4 patterns and ALGEBRA

Will the planets collide?

Algebra allows scientists to predict to an amazing degree of accuracy what happens in our night sky.

Halley's comet appeared twice last century. Scientists warned that the second viewing would be the worst in 2000 years. Better luck next time in 2061! We can also predict solar and lunar eclipses, the best location for viewing, and how long they will last. An Australian applied mathematician, Andrew Prentice, used mathematical calculations involving algebra to predict that Uranus and Neptune had many more moons than had been detected from Earth. These moons have since been confirmed.

So what about more extreme predictions? Pluto and Neptune trade places as the farthest planet in our solar system, so will they ever collide? Whatever the answer, it will all be there in the algebra.

e hi.com.au

outcomes

After completing this chapter you will be able to:

- describe a variety of patterns
- use algebraic expressions to describe patterns
- substitute into formulae
- use algebra to solve problems.

prepzone 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

Worksheet R4.1

1 The next four consecutive numbers after 87 are 88, 89, 90 and 91.

(a) What are the next four consecutive numbers after:
(i) 58 **(ii)** 1999?

(b) What are the next four consecutive *even* numbers after:
(i) 76 **(ii)** 94?

(c) Write down the four consecutive odd numbers between 44 and 52.

Worksheet R4.2

2 **(a)** Multiply each of these by 11 and subtract 3 from your answer.
(i) 4 **(ii)** 10 **(iii)** 200

(b) Multiply each of these by itself and then take 6 from your answer.
(i) 4 **(ii)** 10 **(iii)** 8

Worksheet R4.3

3 Write these sentences out using only numbers, the four operation signs and the equals sign.

(a) The sum of eleven and seventeen is twenty-eight.

(b) The difference between nine and seven is two.

(c) The product of three and four is twelve.

(d) The quotient of sixteen and eight is two.

Worksheet R4.4

4 Write these sentences out using only numbers, the four operation signs and brackets. Then use order of operations to find the answer.

(a) Take the number six, multiply it by two, and then subtract four from it.

(b) Add five to the number ten, then divide what you get by five.

(c) Multiply the sum of six and five by three.

(d) The difference between twenty-three and twenty-one is multiplied by twelve.

KEY WORDS

algebra	pattern	rule	table
formula	pronumeral	substitute	term

RETURN TO MAIN MENU

4.1 Number patterns

A number **pattern** is a list of numbers in which each number is related in some way to previous numbers. The numbers that make up the pattern are called **terms**.

eTutorial

exercise 4.1 Number patterns

Preparation: Prep Zone Q1 and 2

Core

1 Copy these number patterns and write down the next three terms in each.

(a) 1, 3, 5, …
(b) 2, 5, 2, 5, …
(c) 24, 22, 20, …
(d) 1, 0, 0, 1, 0, …
(e) −10, −15, −20, …
(f) 1, 2, 4, 8, …
(g) 64, 32, 16, …
(h) 8, 1, 7, 2, 6, …
(i) 54, 8, 56, 6, 58, 4, …
(j) 1, 2, 4, 7, …
(k) 34, 33, 31, 28, …
(l) 12, 21, 23, 32, 34, …
(m) 12, 22, 33, 45, …
(n) 5, 7, 6, 8, 7, …
(o) 1, 1, 2, 3, 5, …
(p) 3, 1, 4, 5, 9, …
(q) 1, 2, 5, 14, …
(r) 3, 4, 7, 16, …

Hint

2 Copy these number patterns and fill in the missing terms.

(a) 46, ____, 42, 40, ____, ____
(b) 13, ____, 19, 22, ____, ____
(c) ____, ____, 88, 85, 82, ____
(d) ____, 30, ____, 70, 90
(e) 1, 3, ____, 27, ____, ____
(f) ____, ____, 23, 32, 41, ____
(g) 2, ____, 5, 5, 8, ____, ____
(h) 3, ____, 7, 8, 11, ____, ____, 16
(i) 2, 3, ____, ____, 11, ____, 17, 19
(j) ____, 88, 99, 101, ____, ____, 131, 141, ____, ____, ____, ____, ____, 202

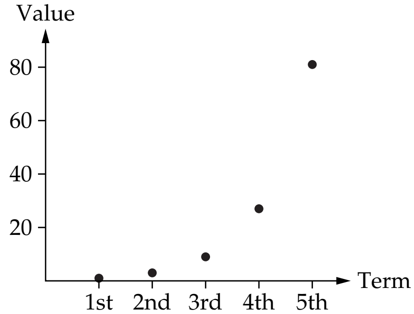
Hint

3 Write down the first five terms of the number patterns which follow these rules.

(a) Start with 5 and add 3 each time.
(b) Start with 29 and take away 4 each time.
(c) Start with 1 and multiply by 3 each time.
(d) Start with 80 and divide by 2 each time.
(e) Start with 3 and multiply by 2 and add 1 each time.
(f) Start with 1 and multiply by 4 and subtract 2 each time.

Hint

4 This grid shows the pattern made by the numbers in Question **3(c)**.



(a) Draw a similar graph for the other parts of Question **3**.

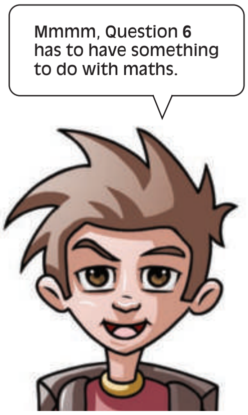
(b) **(i)** Identify and compare the graphs representing increasing number patterns.

(ii) Identify and compare the graphs representing decreasing number patterns.

(iii) Describe how the paths of the points differ and how this relates to the rule of the number pattern.

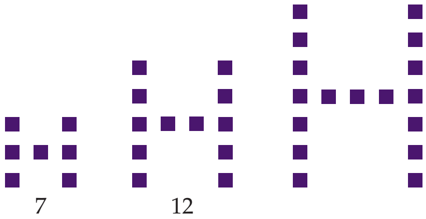
(c) For each of the graphs **3 (a)–(d)**, write two more rules (like those in Question **3**) that would create a similar pattern.

5 Kate writes a sequence of numbers that starts with a number between 10 and 15 and has three added each time. Write a possible sequence of the first four numbers in Kate's pattern.



6 What is the next term in this pattern?
O, T, T, F, F, S, S, E, N, …

7 Mr Heinemann has decided that he wants to use tiles to form large letter Hs which are embedded into the brickwork of the walls of his publishing company offices. He needs to work out how many tiles are needed to make different size Hs.



Copy and complete the table below. See if you can find the pattern to save time.

