angles
- classify angles
- determine properties of angles
- calculate the size of complementary and supplementary angles, vertically opposite angles and angles in a revolution.

# prepzone 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

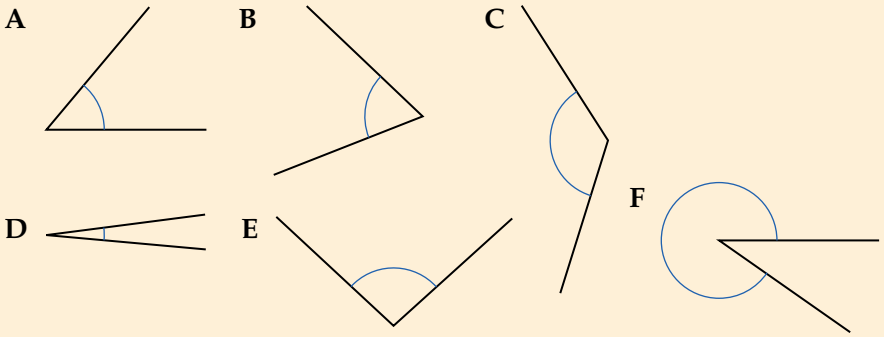
Worksheet R5.1

**1** State the value indicated by each arrow in the following diagram.



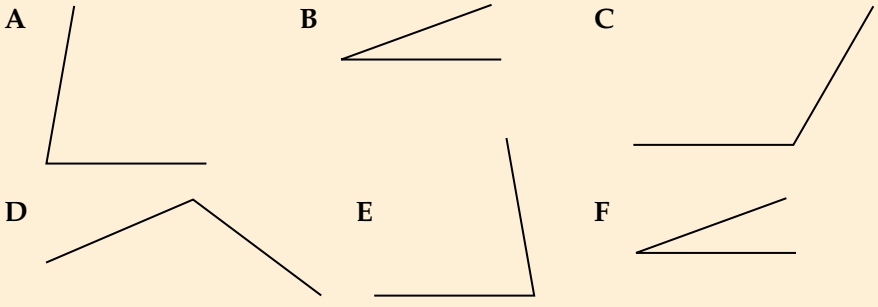

Worksheet R5.2

**2** Arrange the following angles in order from smallest to largest.




Worksheet R5.3

**3** List the pairs of angles which look to be the same size.

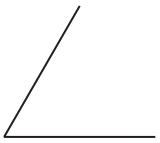



## KEY WORDS

| | | | |
|---|---|---|---|
| acute angle | obtuse angle | right angle | vertex |
| complement | protractor | straight angle | vertically opposite |
| complementary | reflex angle | supplement | |
| degree | revolution | supplementary | |

# 5.1 Measuring angles

An angle is formed whenever two or more lines meet (or intersect). The corner formed where the lines meet is called the **vertex**.



Two angles are shown below.

One is less than $\frac{1}{4}$ of a turn and the other is greater than $\frac{3}{4}$ of a turn.

If you are going to measure the size of the angle it is important to be clear about which of the two angles you are going to measure. That is why it is important to mark the actual angle required. The lines which meet to make the angle are sometimes called the arms of the angle.

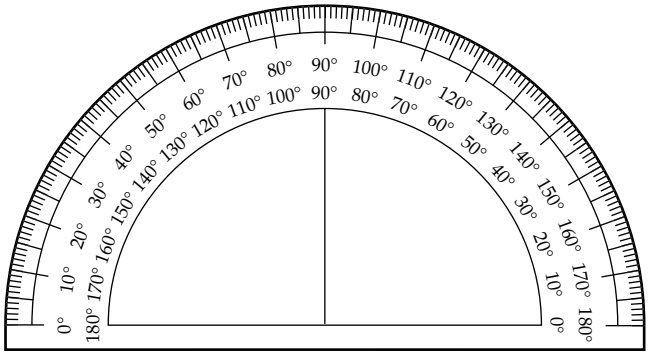
The size of an angle is the amount of turn from one arm of the angle to the other. We use the unit **degree** (°) to describe the size of an angle. There are 360° in a full circle. Other useful angles to remember are a quarter turn (of a full circle) which is 90° and a half turn (of a full circle) which is 180°.

We use a **protractor** to measure the size of angles.

eTutorial

## Using a semi-circular protractor

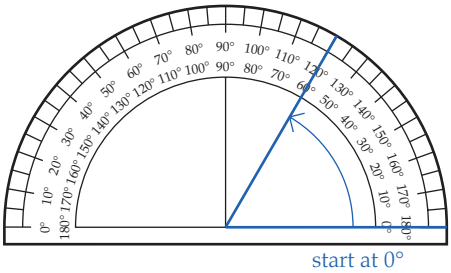
Some protractors are semi-circular, and have two scales marked from 0° to 180° only. The two scales are for measuring angles facing different ways, but can be confusing.



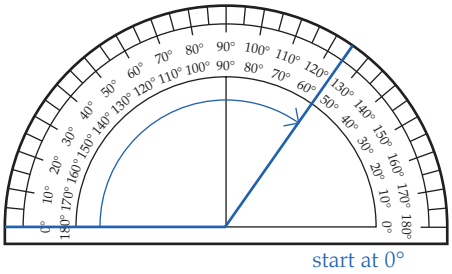

Using a semi-circular protractor is slightly complicated, and the steps followed depend on the type of angle. You must also decide which of the two scales to use.

*For angles less than 180°:*

1. Place the centre of the protractor's base on the vertex.
2. Line up the base of the protractor with one of the arms of the angle.
3. Use the scale on the protractor base *that starts at 0°,* and read the number of degrees the angle spans. (It may be necessary to extend the angle's arms.)



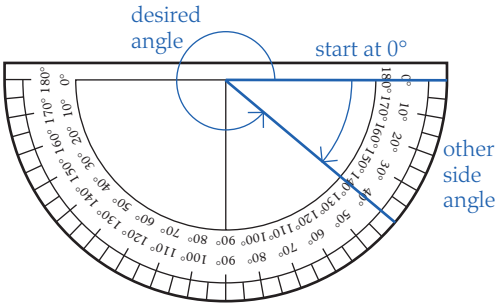

**A 60° angle. Notice how the inner scale is used, because this is the one that has 0° on the bottom arm of the angle.**




**A 125° angle facing the other way. This time the outer scale is used.**

*For angles greater than 180°:*

1. Measure the 'other side angle' as shown using the method described above. The other side angle in our example here measures 40°.




2. Subtract this other side angle from 360° to find the desired angle.

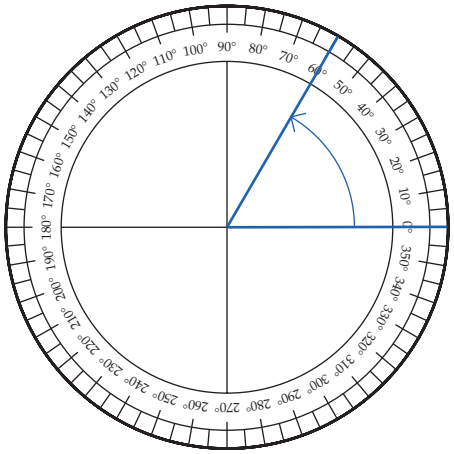
$$\begin{array}{r} 360° \\ -\ 40° \\ \hline 320° \end{array}$$

## Using a circular protractor

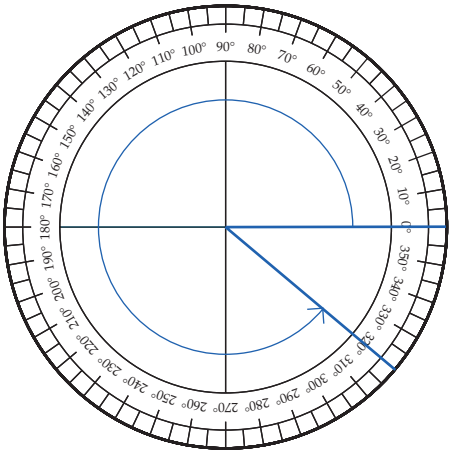
A circular protractor marked from 0° to 360° is shown opposite.

To measure an angle with a circular protractor:

1. Place the centre dot of the protractor on the vertex.
2. Line up the horizontal 0° centre-line of the protractor with one of the arms of the angle. (You may have to turn around the angle to do this.)



3. Read where the other arm of the angle cuts the outer scale of the protractor. (It may be necessary to extend the angle's arms.)




**dangerzone**

**It is easy to make a mistake when using a protractor and use the wrong scale. You should always make an estimate of the angle before measuring it. You can then make sure your measurement of the angle is reasonable.**

Sometimes when you go to measure the size of an angle you find that the arm of the angle is too short to pass through the scale on the protractor. When this occurs you can just extend the length of the arm. This does not change the size of the angle. Use a ruler and be careful that the line is straight.

eTutorial

## exercise 5.1 Measuring angles

Preparation: Prep Zone Q1

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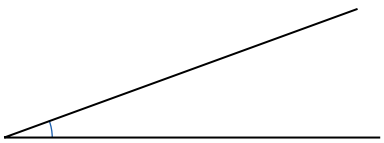
### Core

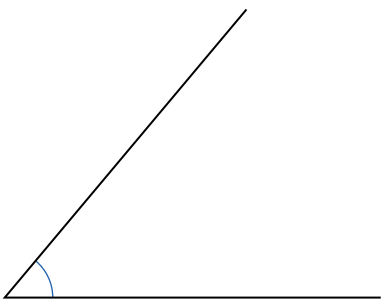
**1** Use a protractor to measure the size of the following angles. Write your answer to the nearest degree.

Hint

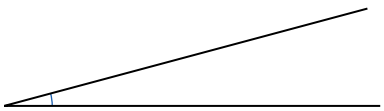

**(a)**



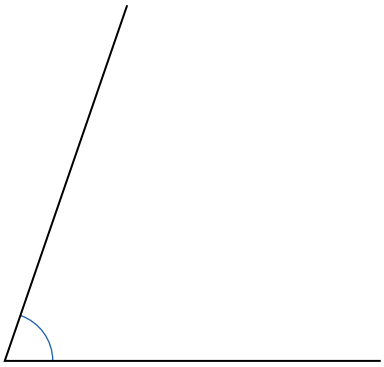
**(b)**



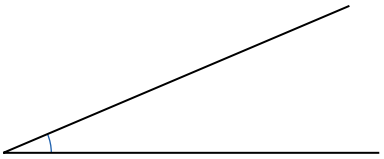
**(c)**



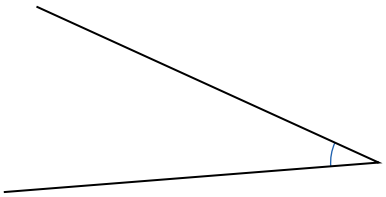
**(d)**



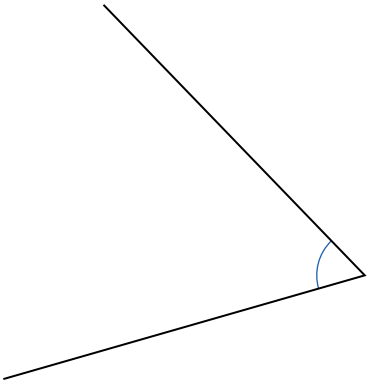
**(e)**



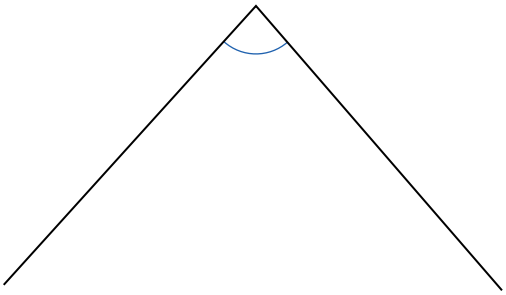
**(f)**



**(g)**



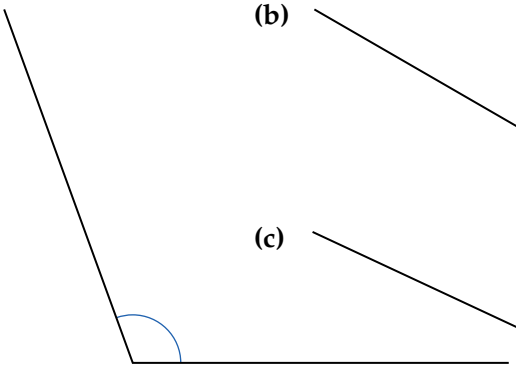
**(h)**



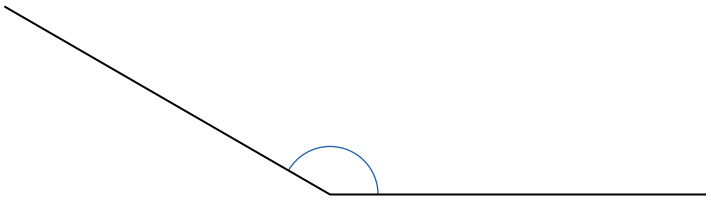
**2** Use a protractor to measure the size of the following angles. Write your answer to the nearest degree.