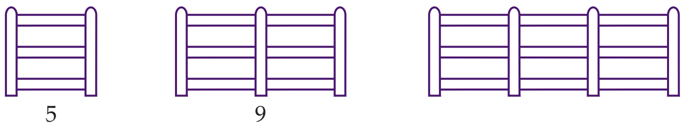
hi.com.au

Hint

Number of tiles in middle of H	1	2	3	4	5	6	7	8
Total number of tiles needed	7	12						

8 Frances wants to work out how many pieces of wood she needs to make a fence around her farm. She decides to look at sections of the fence to help her decide.

Hint



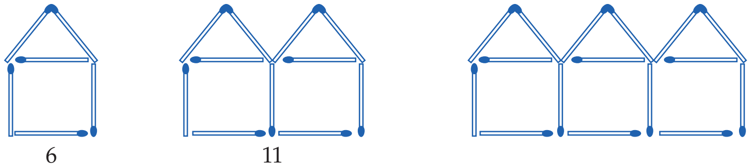
Copy and complete the table below. See if you can save time by finding the pattern.

Number of fence sections	1	2	3	4	5	6	7	8
Number of pieces of wood needed	5	9						

Frances decides she needs 16 fence sections to go all around her farm. How many pieces of wood does she need?

RETURN TO MAIN MENU

9 Count how many matchsticks it takes to make each of these house patterns.



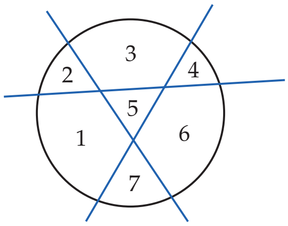
Draw the next house pattern and count the number of matchsticks needed. Copy and complete the table below. See if you can find the pattern to save time.

Pattern	1st	2nd	3rd	4th	5th	6th	7th	8th
Number of matchsticks	6	11						

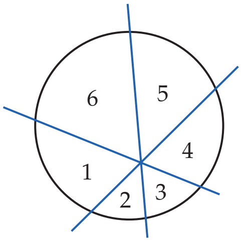
Extension

10 What is the greatest number of sections you can divide a circle into using three straight lines?

From the diagram, the answer is 7.



Note:

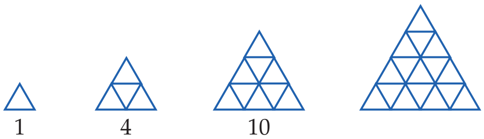


is not the best effort, since only 6 sections are created. You have to get the *maximum number possible*.

What is the maximum number of sections that can be created with eight lines? To find out, copy and complete the table below. Again, try to discover a pattern by drawing diagrams for the first few cases, and use the pattern to complete the table.

Number of lines	1	2	3	4	5	6	7	8
Maximum number of sections			7					

11 Count *all* the triangles that have an outside edge in each of the patterns below.



How many triangles do you think will have an outside edge in the fifth large triangle?

Copy and complete the following table. Try to find the pattern to help you.

Large triangle	1st	2nd	3rd	4th	5th	6th
Number of triangles	1	4	10			

Hint

This one is trickier than it looks. Don't just count the little triangles.

Worksheet A4.1

Worksheet A4.2

eQuestions

Working mathematically

problem solving

Odd sums

Find the sum of the first one hundred odd numbers.

Look for a pattern.

4.2 Algebra rules and formulae

Algebra is the language of mathematics that is used to describe rules and relationships.

When you are working with algebra, it helps if you can picture in your mind the Algebra Computer.

The Algebra Computer changes numbers that are put into it. It can be programmed to work according to any rule, and it will change any number you put into it according to that rule.

For example, a rule could be 'multiply each IN number by 3'. In this case the Algebra Computer would print out a table of values as shown.

IN	*OUT*
6	18
4	12
10	30
3	9
15	45
674	2022

No matter what number you put into the Algebra Computer, it will always multiply the number by 3 and give you the answer as the OUT number.

RETURN TO MAIN MENU

worked example 1

Use the rule 'add 7 to each IN number' to find out what the OUT numbers on the Algebra Computer would be.

IN	OUT
1	
22	
13	
2	
100	
38	

Steps

1. Take IN = 1
2. Take IN = 22
3. Take IN = 13
4. Take IN = 2
5. Take IN = 100
6. Take IN = 38
7. Complete the table of values.

Solution

OUT = 1 + 7 = 8

OUT = 22 + 7 = 29

OUT = 13 + 7 = 20

OUT = 2 + 7 = 9

OUT = 100 + 7 = 107

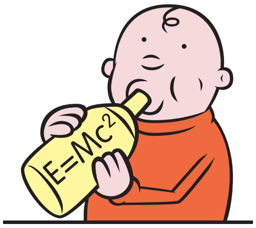
OUT = 38 + 7 = 45

IN	OUT
1	8
22	29
13	20
2	9
100	107
38	45

Changing rules to formulae

Much of algebra deals with shortcuts. Usually the rules that we have been using are written as formulae (or formulas). A **formula** is a rule expressed in symbols.

We need to be careful when there is more than one step in the rule. It helps to use brackets in these cases to make sure the correct operation is done first.



worked example 2

Rewrite each of these rules as formulae using IN and OUT.

(a) Subtract two from each IN number.

(b) Multiply each IN number by three and then add five

(c) Add thirteen to each IN number, then divide by two.

Steps

(a) Rewrite as a formula: to get the OUT number, subtract 2 from the IN number.

Solutions

(a) OUT = IN – 2

RETURN TO MAIN MENU

(b) Rewrite as a formula: to get the OUT number, multiply the IN number by 3, then add 5 to this. The brackets are used to show that we perform the operation inside them first.

(b) OUT = (IN × 3) + 5

(c) Rewrite as a formula: to get the OUT number, add 13 to the IN number, then divide this by 2.

(c) OUT = (IN + 13) ÷ 2

exercise 4.2 Algebra rules and formulae

Preparation: Prep Zone Q3 and 4

Core

Worksheet C4.1

Hint

1 Copy and complete the following tables. Use the rule given in each case to work out what the OUT numbers on the Algebra Computer would be.

(a) Add two to each IN number.

IN	OUT
13	
11	
7	
28	
–1	
65	

(b) Take five from each IN number.

IN	OUT
6	
18	
9	
85	
5	
101	

(c) Subtract eight from each IN number.

IN	OUT
19	
8	
15	
47	
–52	
100	

(d) Multiply each IN number by ten.

IN	OUT
3	
7	
20	
10	
55	
87	

(e) Multiply each IN number by five.

IN	OUT
4	
-8	
2	
12	
5	
1000	

(f) Double each IN number.

IN	OUT
2	
5	
10	
8	
11	
101	

(g) Multiply each IN number by two, then add three.

IN	OUT
12	
11	
7	
9	
100	
18	

(h) Take four from each IN number, then multiply by two.

IN	OUT
4	
5	
14	
-6	
47	
79	

(i) Multiply each IN number by itself, then add seven.

IN	OUT
2	
1	
6	
12	
300	
7	

2 Copy and complete the following tables. Use the rule given in each case to work out what the missing IN and OUT numbers on the Algebra Computer would be.

Hint

(a) Add one to each IN number.

IN	OUT
45	
58	
–9	
	5
	12
	51

(b) Take nine from each IN number.

IN	OUT
17	
12	
	12
	–20
	9
	80

(c) Divide each IN number by three.

IN	OUT
18	
–12	
	5
	11
	0
	20

(d) Multiply each IN number by itself.

IN	OUT
3	
11	
–7	
	36
	81
	100

(e) Add each IN number to itself.

IN	OUT
7	
13	
41	
	–20
	90
	24

(f) Double each IN number, then add one.

IN	OUT
6	
1	
13	
	1
	21
	101

3 Rewrite these rules as formulae using IN and OUT.

Hint

(a) Subtract eighteen from each IN number.
(b) Multiply each IN number by sixty.
(c) Divide each IN number by seven.
(d) Add forty-three to each IN number, then multiply by twenty.
(e) Multiply each IN number by one hundred, then subtract fifty.
(f) Divide each IN number by sixteen, then add thirteen.
(g) Multiply each IN number by itself.
(h) Divide each IN number by itself.
(i) Subtract twelve from each IN number, then divide by nine.
(j) Multiply each IN number by itself, then take away thirty-seven.

4 Choose the correct answer.
Which formula matches up to the rule given in each case?

(a) Multiply each IN number by twelve.
A OUT = IN + 12 **B** IN = OUT × 12
C OUT = 12 × IN **D** OUT = IN – 12

(b) Subtract fifty from each IN number.
A OUT = IN – 50 **B** OUT = 50 – IN
C OUT = IN ÷ 50 **D** OUT = IN + 50

(c) Add thirteen to each IN number, then multiply by nine.

Hint

A OUT = (IN + 9) × 13 **B** OUT = IN + 13 × 9
C OUT = (IN + 13) × 9 **D** OUT = (IN × 9) + 13

RETURN TO MAIN MENU

(d) Multiply each IN number by eight, then subtract fifteen.
A OUT = (IN × 8) + 15 **B** OUT = 15 – (IN × 8)
C OUT = IN – 15 × 8 **D** OUT = (8 × IN) – 15

(e) Multiply each IN number by itself, then add twenty-five.
A OUT = IN × 25 **B** OUT = IN × IN + 25
C OUT = IN + IN × 25 **D** OUT = IN × 2 + 25

Extension

5 Choose the correct answer.
Look at the tables and see if you can work out which rule the Algebra Computer has been using.

(a)

IN	*OUT*
4	10
22	28
5	11
-3	3
39	45
94	100

A Multiply each IN number by two, then add two.
B Add sixteen to each IN number, then divide by two.
C Add six to each IN number.
D Subtract six from each IN number.

Hint

(b)

IN	*OUT*
2	4
1	1
7	49
10	100
100	10 000
-13	169

A Multiply each IN number by two.
B Add two to each IN number.
C Add one to each IN number, then subtract one.
D Multiply each IN number by itself.

6 (a) Which of the tables **A–F** can be described using the following rules?
(i) OUT = IN +/– ___ **(ii)** OUT = IN ×/÷ ___
(iii) OUT = IN ×/÷ ___ +/– ___

Interactive

A

IN	*OUT*
13	9
46	42
31	27
9	5
7	3
28	24

B

IN	*OUT*
82	41
8	4
6	3
-12	-6
36	18
44	22

C

IN	*OUT*
5	50
-8	-20
10	15
12	24
50	500
100	70

D

IN	*OUT*
4	21
74	91
39	56
0	17
23	40
200	217

E

IN	*OUT*
90	44
100	49
6	2
2	0
12	5
60	29

F

IN	*OUT*
55	165
17	51
0	0
12	36
-4	-12
50	150

(b) Which table didn't fit any of the three forms?
(c) For each of the tables that did fit a rule, work out the rule being used.

Homework 4.1

4.3 Pronumerals and substitution

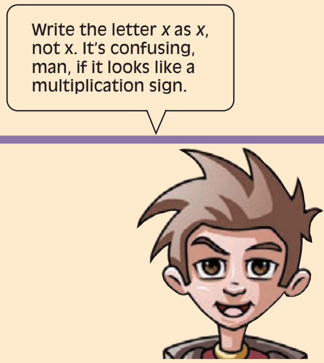
Because much of algebra deals with writing things in as short a way as possible, we usually don't use the words IN and OUT in the formulae. Instead we use letters. It doesn't matter which letters we use. It could be I for IN and O for OUT, but it could just as easily be x for IN and y for OUT.

Letters that take the place of numbers are called **pronumerals**.

worked example 3

Write out these formulae using pronumerals.

(a) OUT = IN – 4 **(b)** OUT = (IN + 7) ÷ 4



Steps

(a) Replace IN by a letter, say x, and OUT by another letter, say, y.

(b) Replace IN by a letter, say u, and OUT by another letter, say, j.

Solutions

(a) OUT = IN – 4
$y = x - 4$

(b) OUT = (IN + 7) ÷ 4
$j = (u + 7) \div 4$

To save even more time, when we are working with pronumerals we don't write the × sign. We also write the number before the letter.

For example, $b = a \times 5$ is written as $b = 5a$
$m = s \times 17 + 6$ is written as $m = 17s + 6$
$d = (k + 8) \times 9$ is written as $d = 9(k + 8)$

Substituting values into formulae

Once you've found the formula for a certain rule, you can use the formula to find what happens to other numbers that aren't in the table.

For example, suppose we had the following table:

a	b
2	15
5	45
11	105
8	75
10	95
20	195

The formula for this is $b = 10a - 5$

RETURN TO MAIN MENU

If the a value is 7, what would the b value be?

To answer this we **substitute** 7 for a in the formula and then work out the answer. Remember $10a$ means $10 \times a$.

$$\begin{aligned} b &= 10 \times 7 - 5 \\ &= 70 - 5 \\ &= 65 \\ \text{So} \quad b &= 65 \end{aligned}$$

worked example 4

For the formula $y = 8(x + 1)$:

(a) Find the value for y when x is 10.

(b) If the x value is 0, what would the y value be?