Hint

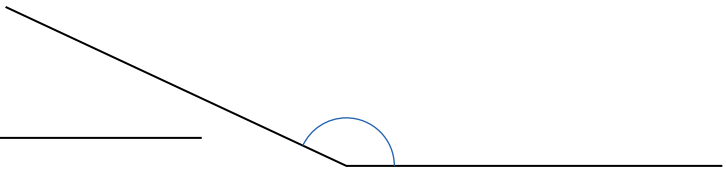
**(a)**



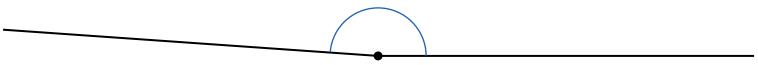
**(b)**



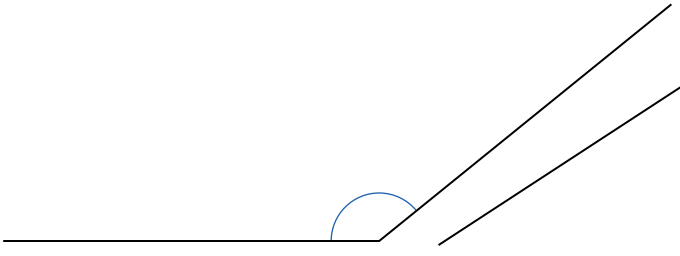
**(c)**



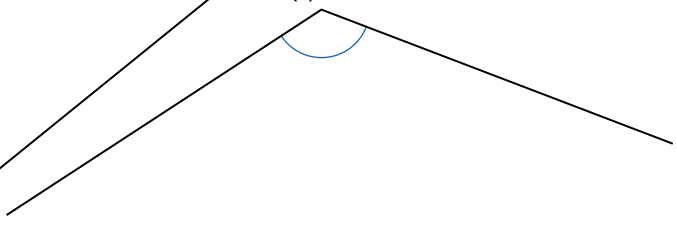
**(d)**



**(e)**



**(f)**

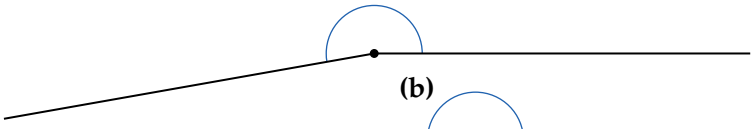


**3** Use a protractor to measure the size of the following angles. Write your answer to the nearest degree.

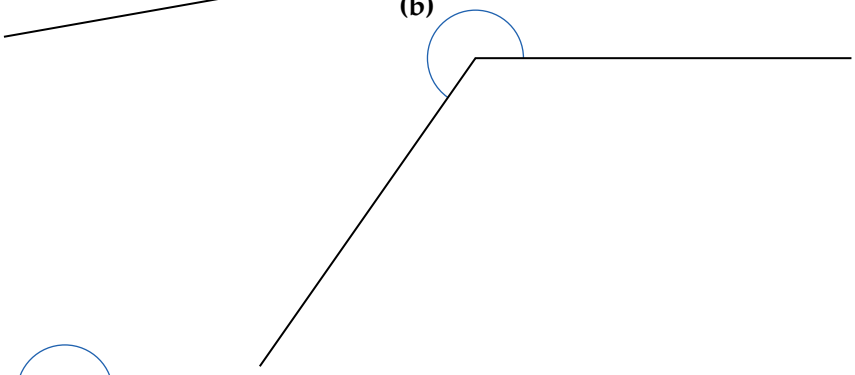
Hint

Worksheet C5.1

**(a)**



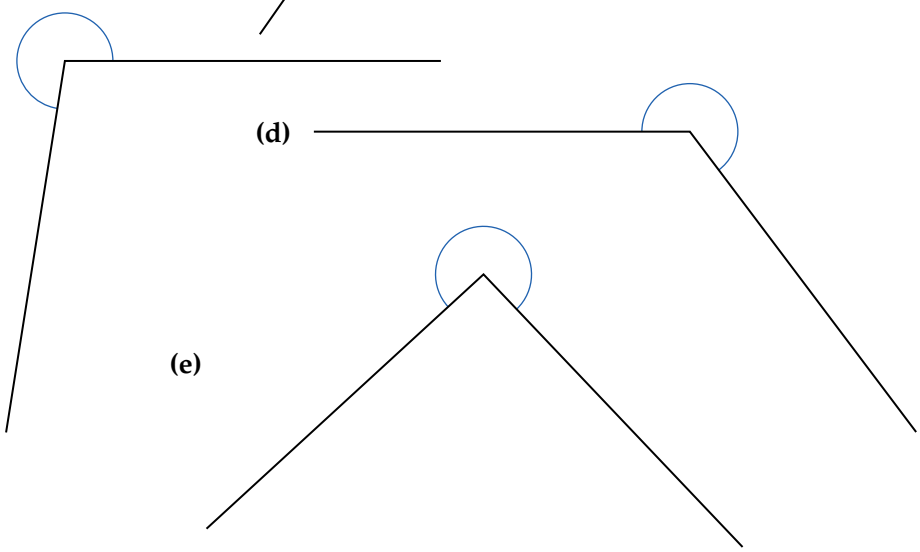
**(b)**



**(c)**

**(d)**

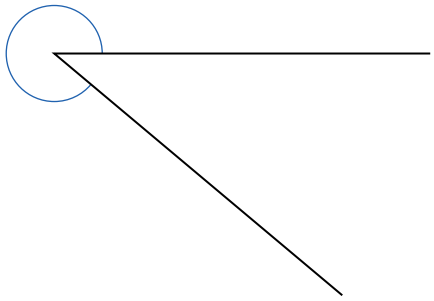
**(e)**



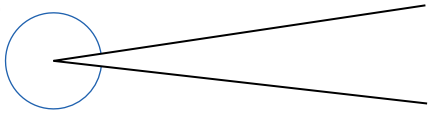
**4** Use a protractor to measure the size of the following angles. Write your answer to the nearest degree.

Hint

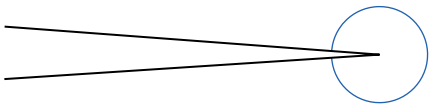
**(a)**



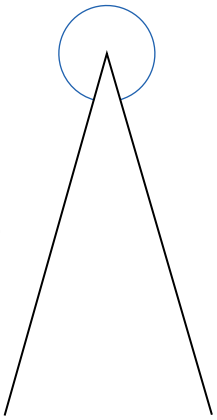
**(b)**



**(c)**

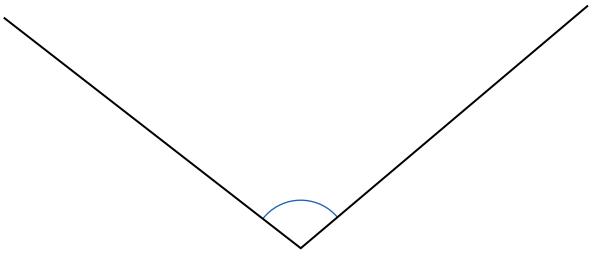


**(d)**

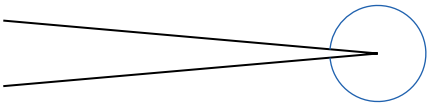


**5** Use a protractor to measure the size of the following angles. Write your answer to the nearest degree.

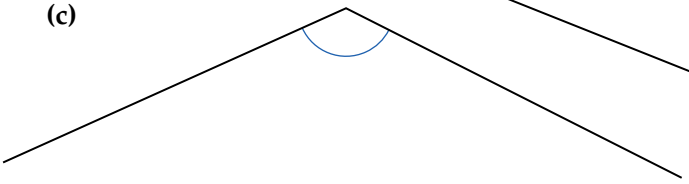
**(a)**



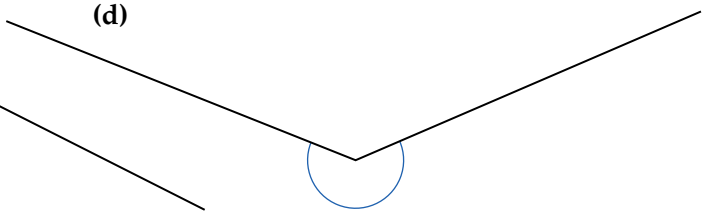
**(b)**



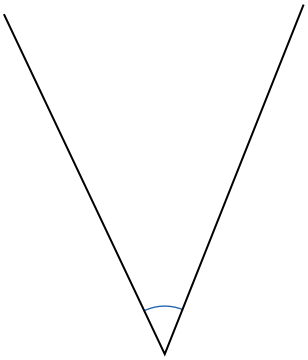
**(c)**



**(d)**

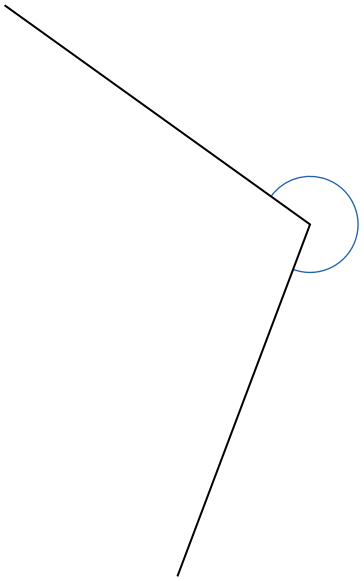


**(e)**

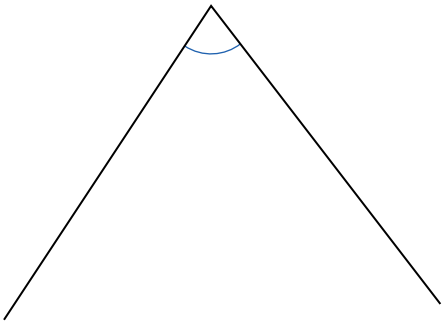


**(f)**

**(g)**



**(h)**



## Extension

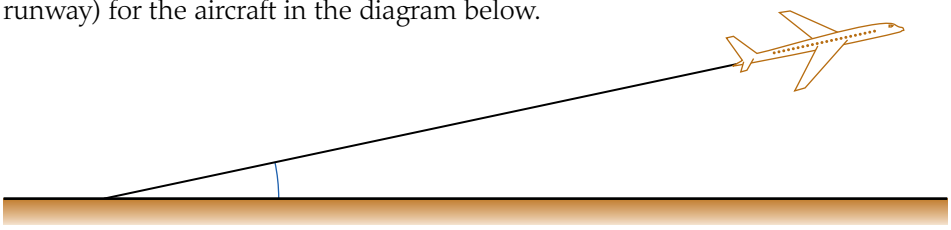
**6** Choose the correct answer.
When you measure the size of the angle between the ladder and the wall in the photo, you find it is:

**A** 150°
**B** 48°
**C** 33°
**D** 27°

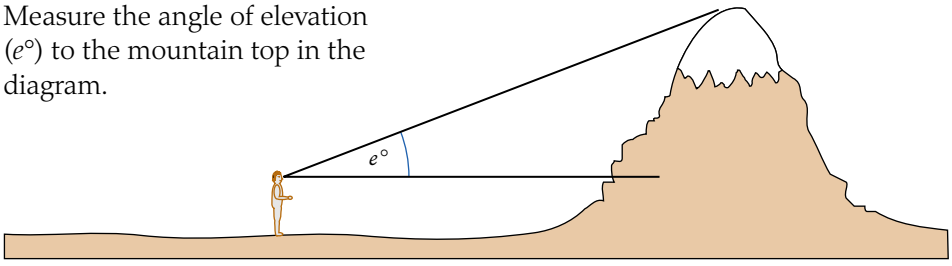
Hint

**7** Measure the angle of ascent (the angle between the flight path and the runway) for the aircraft in the diagram below.