Steps	Solutions
(a) 1. Put in the multiplication sign.	**(a)** $y = 8 \times (x + 1)$
2. Substitute 10 for x in the formula.	$y = 8 \times (10 + 1)$
3. Calculate the value of y. Remember the order of operations: brackets first.	$y = 8 \times 11$ $y = 88$ When $x = 10$, $y = 88$.
(b) 1. Put in the multiplication sign.	**(b)** $y = 8 \times (x + 1)$
2. Substitute 0 for x in the formula.	$y = 8 \times (0 + 1)$
3. Calculate the value of y.	$y = 8 \times 1$ $y = 8$ When $x = 0$, $y = 8$.

eTutorial

exercise 4.3 Pronumerals and substitution

Preparation: Prep Zone Q3 and 4, Ex 4.2

Core

Worksheet C4.2

Hint

1 Write out these formulae using pronumerals. Use the letter x for IN and y for OUT in each case.

(a) OUT = IN – 18 **(b)** OUT = IN + 42

(c) OUT = 2 × IN **(d)** OUT = IN × 9

(e) OUT = IN × 6 **(f)** OUT = 12 × IN

(g) OUT = (IN × 4) – 7 **(h)** OUT = (IN × 13) + 50

(i) OUT = 7 + (IN × 6) **(j)** OUT = 100 – (3 × IN)

(k) OUT = (IN × 5) + (7 × IN) **(l)** OUT = (20 × IN) – (IN × 6)

(m) OUT = (IN + 20) × 9 **(n)** OUT = (IN + 100) × 2

(o) OUT = 6 × (IN – 4) **(p)** OUT = (IN – 2) × 4

eTester

Hint

2 For each of the following formulae, find the value of y for each x value.

(a) $y = x + 3$

(i) $x = 4$ **(ii)** $x = -6$ **(iii)** $x = 10$ **(iv)** $x = 9$

RETURN TO MAIN MENU

(b) $y = x - 1$
(i) $x = 5$ **(ii)** $x = 7$ **(iii)** $x = 30$ **(iv)** $x = -8$

Worksheet C4.3

(c) $y = 4x$
(i) $x = -2$ **(ii)** $x = 4$ **(iii)** $x = 10$ **(iv)** $x = 7$

(d) $y = 11x$
(i) $x = 3$ **(ii)** $x = 5$ **(iii)** $x = -7$ **(iv)** $x = 9$

(e) $y = 3x + 7$
(i) $x = 6$ **(ii)** $x = -9$ **(iii)** $x = 11$ **(iv)** $x = 2$

(f) $y = 2x - 1$
(i) $x = 3$ **(ii)** $x = 8$ **(iii)** $x = -50$ **(iv)** $x = 4$

(g) $y = 25 - 3x$
(i) $x = 7$ **(ii)** $x = 1$ **(iii)** $x = 8$ **(iv)** $x = -2$

(h) $y = 50 - 7x$
(i) $x = -7$ **(ii)** $x = 5$ **(iii)** $x = 1$ **(iv)** $x = 0$

(i) $y = 3(x + 5)$
(i) $x = 1$ **(ii)** $x = 6$ **(iii)** $x = 7$ **(iv)** $x = -5$

(j) $y = 11(10 - x)$
(i) $x = 4$ **(ii)** $x = 5$ **(iii)** $x = -10$ **(iv)** $x = 9$

(k) $y = 5(20 - x)$
(i) $x = 13$ **(ii)** $x = 19$ **(iii)** $x = -10$ **(iv)** $x = 0$

(l) $y = 2(x - 9)$
(i) $x = 19$ **(ii)** $x = -1$ **(iii)** $x = 109$ **(iv)** $x = 49$

(m) $y = 3(x + 4) + 2$
(i) $x = 7$ **(ii)** $x = 0$ **(iii)** $x = 1$ **(iv)** $x = -4$

(n) $y = 2(x - 10) - 1$
(i) $x = 13$ **(ii)** $x = 9$ **(iii)** $x = 110$ **(iv)** $x = 11$

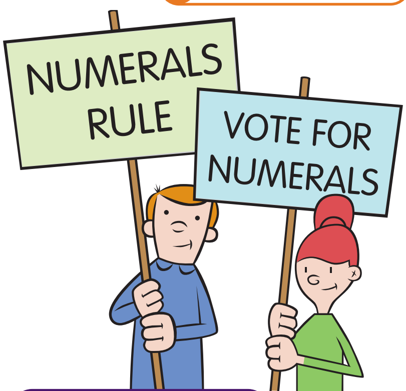
(o) $y = 2x + 5x$
(i) $x = 1$ **(ii)** $x = -1$ **(iii)** $x = 10$ **(iv)** $x = 4$

(p) $y = 3x + 10x$
(i) $x = 1$ **(ii)** $x = 5$ **(iii)** $x = 10$ **(iv)** $x = 0$

3 Answer TRUE or FALSE for each of these statements.

(a) If we substitute $a = 4$ into $b = 5a$ we get $b = 20$
(b) If we substitute $a = -9$ into $b = a + 11$ we get $b = 2$
(c) If we substitute $a = 2$ into $b = 16 + a$ we get $b = 32$
(d) If we substitute $a = 7$ into $b = 7 - a$ we get $b = 1$
(e) If we substitute $u = -4$ into $v = 6u + 1$ we get $v = 3$
(f) If we substitute $q = 10$ into $k = 13q - 8$ we get $k = 122$
(g) If we substitute $x = 6$ into $y = 4(x - 5)$ we get $y = 19$
(h) If we substitute $x = 12$ into $y = 5(14 - x)$ we get $y = 10$

Hint



A pronumeral rally

RETURN TO MAIN MENU

4 Use each of the following rules to complete the tables.

Hint

(a) $y = 7x$

x	y
6	
4	
10	
20	
–8	
101	

(b) $b = 4a$

a	b
11	
–20	
5	
9	
100	
50	

(c) $n = 6m$

m	n
1	
–2	
10	
6	
100	
5	

(d) $k = 8j - 2$

j	k
2	
–5	
11	
10	
9	
100	

(e) $q = 2p - 10$

p	q
11	
15	
–10	
20	
50	
100	

(f) $s = 4r + 7$

r	s
1	
2	
3	
–7	
10	
200	

(g) $v = 4(u - 1)$

u	v
5	
3	
–2	
12	
201	
6	

(h) $n = 3(m - 2) - 5$

m	n
5	
–2	
11	
22	
102	
52	

(i) $y = 7x + 2x$

x	y
10	
11	
3	
–2	
1	
100	

5 By looking at your completed tables, write the rules for Question **4 (g)–(i)** in a different way.

6 Make up a formula that uses addition and multiplication and use it to complete the following table.

x	y
7	
–20	
13	
101	

7 Look at the following tables and use the pronumerals given in each case to write the rule out as a formula.

Worksheet C4.4

Hint

(a)

x	y
7	4
4	1
3	0
10	7
12	9
100	97

(b)

c	d
9	20
2	13
1	12
15	26
40	51
0	11

(c)

p	q
6	–2
11	3
100	92
0	–8
1	–7
7	–1

(d)

r	s
10	50
5	25
1	5
4	20
9	45
20	100

(e)

m	n
12	–36
9	–27
3	–9
10	–30
1	–3
–6	18

(f)

a	b
8	32
15	60
100	400
60	240
1	4
4	16

RETURN TO MAIN MENU

8 The formula $y = 3(x + 2) - 1$ gives the same results as $y = 3x + 5$.

(a) Substitute three different x values into each equation to show they give the same result.

(b) Which equation is better to use? Why?

Extension

9 Look at the following tables and use the pronumerals given in each case to write the rule out as a formula.

Hint

(a)

p	q
1	6
2	11
3	16
4	21
5	26
6	31

(b)

x	y
10	16
2	0
3	2
4	4
5	6
6	8

(c)

s	t
3	14
10	28
100	208
1000	2008
0	8
1	10

(d)

j	k
100	303
5	18
7	24
10	33
1	6
20	63

(e)

m	n
6	40
7	45
0	10
10	60
100	510
200	1010

(f)

c	d
5	14
9	26
1	2
3	8
10	29
2	5

(g)

d	z
1	14
6	49
5	42
100	707
9	70
4	35

(h)

c	q
20	140
10	40
36	300
6	0
7	10
9	30

(i)

f	v
9	800
1	0
2	100
7	600
50	4900
100	9900

eQuestions

speedingzone

Do these in your head as quickly as you can and write down the answers.

Time target: 2 minutes

1 7×25

2 \$12.00 + \$19.25

3 $3000 \div 150$

4 5 squared plus 2 cubed

5 $\frac{1}{2}$ of 250 m

6 $-120 - 81$

7 $\sqrt{6400}$

8 91×200

9 > or <: $\frac{1}{5}$ __ $\frac{1}{4}$

10 $278 - 300$

RETURN TO MAIN MENU

laughzone

Answer the following, showing your working, and then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

For each of the following formulae, find the value of y for each x value.

$y = 3x$

$x = 2$	**S**	$x = 0$	**X**	$x = -3$	**A**	$x = 11$	**G**

$y = 4x - 3$

$x = 5$	**N**	$x = -1$	**T**	$x = 2$	**I**	$x = 100$	**B**

$y = 2(x + 9)$

$x = -4$	**E**	$x = 0$	**L**	$x = -10$	**R**	$x = 191$	**H**

‘

−9	18	33	10	397	−2	−9

5	6

0

-

6	5	33	400	−7	5	17	33

’

RETURN TO MAIN MENU

Working mathematically

problem solving

Cutting string

1 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces of string again, how many pieces will you have? Check your answer using some string and scissors, then complete the following table.

Number of cuts	0	1	2	3	4	5	6	*c*
Total number of pieces								

2 This time, fold the piece of string in half each time before you cut it, as Minh is doing. How many pieces at each cut do you have now? *Remember:* Cut only one piece at a time.

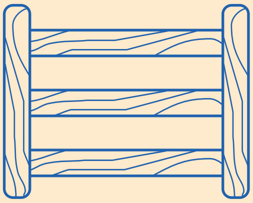
Number of cuts	0	1	2	3	4	5	6	*c*
Total number of pieces								

4.4 Using algebra in problems

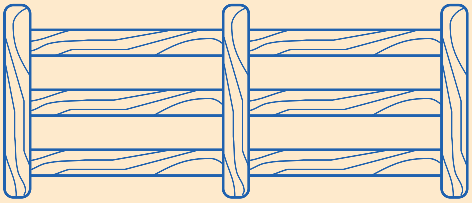
Algebra is useful for solving problems because it helps you to understand and work with rules that are behind patterns.

worked example 5

Frederico is building a fence around Farside Farm. The fences are made up of one-metre pieces of timber as shown.



1 section
5 pieces



2 sections
9 pieces

It is useful to find a rule relating the number of sections of fence to the number of pieces of wood needed. This rule can then be used to find how many pieces of wood would be needed for a larger number of sections.

(a) Choose pronumerals to represent the two different quantities in the relationship.

(b) Draw up a table of values and use the diagrams to fill in the values. Draw two or three more diagrams and add these values to the table.

(c) By looking at the table, find the formula connecting the two pronumerals.

(d) Use the formula to find the number of pieces of timber needed for a fence made up of 50 sections.

Steps

(a) Choose pronumerals. It is easier if you select letters that have the same first letter as the quantities.

(b) 1. Draw a table. The headings of the columns are the letters you chose in part **(a)**.

2. Fill in any values you know from the diagrams.

3. Draw more diagrams to find more values.

4. Add these values to the table.

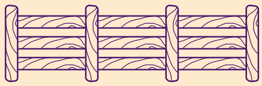
Solutions

(a) Let the number of sections be S and the number of pieces of timber be P.

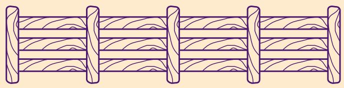
(b)

S	P

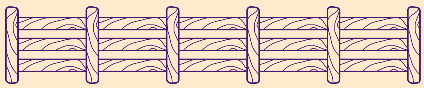
S	P
1	5
2	9



3 sections
13 pieces



4 sections
17 pieces



5 sections
21 pieces

S	P
1	5
2	9
3	13
4	17
5	21

(c) 1. Find the rule. By looking at the table for a while, you should be able to work out that the rule is 'multiply each section by four, then add one'. Write this in words if it helps.

(c) number of pieces
= (number of sections × 4) + 1

2. Write the formula using pronumerals.

$P = S \times 4 + 1$

(d) 1. Write what you are trying to find in algebra.

(d) Find P when $S = 50$.

2. Substitute the value $S = 50$ into the formula.

$P = 4S + 1$
$= 4 \times 50 + 1$

3. Evaluate.

$P = 200 + 1$
$= 201$

4. Write the answer in words.

To make a fence of 50 sections, 201 pieces of timber are needed.