Hint

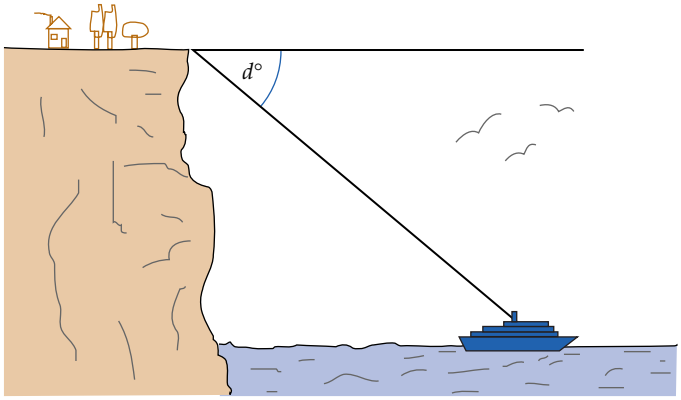


**8** Measure the angle of elevation ($e°$) to the mountain top in the diagram.



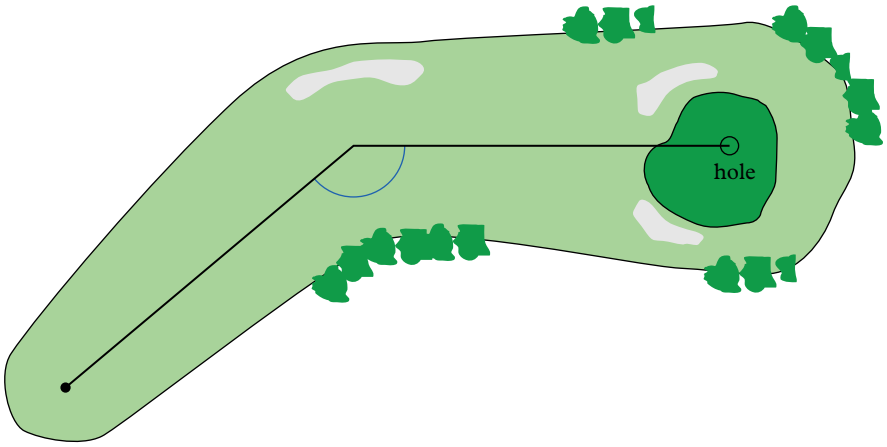

**9** Choose the correct answer.
When you measure the angle of depression ($d°$) to the ship in the diagram, you find it is:

**A** 50°
**B** 40°
**C** 140°
**D** 130°



**10** Tiger plans his assault on a particular golf hole by drawing the ideal path that his ball should take.

Hint




eQuestions

What is the size of the angle between the two ideal paths?

eQuestions

Working mathematically

## problem solving

### Count the angles

How many angles can you count in the diagram? Measure each one.

There are more angles here than you may think. Don't forget the big ones.

## speedingzone

Do these in your head as quickly as you can and write down the answers.

Time target: 2 minutes

**1** $12 \times 50$ **2** $4000 \div 80$ **3** $300 - 4 \times 60$

**4** $210 - 400$ **5** $\sqrt{49} + 3^2$ **6** $\frac{1}{4}$ of 36

**7** What is the size of each angle after a straight angle has been divided into 3 equal angles?

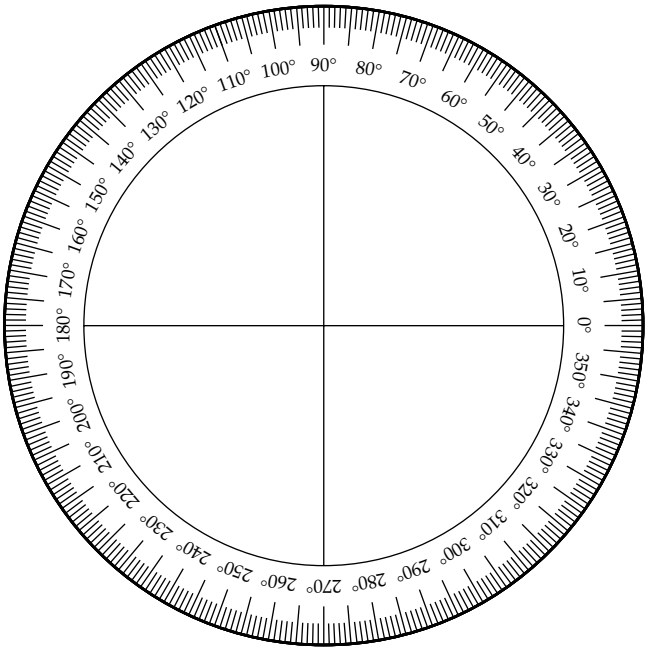
**8** How many 25c lollies can you buy for $4.00?

**9** If you sleep for 7 hours, how many minutes do you sleep for?

**10** If $a = 5$, find $a^2 - 3$.

# 5.2 Estimating angles

Now that you have had experience using a protractor to measure angles, you should be better at giving rough estimates of the size of angles without using a protractor. When estimating angles, try to visualise a 360° circular protractor and think about how much of the complete circle you have covered.




eTutorial

## exercise 5.2 Estimating angles

Preparation: Prep Zone Q1–3, Ex 5.1

Interactive

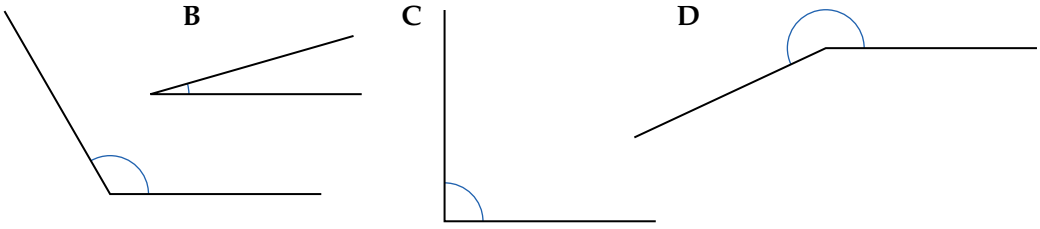
In this exercise, *do not use a protractor* unless specifically asked to. Refer to a diagram of a circular protractor if necessary to help you.

### Core

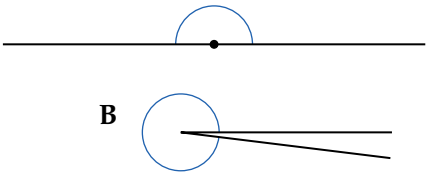
Hint

**1** List the following angles in order of increasing size (i.e. smallest to largest).

**(a)** **A** **B** **C** **D**



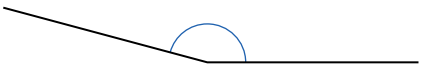
**(b)** **A**



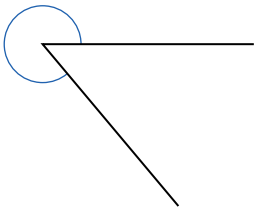
**B**



**C**



**D**

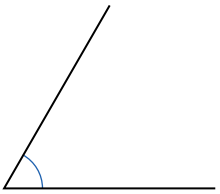


**2** Choose the correct answer.

**(a)** The angle shown here is about:

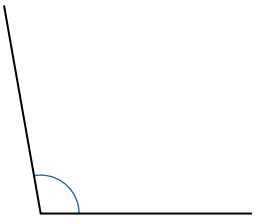
**A** 20°
**B** 45°
**C** 60°
**D** 85°

Hint



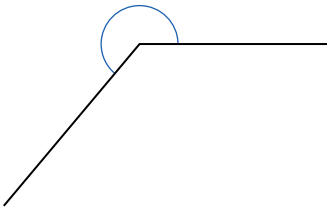
**(b)** The angle shown here is about:

**A** 100°
**B** 145°
**C** 185°
**D** 200°



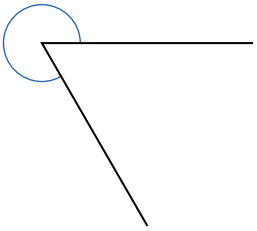
**(c)** The angle shown here is about:

**A** 190°
**B** 230°
**C** 280°
**D** 310°



**(d)** The angle shown here is about:

**A** 60°
**B** 120°
**C** 240°
**D** 300°

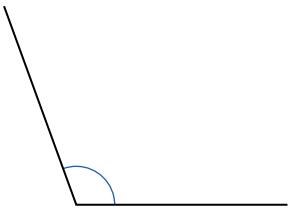


**3** Choose the correct answer.

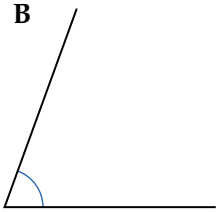
**(a)** Which of the angles below is about 170°?

Hint

**A**



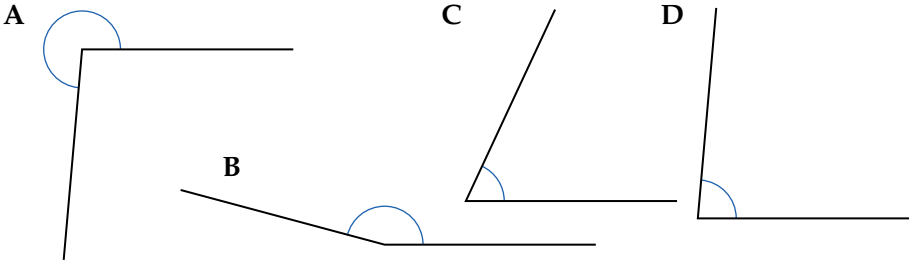
**B**



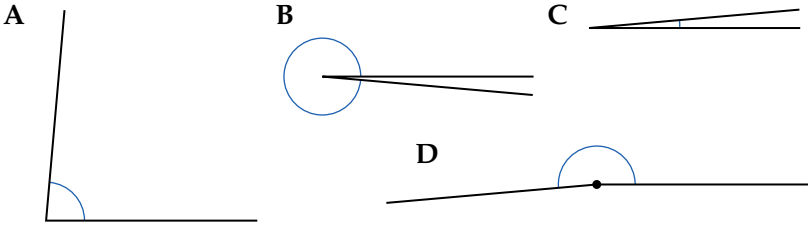
**C**

**D**

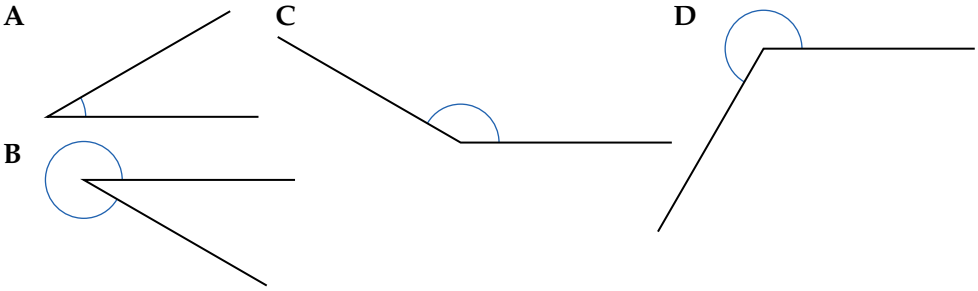
**(b)** Which of the angles below is about 265°?




**(c)** Which of the angles below is about 5°?

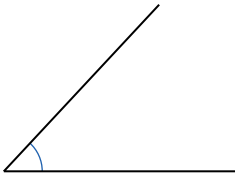



**(d)** Which of the angles below is about 330°?

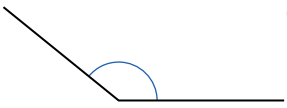



**4** Estimate the size of each of these angles. Copy the table on the following page and complete only the first column.