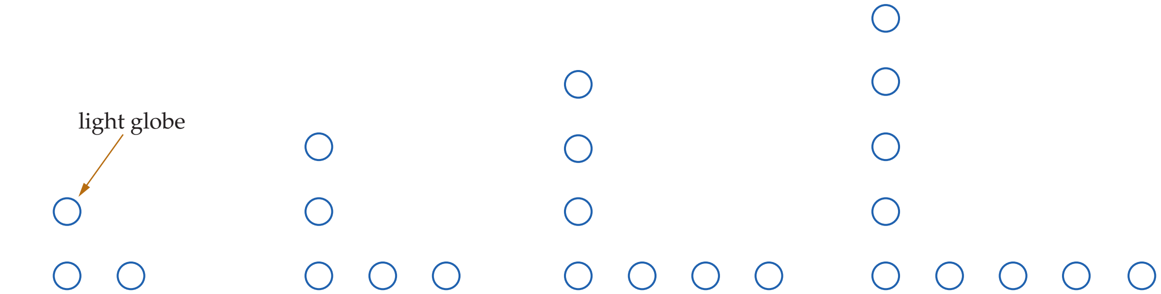
exercise 4.4 Using algebra in problems

p Preparation: Prep Zone Q1–4, Exs 4.1–4.3

e hi.com.au

Core

1 Lightworks International Co. (motto: 'Watch us shine!'), designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.



height of L = 2
number of globes = 3

height of L = 3
number of globes = 5

height of L = 4
number of globes = 7

height of L = 5
number of globes = 9

(a) Copy and complete: Let H represent the height of the letters Ls, and G represent the ____________ needed to make the sign.

e Animation

(b) Copy and complete the table. Draw the next two diagrams and add these values to the table.

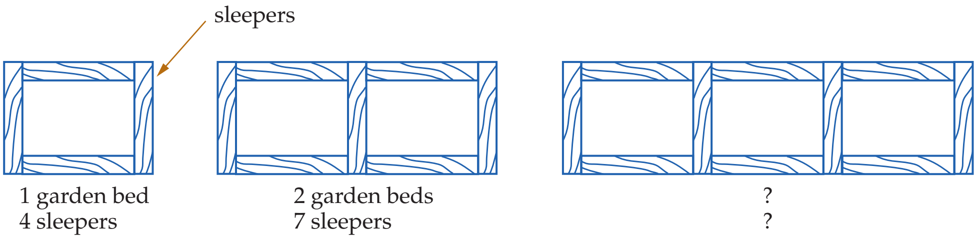
H	G
2	3
3	5
4	
5	

(c) By looking at the table in part **(b)**, find a formula connecting G to H.

(d) Substitute $H = 120$ into the formula to find out how many globes would be needed to make an L with a height of 120.

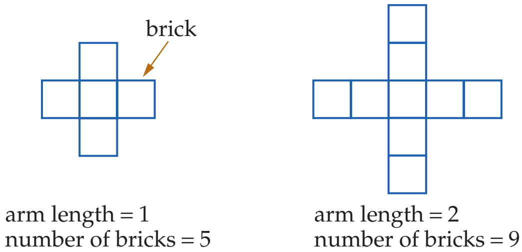
2 Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds such as in the examples below. The city council has contracted her to divide several large areas of the City Central Gardens into different beds.

Hint



(a) Choose pronumerals to represent garden beds and sleepers.
(b) Complete a table for values up to 5 garden beds.
(c) Find the formula connecting the two pronumerals.
(d) Find how many sleepers would be needed to divide the garden into a section made up of 21 beds, one made up of 40 beds and one made up of 111 beds.

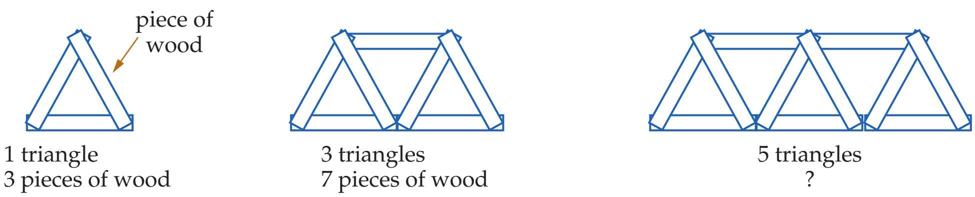
3 The White Cross charity organisation has a white cross as its emblem. Their buildings all have a white cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.



(a) Choose pronumerals to represent arm length and number of bricks.
(b) Complete a table for values up to 4 arm lengths.
(c) Find the formula connecting the two pronumerals.
(d) Find how many white bricks would be needed to make a cross which has an arm length of 52.

4 Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the ones shown below, running across the walls.

Hint



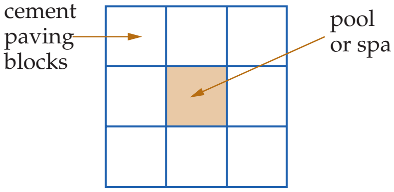
(a) Choose pronumerals to represent triangles and pieces of wood.
(b) Complete a table for values up to 9 triangles.
(c) Find the formula connecting the two pronumerals.
(d) Find how many pieces of wood would be needed to make a total of 203 triangles.

RETURN TO MAIN MENU

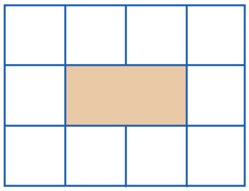
Extension

Hint

5 Plunge & Co., the Pool Paving People (motto: 'We get around!'), specialise in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.



length of spa = 1
number of paving blocks = 8



length of spa = 2
number of paving blocks = 10

(a) Choose pronumerals for spa length and number of paving blocks.

(b) Complete a table for values up to 4 spa lengths.

(c) Find the formula connecting the two pronumerals.

(d) Plunge & Co. have recently been asked by the Australian Swimmers' Guild to build a single-lane lap pool to help train their long-distance swimmers. They have worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks they will need to pave around the whole pool.

(e) For another job, Plunge & Co. use 20 paving blocks to surround an outdoor spa. What is the length of this spa?

Homework 4.2

Worksheet A4.3

Worksheet A4.4

Working mathematically

investigation

The handshake problem

How many handshakes would it take for a group of ten people at a party to introduce themselves to each other?

1 Andrea and Bill arrive at the same time and shake hands to greet each other. Obviously one handshake is needed. If three people, Andrea, Bill and Cecil (*A*, *B* and *C* for short) arrived together, the handshakes required could be represented as *AB AC BC*, i.e. 3 handshakes required.

Now, a fourth person, Deirdre (*D*), arrives.

(a) Write down the different handshakes that are needed for each person to greet the other three.

(b) How many handshakes are required?

2 How many handshakes are required if there are five people?

3 Copy and complete the table below.

Number of people	1	2	3	4	5	6	7	8	9	10
Number of handshakes	0	1	3							

4 Can you discover a pattern relating the number of handshakes to the number of people? Describe this pattern.

Working mathematically

investigation

Cups and counters

In Chapter 10 you will learn about different ways of writing algebraic expressions. Here you will be introduced to some of these through the use of cups and counters.

In this investigation you will work with a partner. Each counter has the value 1. The cups themselves have no value; they are used to organise the counters.

1 **(a)** You and your partner each need to begin with an empty cup. Place three counters in each cup and one counter on the table next to the cups. How many counters do you have altogether?

(b) Remove a counter from each cup. How many counters do you have altogether now?

(c) Copy and complete the following table.

Number of counters in each cup	1	2	3	4	5	6	n
Total number of counters		5	7				

(d) Copy and complete: Where n represents the number of counters in each cup, the number of counters is ___ + ___.

(e) If n equals 20, how many counters would you have?

2 **(a)** You and your partner each put the same amount of counters in a cup and take 4 additional counters each. If n represents the number of counters in each cup, how many counters do you have altogether?

(b) **(i)** Using n write an expression to show how many counters your partner has.

(ii) Copy and complete: You have the same number of counters as your partner so this means that together you have 2(______) counters.

(c) **(i)** If you each have one counter in your cup, how many counters do you have altogether?

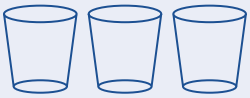
(ii) If you each have no counters in your cup, how many counters do you have altogether?

(d) Copy and complete the following table to show that your answers to parts **(a)** and **(b) (ii)** are equivalent.

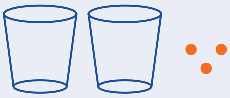
Number of counters in each cup, n	0	1	2	3	4	5	6
$2n + 8$	8						
$2(n + 4)$	8						

3 Write an expression using n, where n represents how many counters there are in each cup, for each situation below.

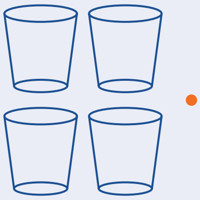
(a)



(b)

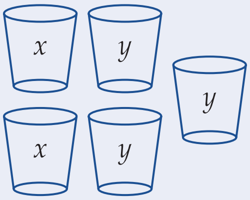


(c)

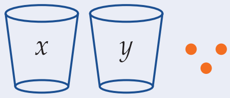


4 In the diagrams below, the cups marked x have x counters in them and the cups marked y have y counters in them.
Write an expression to represent how many counters are in each of the following.
(Use brackets in your answer to part **(c)**.)

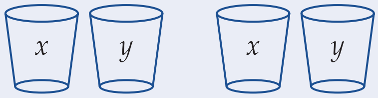
(a)



(b)



(c)



5 Draw a diagram similar to those in Questions **3** and **4** to represent each of the following expressions.

(a) $4x + y + 2$ **(b)** $3(x + y)$ **(c)** $2(x + y) + x$

RETURN TO MAIN MENU

maths@work

Editor: Sally Woollett

Company: self-employed
Qualifications/Experience: Bachelor of Applied Science (Monash University); Graduate Diploma in Editing and Publishing (RMIT)
Related occupations: publisher, author, proofreader

As a freelance editor, my job is to help publishers in the process of turning an author's work into, for example, a printed textbook. This requires careful checking of all parts of the book, such as the order of the information and the way in which it is written. When I edit maths books, I can use my knowledge of maths to spot answers that don't make sense, or perhaps to identify when there is a step missing in a worked example.

My favourite part of mathematics at school was probability. I like to calculate how likely it is that a particular event will happen.

Most books are printed in smaller parts called sections. These are usually 16 pages long. After printing, these sections are all folded and joined together in the correct order to make the final book. Before a book is printed, printers need to know how many sections it will contain. Sometimes I need to include inserts in textbooks. Because of the way books are printed in 16-page sections, an insert can only be made either in the middle of a 16-page section or between 16-page sections. This means I need to do some calculations to see where these inserts can go.

If I let x represent the section number, then $16x$ tells me the page number at the end of the xth section and $16x - 8$ tells me the page number on the left in the middle of section x.

The editor's problem

1 **(a)** Sally is editing a new edition of a textbook, and she wants to include a four-page colour insert in the middle of each of the sections 10, 14 and 20. Use $16x - 8$ to find after what pages she can place the inserts.

(b) In the book Sally is editing, there are 16 pages in the introductory section before Chapter 1 begins (numbered in Roman numerals from i to xvi) and the first page of Chapter 1 is numbered page 1. What are the actual page numbers of the pages after which Sally will place the inserts?

2 The last book that Sally is editing has 18 pages in the introductory section (numbered in Roman numerals from i to xviii). She wants to include a four-page colour insert after section 15. After what page number can she place the insert?

RETURN TO MAIN MENU

languagezone

Summary

Copy and complete the following summary of this chapter using the words from the list. A word may be used more than once.

1 The missing ______ in the number ______ 3, …, 9, 12 is 6.

2 A ______ expressed in symbols is called a ______.

3 A ______ can be a letter or symbol that represents a number.

4 To find the value of y using the ______ $y = 2x + 3$, when $x = 1$, we need to ______ 1 for x.

5 In ______ we try to write things in as short a way as possible.

6 The rule OUT = IN × 6 can be used to complete a ______ of values.

Key words

algebra
formula
pattern
pronumeral
rule
substitute
table
term

Questions

1 Write the meaning of the word 'substitute'.

2 Is every set of numbers a pattern? Explain.

3 The word 'term' has many meanings. Write one mathematical meaning and one non-mathematical meaning of this word.

4 Is $f = e + 7$ an example of a rule or a term? Explain.

5 Try to make at least 10 words, of three letters or more, from the letters of 'pronumeral'.

6 The following words from this chapter are missing their vowels (a, e, i, o, u). Copy and complete the words.

(a) n _ m b _ r **(b)** c _ m p _ t _ r

(c) s _ l v _

7 One way to write the plural of 'formula' is 'formulae'. What is the other way?

Worksheet L4.1

Worksheet L4.2

chapter 4 REVIEW

FAQs

Do all groups of numbers have a pattern?
No. Some groups of numbers can be completely random.
For example, 3, 7, 1, 100, 267, 13.

Can all number patterns be described using algebra?
Yes, but many number patterns are very complicated and beyond what you would be expected to know in Year 7.

Core

1 Copy and complete the following tables. Use the rule given in each case to work out what the missing OUT numbers on the Algebra Computer would be. 4.2

(a) Add three to each IN number.

IN	OUT
57	
34	
–12	
4	
1	
64	

(b) Multiply each IN number by three, then subtract five.

IN	OUT
4	
3	
2	
9	
10	
12	

(c) Subtract eight from each IN number, then multiply by two.