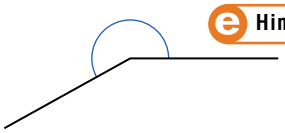
**(a)**



**(b)**

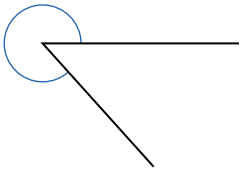


**(c)**

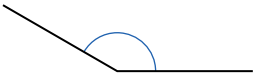


Hint

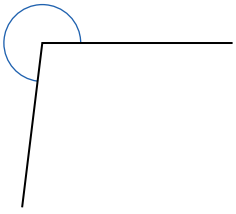
**(d)**



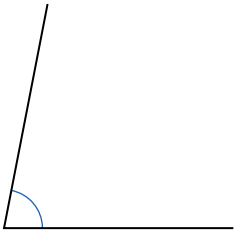
**(e)**



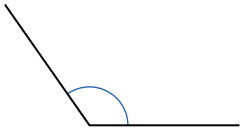
**(f)**



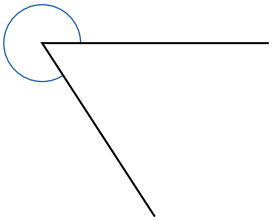
**(g)**



**(h)**



**(i)**



**(j)**

| Angle | Estimate | Measured value | Error |
|---|---|---|---|
| (a) | | | |
| (b) | | | |
| (c) | | | |
| (d) | | | |
| (e) | | | |
| (f) | | | |
| (g) | | | |
| (h) | | | |
| (i) | | | |
| (j) | | | |
| | | Total error | |
| | | Average error (Divide total error by 10) | |

### Extension

**5** Now measure each of the angles in Question **4** using a protractor, and complete the table. You will need to extend the length of the arms. The error is the difference between your estimate and the measured value. It doesn't matter whether the estimate is greater than or smaller than the measured value. How does your angle-estimating ability compare with that of your classmates?

Hint

eQuestions


# 5.3 Drawing angles with a protractor

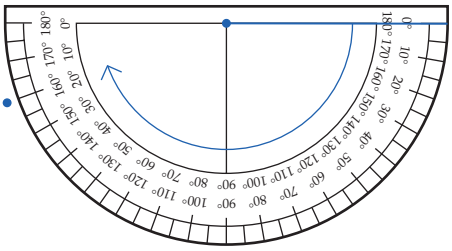
## Using a semi-circular protractor

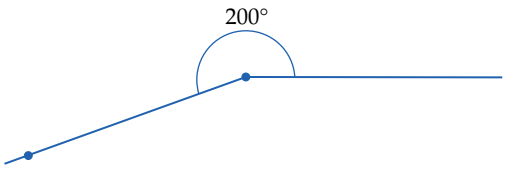
1. If the angle is less than 180°, follow the procedure given for a circular protractor (below).
2. If the angle is greater than 180°, proceed as follows. An example of 200° is used to explain the procedure.
   Calculate the 'other side angle' by subtracting from 360°.

$$\begin{array}{r} 360^\circ \\ -200^\circ \\ \hline 160^\circ \end{array}$$

3. Draw a base line and a vertex dot. (A vertex dot marks the location of the angle's corner.)
4. Place the protractor upside down so its zero is on the base line and its centre is on the vertex dot.
5. Locate 160° (start at 0° on the base line and move clockwise) and mark a small dot on the edge of the scale as shown.
6. Remove the protractor and join the small dot to the vertex dot.
7. Mark the *other side* of the angle drawn, as this is the angle desired.

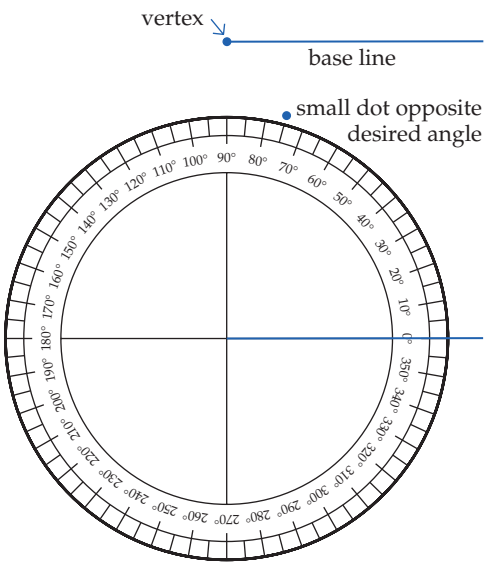




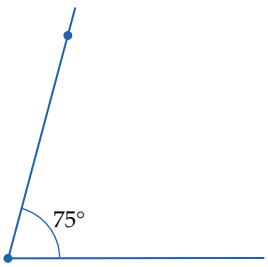



## Using a circular protractor

1. Draw a base line and a vertex dot.
2. Place the zero of the protractor on the base line with its centre on the vertex dot.
3. Locate the desired angle (e.g. 75°) and mark a small dot on the edge of the scale as shown.




4. Remove the protractor and join the small dot to the vertex dot.
5. Mark the angle with a small arc as shown.





A pro-tractor rally

# exercise 5.3 Drawing angles with a protractor

Preparation: Prep Zone Q1, Ex 5.1

*You will need:* A protractor.

## Core

**1** Draw the following angles using a protractor.

**(a)** 10° **(b)** 70° **(c)** 6°
**(d)** 36° **(e)** 55° **(f)** 84°

Hint

**2** Draw the following angles using a protractor.

**(a)** 95° **(b)** 151° **(c)** 130°
**(d)** 100° **(e)** 175° **(f)** 108°

Hint

**3** Draw the following angles using a protractor.

**(a)** 182° **(b)** 260° **(c)** 222°
**(d)** 205° **(e)** 235° **(f)** 257°

Hint

Worksheet C5.2

**4** Draw the following angles using a protractor.

**(a)** 290° **(b)** 318° **(c)** 275°
**(d)** 334° **(e)** 300° **(f)** 247°

Hint

## Extension

**5** Draw the following angles without a protractor.

**(a)** 180° **(b)** 360°

**6** For each of the following draw a circle then divide it into wedges with the following angles at the centre.

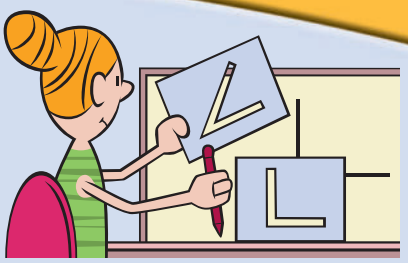
**(a)** 100°, 210°, 50° **(b)** 90°, 90°, 72°, 108°
**(c)** 40°, 140°, 180° **(d)** 32°, 161°, 85°, 82°
**(e)** 215°, 34°, 71°, 40° **(f)** 19°, 28°, 172°, 141°


Working mathematically

## problem solving

### Ella's angles

Ella needs to draw a 60° angle, but she has lost her protractor. She has a stencil that can draw 90° and 40° angles. How can she use these two angle sizes to draw a 60° angle?



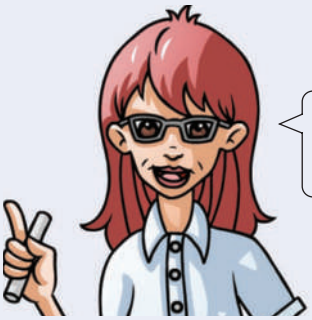
**Ella will need to use a series of steps to construct her angle.**

Working mathematically

investigation

# Dot paper angles

Worksheet A5.1

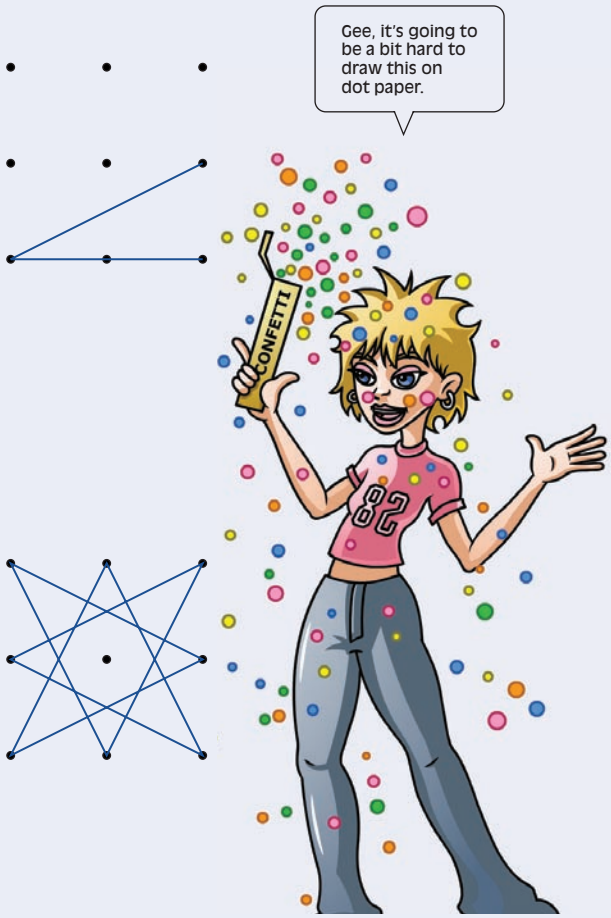



**1** Draw as many *different* angles as you can on separate 3 × 3 dot grids so that each angle starts and ends on a dot and has its corner (vertex) on a dot. One example is shown opposite. Use a protractor to measure each angle and write the number of degrees next to each one. Did you find the ten different possible angles? If you found more than ten, check carefully—some of your angles may actually be the same size. You may like to work with a partner to make sure all possible angles are found.

**2** Choose one of the angles from Question **1** and, on a new 3 cm × 3 cm grid, draw this angle in as many different positions as possible. You may wish to colour the resulting pattern. The pattern for one of the angles is shown opposite.

**3** Try Question **2** for several of the other angles.

**4** Investigate the possible angles for a larger (e.g. 4 × 4) grid, and try to produce some patterns using repeated angles.



# maths in action

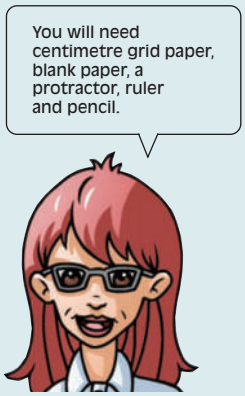
## Billiard ball bounces

Worksheet A5.2


It is often said that a person's skill at billiards or pool is a sign of misspent youth, but top players could claim to have worked hard at their maths.

The game of billiards originated in France in the early 1500s. First a railing was added to the edges, then a cushion in 1835, allowing players to bounce the ball off the sides with speed and accuracy. The object of the game was to use the white ball to hit both the red ball and the spot ball with the same shot—this was called a cannon. The modern game was developed later by the British and uses a 12 foot by 6 foot table (3.66 m by 1.83 m). Important in planning shots is the fact that a ball bounces off a side at the same angle at which it approaches.

### Questions




Imagine a '6 × 4' billiard table with pockets at the corners only, and a ball hit from a corner at an angle of 45° to a side of the table.

Assuming that:

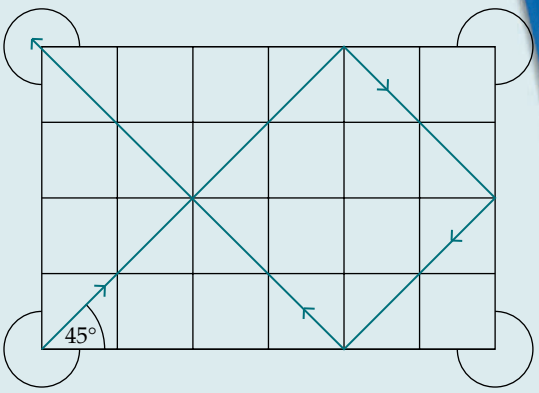
- the rebound angle equals the approach angle for each collision the ball makes with a side of the table, and
- the ball doesn't stop rolling until it falls into a corner pocket,

the path of the ball can be traced as shown.

**1** On square centimetre grid paper, draw tables of the dimensions given, and trace the path of a ball hit from a corner at an angle of 45° to a side until the path ends at a pocket.

**(a)** 1 cm × 4 cm **(b)** 2 cm × 8 cm **(c)** 3 cm × 12 cm **(d)** 3 cm × 4 cm
**(e)** 9 cm × 12 cm **(f)** 5 cm × 6 cm **(g)** 4 cm × 5 cm **(h)** 8 cm × 10 cm

**(i)** 3 cm × 8 cm
**(j)** 5 cm × 7 cm
**(k)** 4 cm × 9 cm
**(l)** 7 cm × 9 cm
**(m)** 6 cm × 11 cm
**(n)** 5 cm × 5 cm




**2** Copy and complete the table below using your diagrams from Question **1**.

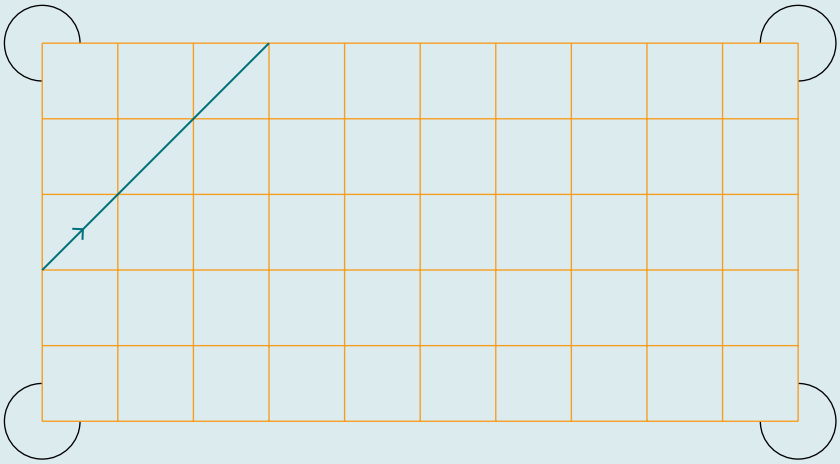
| | *Table dimensions* | *No. bounces* |
|---|---|---|
| **(a)** | 1 × 4 | |
| **(b)** | 2 × 8 | |
| **(c)** | 3 × 12 | |
| **(d)** | 3 × 4 | |
| **(e)** | 9 × 12 | |
| **(f)** | 5 × 6 | |
| **(g)** | 4 × 5 | |

| | *Table dimensions* | *No. bounces* |
|---|---|---|
| **(h)** | 8 × 10 | |
| **(i)** | 3 × 8 | |
| **(j)** | 5 × 7 | |
| **(k)** | 4 × 9 | |
| **(l)** | 7 × 9 | |
| **(m)** | 6 × 11 | |
| **(n)** | 5 × 5 | |

**3** If you can, explain how you can predict:
**(a)** whether or not the ball will visit every square
**(b)** how many bounces will occur
**(c)** which pocket the ball will end up in.

**4** If you answered Question **3(b)**, predict how many bounces will occur for a 56 × 42 table.

**5 (a)** On square centimetre grid paper, trace the path of a ball hit from the position shown on a 5 × 10 table.



**(b)** Try starting from different positions and trace the paths.

***Activity***

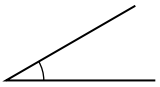
hi.com.au

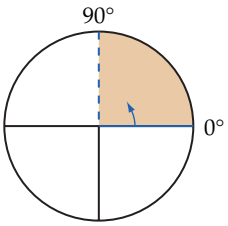
Use your understanding of angles to play a game of online pool at this site.

# 5.4 Describing angles

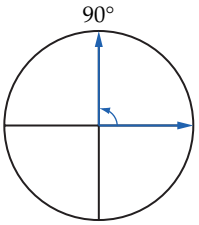
Angles may be classified as one of the following types:

**Acute angle:** less than 90°—smaller than a square corner.

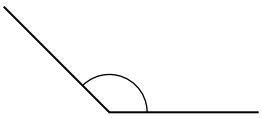


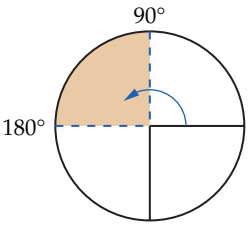



**Right angle:** exactly 90°—a square corner, a $\frac{1}{4}$ turn. A small square drawn in the corner of an angle means it is a right angle.

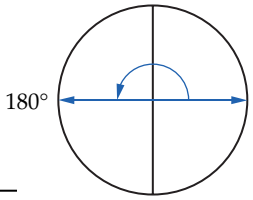



**Obtuse angle:** more than 90° but less than 180°—bigger than a square corner.

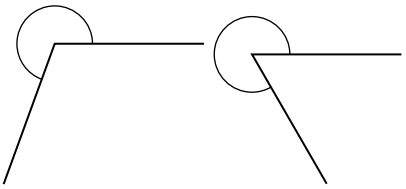


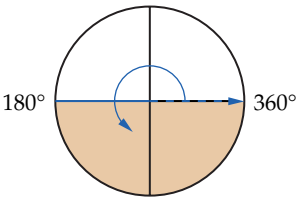



**Straight angle:** exactly 180°—a straight line angle, a half turn.

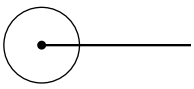


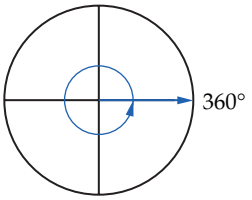

**Reflex angle:** greater than 180° but less than 360°—like the angle on the outside of a bent knee.





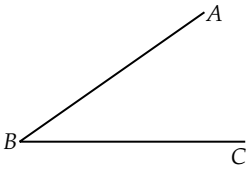

**Revolution:** 360°—a full circle.





## Naming angles

Capital letters may be used to help us describe angles. The angle shown may be described as angle $ABC$, $\angle B$ or $\angle ABC$. ($\angle$ is the symbol for angle.) $\angle CBA$ or $CBA$ is also correct. We can use the same letters to name points, lines and intervals.




Notice that the letter on the corner of the angle (the vertex) goes in the middle of the three letters. In this case the $B$ must be in the middle.

eTutorial

## exercise 5.4 Describing angles

Preparation: Prep Zone Q1 and 2, Ex 5.2

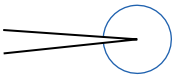
### Core

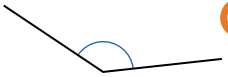
**1** State the type of angle shown in each case.

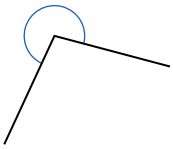
Worksheet C5.3

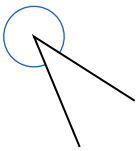
Hint

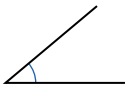
**(a)** 

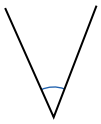
**(b)** 

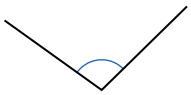
**(c)** 

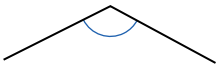
**(d)** 

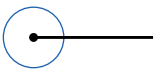
**(e)** 

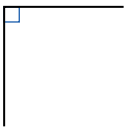
**(f)** 

**(g)** 

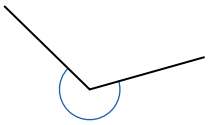
**(h)** 

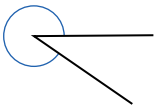
**(i)** 

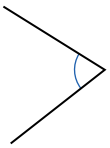
**(j)** 

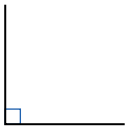
**(k)** 

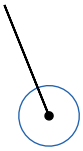
**(l)** 

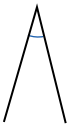
**(m)** 

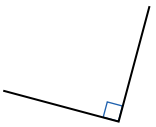
**(n)** 

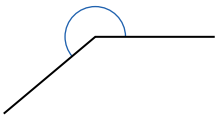
**(o)** 

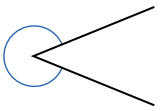
**(p)** 

**(q)** 

**(r)** 

**(s)** 

**(t)** 

**(u)** 

**2** State the type of angle given in each case.

**(a)** 23° **(b)** 117° **(c)** 275° **(d)** 360°

**(e)** 180° **(f)** 75° **(g)** 90° **(h)** 162°

**3** Draw two examples of:

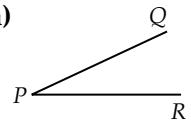
**(a)** an acute angle **(b)** a reflex angle **(c)** an obtuse angle.

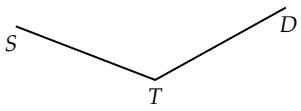
Hint

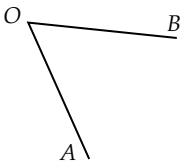
**4** Give a letter name for each of the following angles.

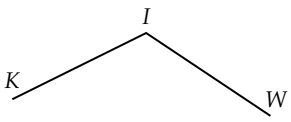
Worksheet C5.4

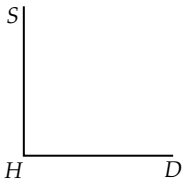
Hint

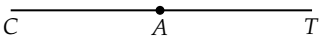
**(a)** 


**(b)** 


**(c)** 


**(d)** 


**(e)** 


**(f)** 


**5** Draw any angle which may be named:

**(a)** $\angle PQR$ **(b)** $\angle G$ **(c)** $\angle K$ **(d)** $\angle DEF$

Hint

## Extension

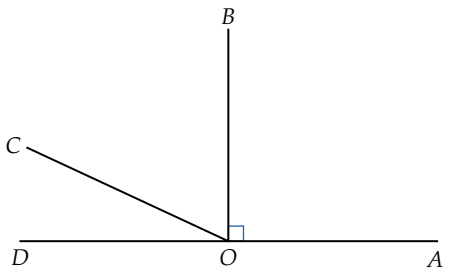
**6** Give the letter name of:

**(a)** each acute angle in the diagram, using three letters

**(b)** each obtuse angle in the diagram, using three letters

**(c)** the point where *CO* touches *DA*, using one letter

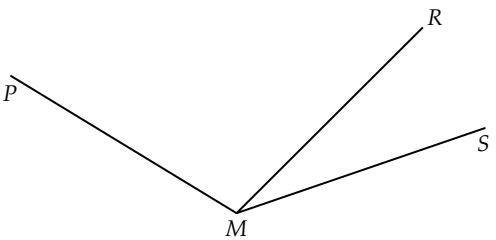
**(d)** the line that meets *DA* at right angles, using two letters.




**7 (a)** Measure the reflex angle *RMS* in the diagram.

**(b)** Measure the reflex angle *PMS* in the diagram.

**(c)** Why is it necessary to specify 'reflex' in parts **(a)** and **(b)** above?




eQuestions

Homework 5.1

Worksheet A5.3

Worksheet A5.4

Working mathematically

## computer investigation

Cabri

### Complementary and supplementary angles

Angles on a line and angles in a right angle have certain properties. To answer the following questions, click on the icons to the right to open the Cabri Geometry files on complementary and supplementary angles. Observe what happens to the sizes of the angles when you drag the point.

You will need to download Cabri Geometry from the eMaths Zone CD if you haven't already done so.

**1** What can you say about angles on a straight line? These are called supplementary angles. e Interactive

**2** What can you say about angles in a right angle? These are called complementary angles. e Interactive

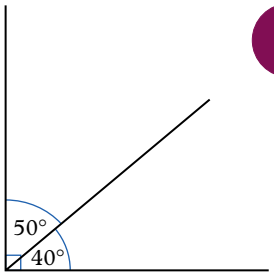
# 5.5 Complementary and supplementary angles

**Complementary** angles add up to **90°** (a right angle).

40° and 50° are complementary angles because together they add up to 90°.

If two angles are complementary, we say each is the **complement** of the other. For example, 40° is the complement of 50°.