IN	OUT
38	
9	
10	
58	
–2	
408	

2 Copy and complete the following tables. Use the rule given in each case to work out what the missing IN and OUT numbers on the Algebra Computer would be. 4.2

(a) Divide each IN number by five.

IN	OUT
15	
35	
100	
	1
	0
	6

(b) Multiply each IN number by twelve.

IN	OUT
12	
–8	
	36
	12
	0
	60

(c) Divide each IN number by two, then add one.

IN	OUT
10	
14	
6	
	3
	11
	26

RETURN TO MAIN MENU

3 Choose the correct answer. **4.2**
Which formula matches up to the rule given in each case?

(a) Add six to each IN number.
A OUT = IN – 6 **B** OUT = IN + 6
C OUT = IN × 6 **D** IN = OUT + 6

(b) Subtract five from each IN number, then multiply by three.
A OUT = (IN – 3) × 5 **B** OUT = (IN + 5) × 3
C OUT = (IN – 5) × 3 **D** OUT = IN – 5 × 3

(c) Divide each IN number by four, then add thirteen.
A OUT = (IN ÷ 4) + 13 **B** OUT = (IN + 13) ÷ 4
C OUT = (IN ÷ 13) + 4 **D** OUT = IN ÷ (4 + 13)

4 **(a)** Substitute the given values in the following formula to find p. **4.3**
$p = 20a - 6$
(i) $a = 3$ **(ii)** $a = 4$ **(iii)** $a = 10$ **(iv)** $a = -5$

(b) Substitute the given values in the following formula to find the value of k.
$k = 2(c - 4) + 11$
(i) $c = 6$ **(ii)** $c = 4$ **(iii)** $c = -6$ **(iv)** $c = 14$

5 Choose the correct answer. **4.3**

(a) If you substitute $x = 3$ into the formula $y = 5x - 2$, the value of y is:
A $y = 3$ **B** $y = 5$ **C** $y = 13$ **D** $y = 17$

(b) If you substitute $g = 5$ into the formula $k = 7(9 - g)$, the value of k is:
A $k = 28$ **B** $k = 35$ **C** $k = 58$ **D** $k = 63$

6 Use the following rules to complete the tables. **4.3**

(a) $y = x - 7$

x	y
7	
9	
12	
20	
-8	
107	

(b) $y = 10(x - 3)$

x	y
4	
5	
13	
-7	
78	
54	

(c) $y = 3(x + 5) - 2$

x	y
0	
-5	
15	
5	
4	
95	

7 For each of the tables on the following page, work out which rule is being used and write it as a formula using the pronumerals. **4.3**

(a)

x	*y*
76	69
54	47
8	1
28	21
–9	–16
103	96

(b)

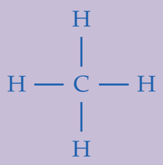
a	*b*
15	34
10	24
2	8
7	18
9	22
22	48

(c)

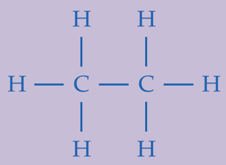
v	*w*
5	499
10	999
3	299
1	99
8	799
56	5599

Extension

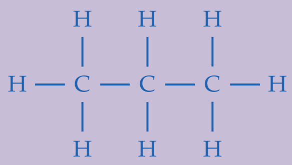
8 Atoms of carbon (C) combine with atoms of hydrogen (H) to make various hydrocarbon molecules. These diagrams show the structures of some 'straight-chain' hydrocarbons known as alkanes. The letters represent atoms and the lines joining atoms represent bonds. **4.1**



Methane



Ethane



Propane

How many hydrogen atoms would be needed to make decane (ten carbon atoms)? Copy and complete the table to answer this question. Try to discover a pattern to save time.

Name of alkane	*Number of carbon atoms*	*Number of hydrogen atoms*
Methane	1	4
Ethane	2	6
Propane	3	
Butane	4	
Pentane	5	
Hexane	6	
Heptane	7	
Octane	8	
Nonane	9	
Decane	10	

9 Consider the hydrocarbon alkanes described in Question **8**. **4.4**

(a) Draw up a table of values showing a column headed *C* which lists the number of carbon atoms, and a column headed *B* which lists the number of bonds needed to make the alkane. Use the pictures above to fill in values.

(b) Draw a diagram to work out how many bonds would be needed to form butane (four carbon atoms). Then draw another to work out how many are needed to make pentane (five carbon atoms). Enter these two sets of values into the table.

RETURN TO MAIN MENU

(c) By looking at the table, find a formula connecting B to C.
(d) How many bonds would be needed to make decane (ten carbon atoms) and hectane (100 carbon atoms)?

REPLAY

1 Set out these calculations in your normal way and work out the answers.
(a) 378 – 152 **(b)** 6780 – 496 **(c)** 1952 – 1078

Worksheet R4.5

2 Copy and complete the following number patterns.
(a) 1, 1.5, 2, __, __, __ **(b)** 0.01, 0.03, 0.05, __, __, __
(c) 2.4, 2.3, 2.2, __, __, __

Worksheet R4.6

3 State whether each of the following is a proper fraction, an improper fraction or a mixed number.
(a) $\frac{7}{5}$ **(b)** $1\frac{3}{8}$ **(c)** $\frac{7}{22}$ **(d)** $\frac{1}{100}$ **(e)** $\frac{25}{2}$ **(f)** $10\frac{1}{11}$

Worksheet R4.7

4 Write the following numbers using the Babylonian system.
(a) 12 **(b)** 64 **(c)** 131

1.1

5 Use rounding to the first digit to find estimates for these multiplications.
(a) 207×38 **(b)** 5780×95 **(c)** $6 \times 42\,874$

1.5

6 Simplify:
(a) $18 \div 6 + 3$ **(b)** $4 \times 6 \div 2 \times 3$ **(c)** $(33 - (7 + 6)) \times 5$

1.6

7 Write a directed number to represent each of the following.
(a) A loss of $500 **(b)** A weight gain of 2.7 kg
(c) 348 m below sea level

2.1

8 Calculate:
(a) $5 - 12$ **(b)** $^{-}17 + 10 - 13$ **(c)** $100 - 150 + 86$

2.5

9 Calculate:
(a) $^{-}56 \div 8$ **(b)** $\frac{^{-}60}{^{-}12}$ **(c)** $2400 \div {^{-}60}$

2.7

10 List the first five multiples of:
(a) 4 **(b)** 6 **(c)** 20

3.1

11 List all numbers less than 10 that have exactly three factors.

3.3

12 Evaluate the following without using a calculator.
(a) $\sqrt{64}$ **(b)** $\sqrt{100}$ **(c)** $\sqrt{4900}$

3.7

Assignment 4