A pair of complementary angles

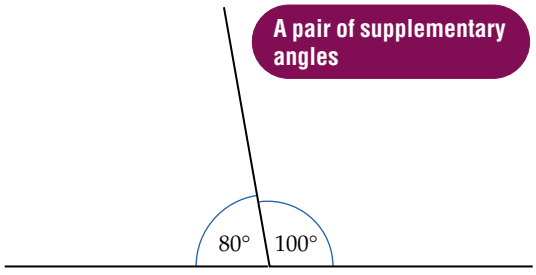



**Supplementary** angles add up to **180°** (a straight angle).

100° and 80° are supplementary angles because together they add up to 180°.

If two angles are supplementary, we say each is the **supplement** of the other. For example, 100° is the supplement of 80°.

A pair of supplementary angles

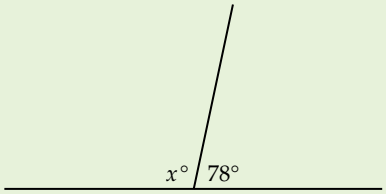


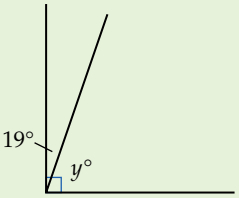
One way of avoiding getting mixed up with these definitions is to remember that, when counting, 90 comes before 180, and when reciting the alphabet, C comes before S.

C 90° → S 180°

## worked example 1

**(a)** What is the complement of 11°?
**(b)** What is the supplement of 57°?
**(c)** Find the size of angle $x°$.
**(d)** Find the size of angle $y°$.




### Steps

**(a)** An angle and its complement must add up to 90°, so we subtract 11° from 90° to find the complement.

**(b)** An angle and its supplement add up to 180°, so we subtract 57° from 180° to find the supplement.

**(c)** $x°$ and 78° are shown as supplementary angles, so we subtract 78° from 180° to find $x°$.

**(d)** $y°$ and 19° are shown as complementary angles, so we subtract 19° from 90° to find $y°$.

### Solutions

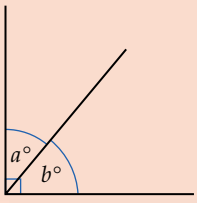
**(a)**
$$\begin{array}{r} 90° \\ -\ 11° \\ \hline 79° \end{array}$$
The complement of 11° is 79°.

**(b)**
$$\begin{array}{r} 180° \\ -\ 57° \\ \hline 123° \end{array}$$
The supplement of 57° is 123°.

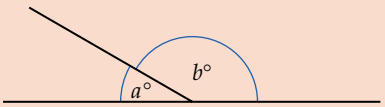
**(c)**
$$\begin{array}{r} 180° \\ -\ 78° \\ \hline 102° \end{array}$$
$x° = 102°$

**(d)**
$$\begin{array}{r} 90° \\ -\ 19° \\ \hline 71° \end{array}$$
$y° = 71°$

Complementary angles add up to 90°




Supplementary angles add up to 180°




C 90° → S 180°

eTutorial

# exercise 5.5 Complementary and supplementary angles

Preparation: Ex 5.4

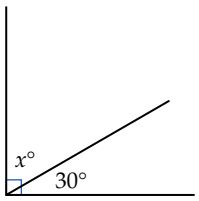
## Core

**1** Find the size of angle $x°$ in each case.

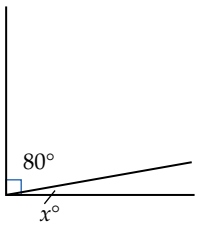
eTester

Hint

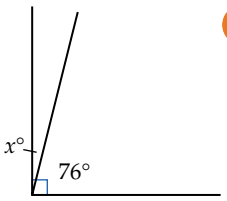
**(a)**



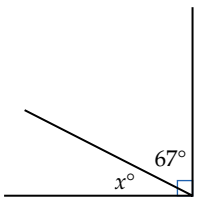

**(b)**



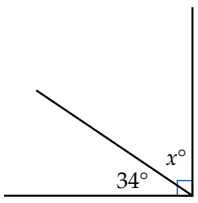

**(c)**



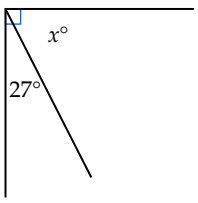

**(d)**



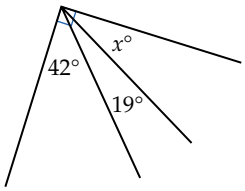

**(e)**



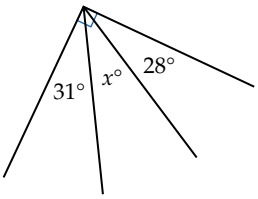

**(f)**



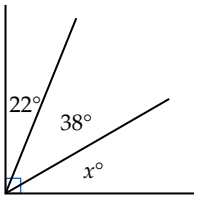

**(g)**




**(h)**




**(i)**




**2** Find the complement of the following angles.

**(a)** 27° **(b)** 45° **(c)** 68° **(d)** 15°

**3** Choose the correct answer.

Worksheet C5.5

Hint

Worksheet C5.6

**(a)** Which one of the following is a pair of complementary angles?

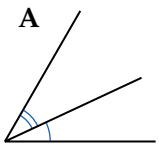
**A** 330° and 30° **B** 30° and 150°
**C** 15° and 75° **D** 60° and 40°

**(b)** Which one of the following is a pair of supplementary angles?

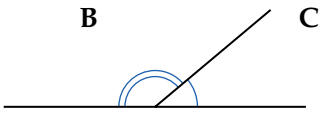
**A** 0° and 90° **B** 45° and 55°
**C** 90° and 90° **D** 200° and 160°

**(c)** Which diagram shows supplementary angles?

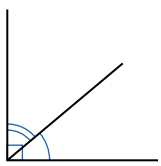
**A**



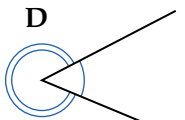
**B**



**C**

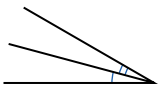


**D**

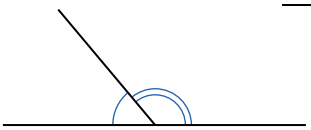


**(d)** Which diagram shows complementary angles?

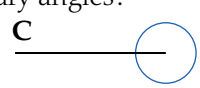
**A**



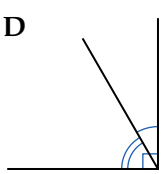
**B**

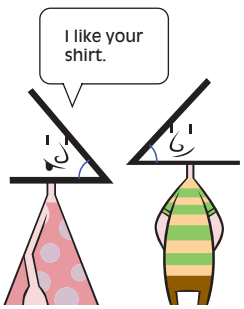


**C**



**D**



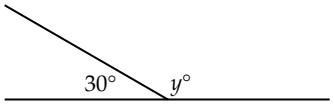



A 'complimentary' angle

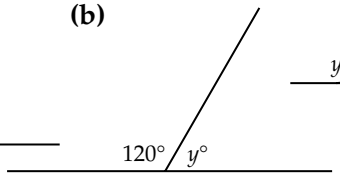
**4** Find the size of angle $y°$ in each case.

Hint

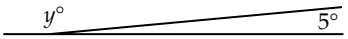
**(a)**



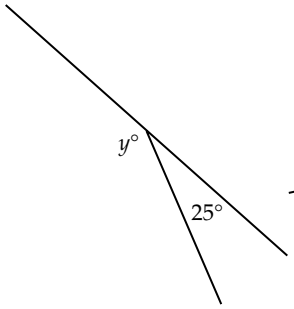

**(b)**



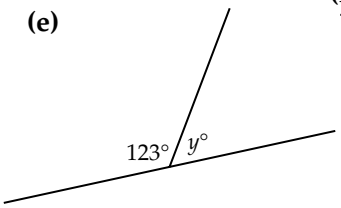

**(c)**



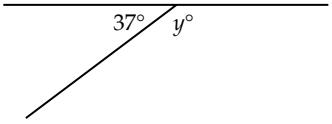

**(d)**



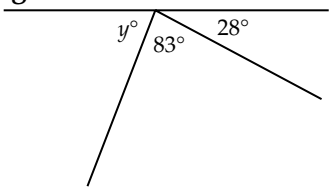

**(e)**



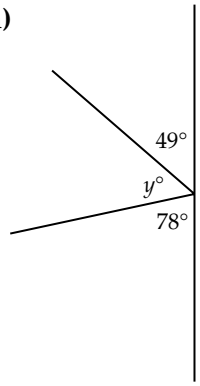

**(f)**



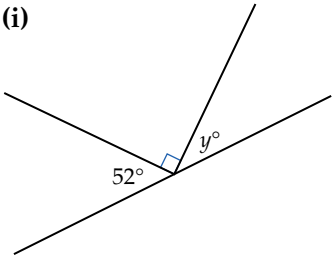

**(g)**




**(h)**




**(i)**




**5** Find the supplement of the following angles.

Hint

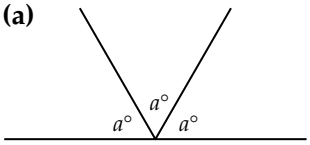
**(a)** 32° **(b)** 90° **(c)** 124° **(d)** 176°

## Extension

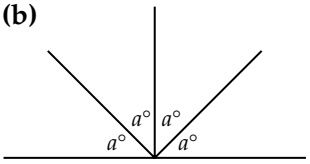
**6** Determine the size of angle $a°$ in each case.

Hint

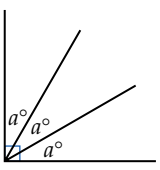
**(a)**



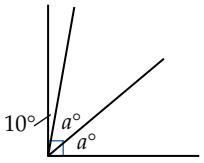

**(b)**



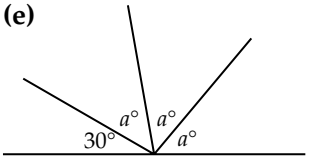

**(c)**



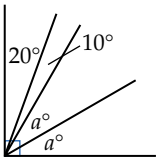

**(d)**




**(e)**




**(f)**



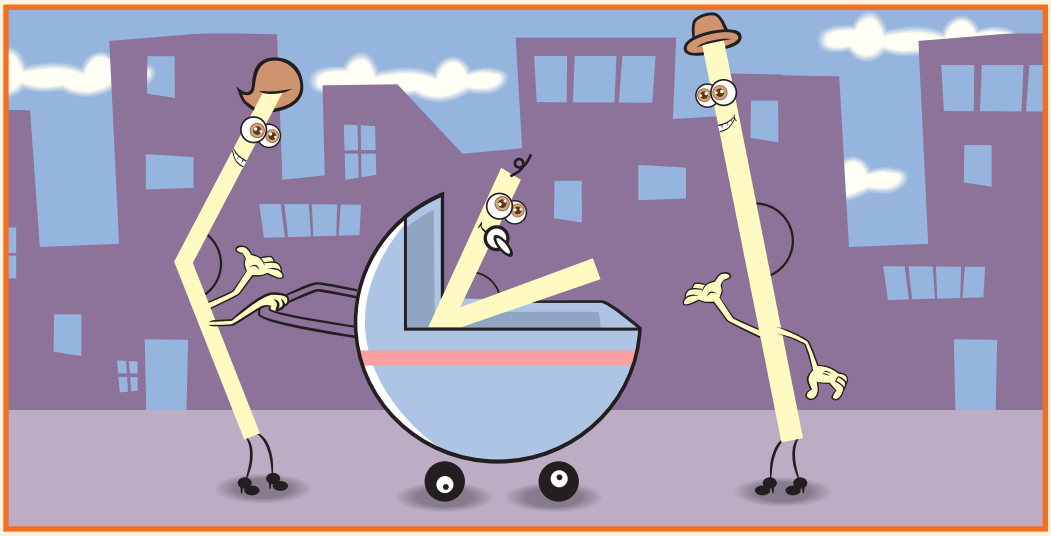

**7** **(a)** Write two examples of pairs of complementary angles.
**(b)** Write two examples of pairs of supplementary angles.

Worksheet C5.7

**8** **(a)** Explain why adjacent angles adding to 90° form a right angle.
**(b)** Explain why adjacent angles adding to 180° form a straight angle.

eQuestions

## laughzone



Answer the following, showing your working, then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

The following are angles in a right angle. Find the value of the pronumeral.

37°, $n$° **N** 56°, $u$° **U** 29°, 36°, $c$° **C** 12°, 57°, $l$° **L**

The following are angles on a straight line. Find the value of the pronumeral.

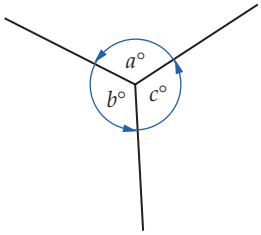
29°, $a$° **A** 142°, $t$° **T** 13°, 59°, $g$° **G** 72°, 53°, $e$° **E**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ‘ | | | | | | | | | | ’ |
| 151° | 25° | 34° | 38° | 55° | 151° | 53° | 108° | 21° | 55° | |

# 5.6 Angles in a revolution

We can see from the circular protractor that there are 360° in a revolution.

If more than one angle can be added together to complete one full revolution (a circle), then the angles must add up to 360°.

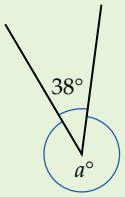



$a° + b° + c° = 360°$

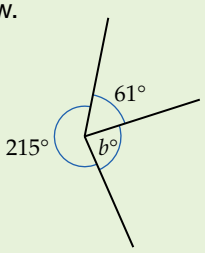
## worked example 2

Find the size of each of the unknown angles below.

**(a)**




**(b)**




### Steps

**(a)** Subtract the known angle from 360° to find the value of $a°$.

**(b)** 1. Add the known angles.

2. Subtract the total of the known angles from 360° to find the value of $b°$.

### Solutions

**(a)**
$$\begin{array}{r} 360° \\ -\ 38° \\ \hline a° = \ 322° \end{array}$$

**(b)**
$$\begin{array}{r} 61° \\ +215° \\ \hline 276° \end{array}$$

$$\begin{array}{r} 360° \\ -276° \\ \hline b° = \ 84° \end{array}$$

eTutorial

## exercise 5.6 Angles in a revolution

Preparation: Ex 5.4

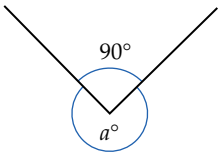
### Core

**1** Find the value of the pronumeral in each case.

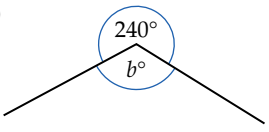
Hint

Worksheet C5.8

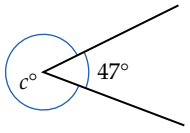
**(a)**



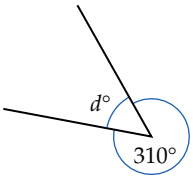

**(b)**



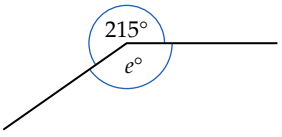

**(c)**



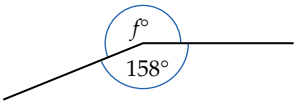

**(d)**



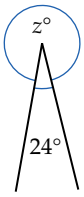

**(e)**



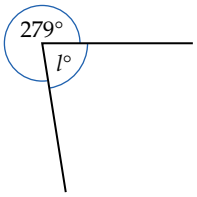

**(f)**



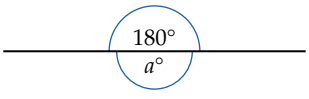

**(g)**




**(h)**

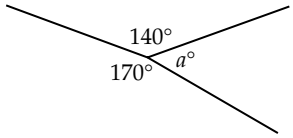



**(i)**

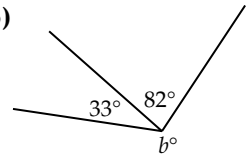


**2** Find the value of the angle in each diagram below.

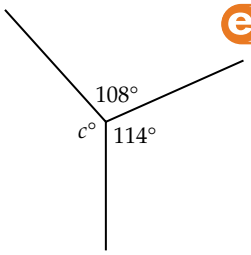
**(a)**




**(b)**

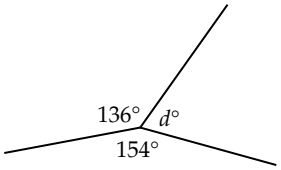



**(c)**

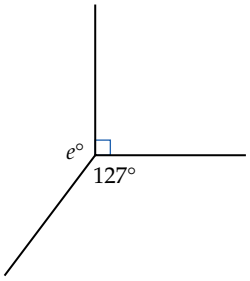



Hint

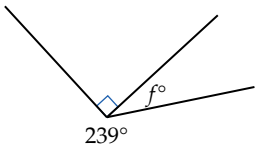
**(d)**




**(e)**

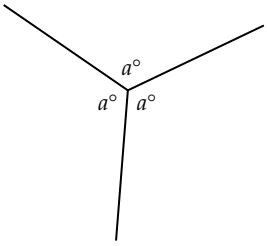



**(f)**

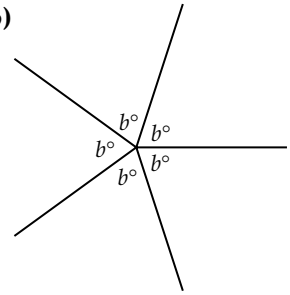



**3** Find the value of the angle in each of the following.

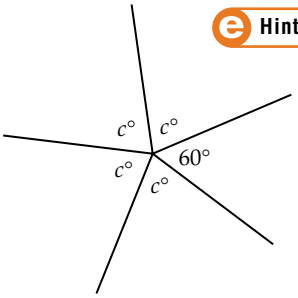
**(a)**




**(b)**




**(c)**




Hint

**4** Write two examples of three angles that add to make a revolution.

**5** Explain why adjacent angles adding to 360° form a revolution.

Worksheet A5.5

eQuestions

## Working mathematically

## computer investigation

Cabri

### Vertically opposite angles

When two lines intersect, four angles are formed. To answer the following question, click on the icon to the right to open the Cabri Geometry files on these angles. Observe what happens to the sizes of the angles when you move any of the points.

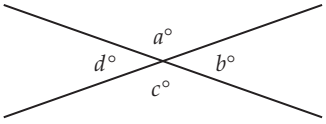
What can you say about vertically opposite angles? Explain why. You might want to refer to supplementary angles in your explanation.

You will need to download Cabri Geometry from the eMaths Zone CD if you haven't already done so.

Interactive

# 5.7 Vertically opposite angles

Whenever two lines intersect, four angles are formed. In the following diagram these angles have been labelled as $a°$, $b°$, $c°$ and $d°$. Pairs of angles like $a°$ and $c°$ are given a special name—they are called **vertically opposite** angles.




There is another pair of vertically opposite angles in the diagram—$b°$ and $d°$.

If you measured $a°$ and $c°$ with your protractor you would find they were the same size. The same is true for $b°$ and $d°$. Check both of these pairs of angles for yourself.

Vertically opposite angles occur on either side of the point where two straight lines intersect. Vertically opposite angles are equal.

eTutorial

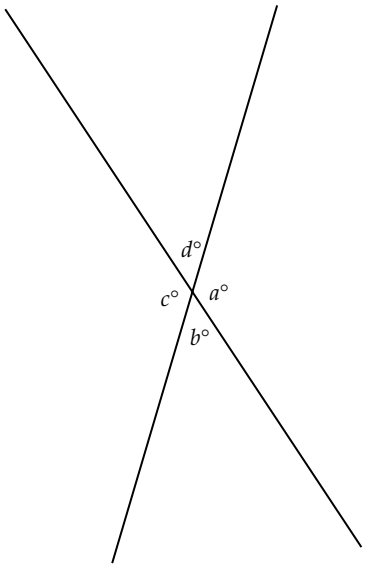
## exercise 5.7 Vertically opposite angles

Preparation: Exs 5.5 and 5.6

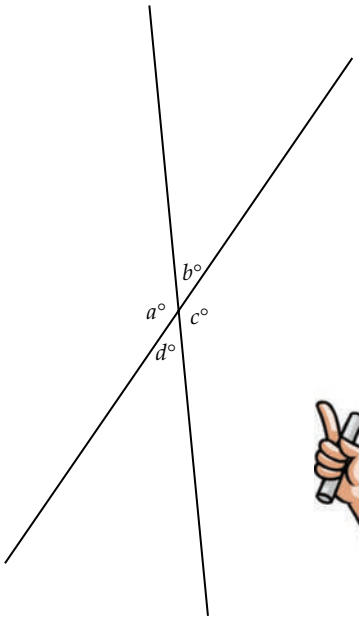
### Core

**1** Below are four pairs of vertically opposite angles labelled with pronumerals. Measure the size of the pairs of the vertically opposite angles.

**(a)**




**(b)**

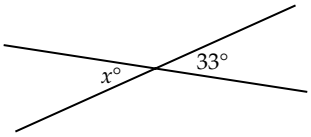



Hint

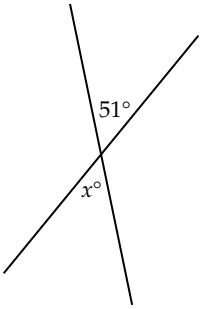
**2** Find the value of $x°$ in each case.

Hint

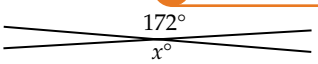
**(a)**



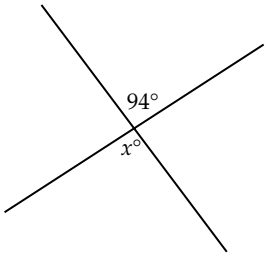

**(b)**



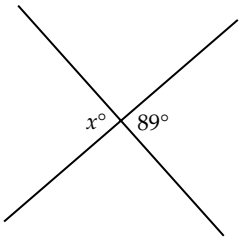

**(c)**



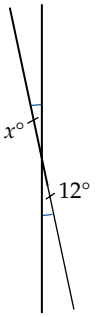

**(d)**




**(e)**



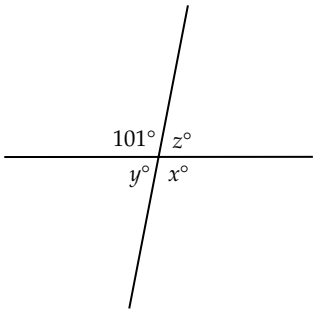

**(f)**



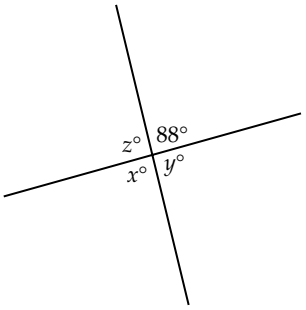

## Extension

**3** Find the value of the pronumerals in each case.

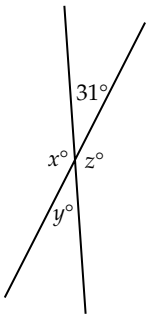
**(a)**




**(b)**




**(c)**




Hint

Worksheet C5.9

eQuestions

Homework 5.2

Worksheet A5.6

# languagezone

## Summary

Copy and complete the following summary of this chapter using the words and phrases from the list. A word or phrase may be used more than once.

**1** To measure an angle, the dot on the p_____ must match up with the _____ of the angle. The lines must match up with the arms of the angle.

**2** An angle of 40° has a _____ of 50°.

**3** _____ angles add to 180 _____.

**4** _____ _____ angles are equal.

**5** The angle names listed in order from largest to smallest are r_____, _____ _____, _____ _____, _____ _____, _____ _____ and _____ _____.

## Questions

**1** Use a dictionary to find the non-mathematical meanings of the words 'acute', 'obtuse' and 'reflex'. (Don't write these definitions down.) Use these definitions to explain why these names are given to certain angles.

**2** Another word for 'revolution' is 'perigon'. Guess what the prefix 'peri-' means, then look it up in the dictionary. Find two words that start with 'peri' and have related definitions. Write their meanings.

**3** If a revolution is split into two angles and one of them is reflex, what kind of angle is the other? Is there more than one option?

**4** A revolution is divided into three equal angles. What kind of angles are they?

**5** Explain the difference between 'complementary' and 'supplementary'.

**6** Unjumble these words:
TMMLOPCEEN NTIORLUEVO ROTCTPRAOR

**7** Write the key words from the list above that have part of the following words in them:
flexible revolve supply comply

## Key words

**acute angle**
**complement**
**complementary**
**degrees**
**obtuse angle**
**protractor**
**reflex angle**
**revolution**
**right angle**
**straight angle**
**supplement**
**supplementary**
**vertex**
**vertically opposite**

e Worksheet L5.1

e Worksheet L5.2

# chapter 5 REVIEW

## FAQs

*I keep getting confused with which protractor scale to use (the inside or outside one) when measuring angles. How can I be sure I am using the right one?*
You need to use the scale that starts at zero on the arm you are measuring from. Also, you should always estimate the size of the angle first. This is a way of checking your answer.

*Is a 90° angle called acute?*
No, a 90° angle is called a right angle. An acute angle must be *less than* 90°.

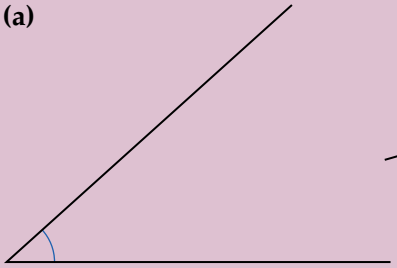
*Is ∠ABC the same as ∠CBA?*
Yes, these angles are the same. Angles can be read from either side, but it is important that the letter on the corner of the angle goes in the middle of the angle name.

### Core

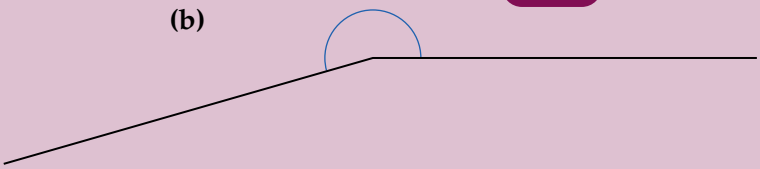
hi.com.au 5.1

**1** Use a protractor to measure each of the following angles.

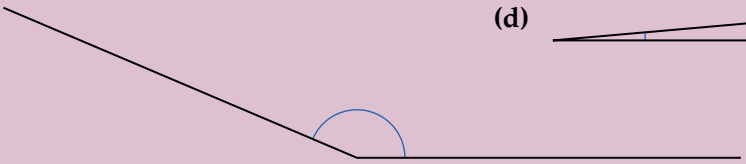
**(a)**



**(b)**

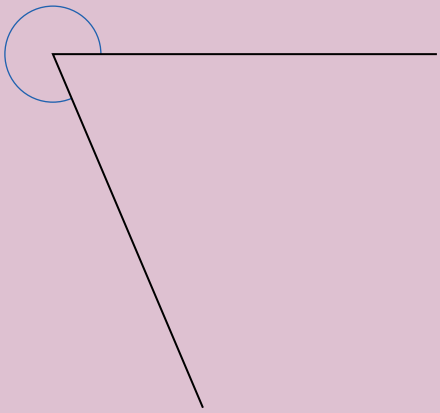


**(c)**

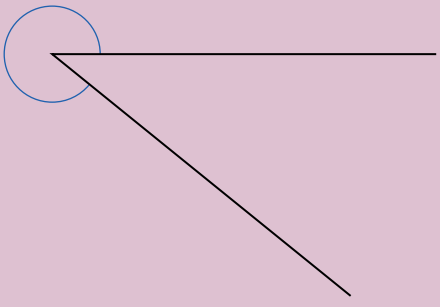


**(d)**

**(e)**



**(f)**

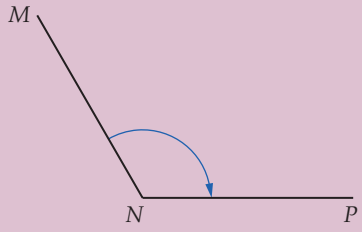


**2** Draw an angle of each size stated. 5.3

**(a)** 56° **(b)** 162° **(c)** 257° **(d)** 304°

**3** Choose the correct answer. 5.4

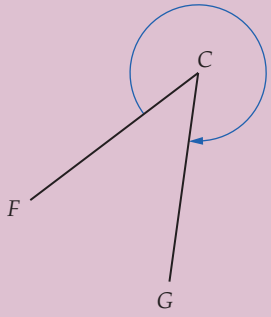
Which of the following is not true of the angle shown?




**A** It is named $\angle NPM$.
**B** It is named $\angle MNP$.
**C** It is approximately 120°.
**D** It is an obtuse angle.

**4** Choose the correct answer. 5.4

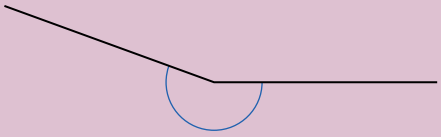
Which of the following is true of the angle shown?



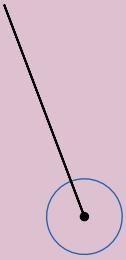

**A** It is obtuse.
**B** It is reflex.
**C** It is less than 180°.
**D** It is called $\angle CFG$.

**5** State the type of each angle below (i.e. acute etc.). 5.4

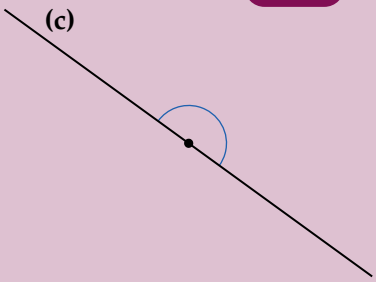
**(a)**



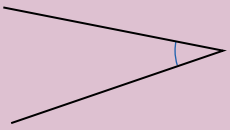
**(b)**



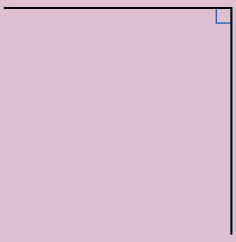
**(c)**



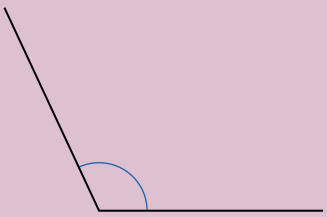
**(d)**



**(e)**

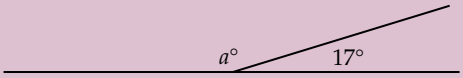


**(f)**

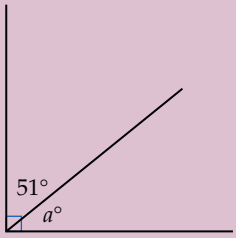


**6** Find the value $a°$ in each of the following diagrams. 5.5

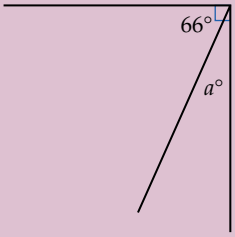
**(a)**



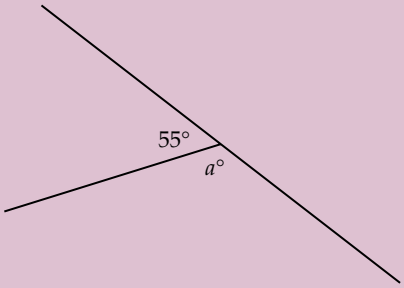

**(b)**




**(c)**

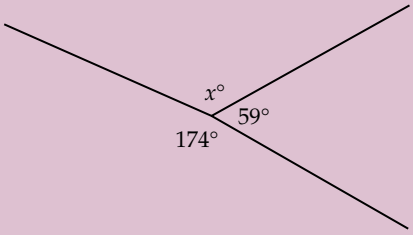



**(d)**

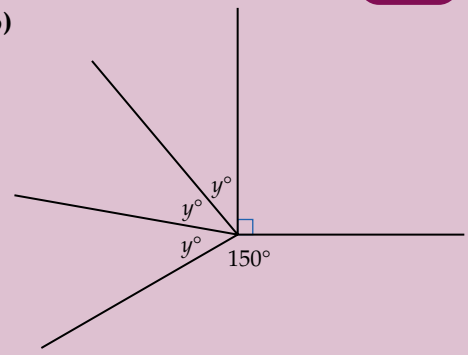



**7** Find the value of the pronumeral in each case. 5.6

**(a)**

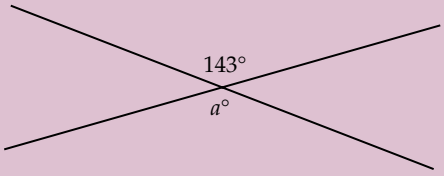



**(b)**

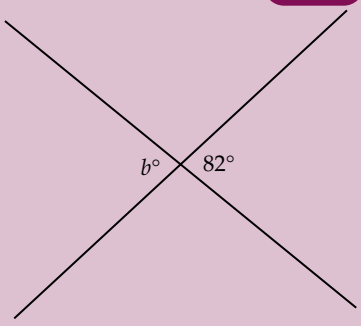



**8** Find the value of the pronumeral in each case. 5.7

**(a)**

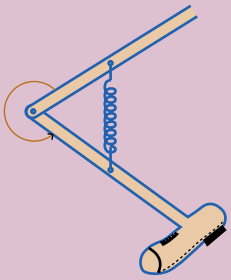



**(b)**



## Extension

**9** Eloise has constructed a mechanical leg to kick a football as part of her physical education investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



5.1

**10** Building the Leaning Tower of Pisa began in about the 12th century in Pisa, Italy. Ever since, it has gradually been tipping sideways, and it would have eventually fallen over if engineers and architects hadn't found a way to stabilise the base.

**(a)** Measure the angle that the tower leans over from vertical.

**(b)** If it has been falling for 800 years, calculate the average angle it moves each year.

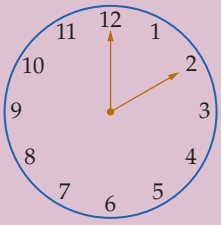
5.1

**11** On the clock face shown, what is the smaller angle made at the centre if the hands are pointing at:

**(a)** 12 and 2

**(b)** 12 and 7

**(c)** 2 and 7?




5.1, 5.6

## REPLAY

**1** Write the following Hindu–Arabic numbers as Roman numerals.

**(a)** 12 **(b)** 49 **(c)** 368 **(d)** 3056

1.1

**2** Copy and complete the following magic squares. 1.3

**(a)**

| | | |
|---|---|---|
| 18 | | 16 |
| | 15 | |
| 14 | | |

**(b)**

| | | |
|---|---|---|
| 10 | 5 | 12 |
| | | |
| 6 | | |

**(c)**

| | | |
|---|---|---|
| 7 | | 15 |
| | 13 | |
| | | 19 |

**3** Calculate: 1.7

**(a)** $(2 + 7) \times 6$ **(b)** $15 \div 5 + 7 \times 4$ **(c)** $600 \div (5 \times (3 + 9))$

**4** Find the next three numbers in each pattern. 2.2

**(a)** -7, -4, -1, __, __, __ **(b)** 16, 8, 0, __, __, __

**(c)** 22, 12, 2, __, __, __

**5** Calculate: 2.5

**(a)** $-56 + 60$ **(b)** $-45 - 120$ **(c)** $-67 + 100 - 12$

**6** Calculate: 2.6

**(a)** $-9 \times 7$ **(b)** $-12 \times -30$ **(c)** $-3 \times 4 \times -5$

**7** Write the first four multiples of each of the following. 3.1

**(a)** 7 **(b)** 12 **(c)** 15

**8** Which of the following numbers are divisible by 9? (Use the correct divisibility test, *not* your calculator.) 3.2
108, 569, 4734, 9512, 18 999, 28 008

**9** Simplify: 3.8

**(a)** $5^2 + 2^3$ **(b)** $2^6$ **(c)** $2^4 \times 10^4$

**10** Copy and complete the following number patterns. 4.1

**(a)** 65, 56, 47, __, __, __ **(b)** 1, 4, 9, __, __, __

**11** Write the following expressions using pronumerals $n$ for the IN number and $m$ for the OUT number. 4.2

**(a)** Multiply the IN number by two to get the OUT number.

**(b)** Add five to the IN number to get the OUT number.

**(c)** Subtract seven from the IN number then divide by two to get the OUT number.

**12** If $y = 2x - 3$, find the value of $y$ when: 4.3

**(a)** $x = 0$ **(b)** $x = 4$ **(c)** $x = -7$