

exploring

The truth is out there in the numbers

Mathematics is sometimes said to be the universal language that we could use to prove to aliens that we are intelligent but would it be a simple task?

What sort of mathematics could we send to aliens? We couldn't just send out some complicated number or number pattern. Our number system is based on tens—mainly because we have 10 fingers. So who knows what an alien society's number system is based on? A pattern in our number system might not be a pattern in another number system. The best numbers to send to show we are an intelligent people would be prime numbers—numbers such as 2, 3, 5, 7, 11 that have exactly two factors. Prime numbers will have this special property in any number system.

outcomes

After completing this chapter you will be able to:

find multiples of numbers

- find factors of numbers
 - express a number as a product of prime factors
 - explore other groups of numbers, such as palindromes, Fibonacci numbers and triangular numbers
 - find squares and square roots
 - find cubes and cube roots
 - use index notation.

prepzone3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your eMaths Zone CD or ask your teacher for the Replay Worksheet. Worksheet R3.1 **1** Copy and complete these within three minutes. (a) $6 \times 7 =$ $6 \times 6 =$ $6 \times 8 =$ $6 \times 4 =$ $6 \times 11 =$ **(b)** $7 \times 11 =$ $7 \times 7 =$ $7 \times 5 =$ $7 \times 2 =$ $7 \times 3 =$ (c) $8 \times 7 =$ $8 \times 8 =$ $8 \times 6 =$ $8 \times 4 =$ $8 \times 10 =$ (d) $9 \times 12 =$ $9 \times 3 =$ $9 \times 5 =$ $9 \times 8 =$ $9 \times 11 =$ (e) $12 \times 7 =$ $12 \times 12 =$ $12 \times 11 =$ $12 \times 6 =$ $12 \times 9 =$ 😑 Worksheet R3.2 **2** (a) List all the digits with which an even number can end. (b) List all the digits with which an odd number can end. 😑 Worksheet R3.3 **3** (a) Rearrange the following numbers in ascending order (i.e. from smallest to largest). 567, 4500, 0, 74, 11 100, 6008, 12, 602 (b) Rearrange the following numbers in descending order (i.e. from largest to smallest). 110, 11 011, 1011, 111, 10, 1001, 1101, 10 111 Worksheet R3.4 **4** Find: (a) $3 \times 2 \times 3$ (b) $5 \times 3 \times 3 \times 2$ (c) $2 \times 2 \times 2$ (d) $10 \times 10 \times 10 \times 10$ e Worksheet R3.5 5 Find: (a) $50\,000 + 6000 + 800 + 90 + 5$ **(b)** $7 \times 1000000 + 2 \times 10000 + 5 \times 1000 + 7 \times 10 + 3$

KEY WORDS

base ex	cpanded form	indices	square number
composite number fac	ctor	multiple	square root
cube number fac	ctor tree	palindrome	triangular number
cube root Fil	bonacci numbers	power	
divisibility test in	dex	prime factor	
divisible in	dex form	prime number	

3.1 Multiples

Multiples of a number can be thought of as the answers to the 'times tables' for that number. For example, the times table for 7 is:

$1 \times 7 = 7$ $2 \times 7 = 14$ $3 \times 7 = 21$ $4 \times 7 = 28$ $5 \times 7 = 35$ and so on.	7, 14, 21, 28, 35 are some of the multiples of 7.
$4 \times 7 = 28$ 5 × 7 = 35 and so on.	some of the multiples of 7.

Another way of finding multiples of 7 is to count by 7s: 7, 14, 21, 28, 35, ...

The list of multiples for any number will continue forever.

To find out if a number is a multiple of another (smaller) number, just divide and see if the answer is a whole number. For example, 300 is a multiple of 12 because $300 \div 12 = 25$, a whole number.

<u>Multiples</u>

Preparation: Prep Zone Q1



Core

exercise 3.1

1	1 Find the first five multiples of each of these.												
	(a)	2	(b)	3	(c)	4	(d)	8	(e)	6	(f)	5	e Hint
	(g)	9	(h)	11	(i)	14	(j)	15	(k)	16	(1)	19	Worksheet C3 1
	(m)	20	(n)	50	(o)	100	(p)	2000					
2	Find	l the firs	t thre	ee multi	ples o	of each o	of the	ese.					
	(a)	70	(b)	75	(c)	86	(d)	123	(e)	345	(f)	99	e Hint
	(g)	738	(h)	815	(i)	1250	(j)	1999	(k)	2005	(1)	3111	
	(m)	8410	(n)	9010	(o)	10004	(p)	10 211					
3	Cho	ose the	corre	ect answ	er.								
	(a)	Which	one o	of the fo	llowi	ng num	bers	is not a	multi	iple of 12	2?		
		$\mathbf{A} \hspace{0.1in} 564$		В	346	5	(C 780		D	1188	Galcul	atos
	(b)	Which	of th	e follow	ing n	umbers	is no	ot a mult	iple o	of 23?			
		A 529		В	943	3	(C 23		D	853		
=,,	tei	nsion											
		131011											
4	Shai	ron and	Rica	rdo start	jogg	ing arou	ınd a	n oval a	t the	same tii	me b	ut in	Animation

4 Sharon and Ricardo start jogging around an oval at the same time but in opposite directions. Sharon takes 5 minutes to run each lap and Ricardo takes 4 minutes. After how many minutes running at that pace would they cross their starting point at the same time?

eQuestions

3.2 Divisibility

We often want to know if a smaller number will divide evenly into a larger number. If this occurs we say the larger number is **divisible** by the smaller number. For example, 10 is divisible by 5. On the other hand, 24 is not divisible by 9 because 24 ÷ 9 gives us a remainder. We can use a calculator to check this but often we can find out more quickly by using some mental shortcuts and divisibility tests.



- (b) What sort of numbers are divisible by 3? Can you find a pattern?
- **5** (a) Which of these numbers are divisible by 9? 81, 679, 2999, 82, 5634, 220 221, 87 984, 16 668, 562
 - (b) What sort of numbers are divisible by 9? Can you find a pattern?
- **6** (a) Which of these numbers are divisible by 4? 516, 7612, 311, 608, 61, 64, 5364, 38 921, 500
 - (b) What sort of numbers are divisible by 4? Can you find a pattern?
- **7** The shortcut to test if a number is divisible by 6 is to see if it is divisible by both 2 and 3.





in each number.





Look back at the pattern you found in Question 4.

The clue is in the last two digits of each number.

80

- (a) Use the test to see which of these numbers are divisible by 6. 436, 321, 132, 741, 8760, 4529, 3528, 705 630, 11 112
- (b) The shortcut for 12 is similar to the shortcut for 6. What do you think the pattern is for seeing if a number is divisible by 12?
- **8** Copy and complete the following divisibility tests:

Number	Divisibility test
2	Look at the digit only. If it is or zero then the original number is divisible by 2.
3	up all the digits and see if the is divisible by 3. If it is then the original number is divisible by 3.
4	Look at the number formed by the last digits only. If this number is divisible by 4, then the number is divisible by 4.
5	Look at the digit. If it is a or a, then the number is by 5.
6	Do two tests. See if the number is divisible by and
9	up all the and see if the is divisible by 9. If it is then the original number is by
10	Look at the If it is then the number is

9 State TRUE or FALSE for the following.

- (a) 346 is divisible by 3.
- (c) 548 348 is divisible by 2.
- (e) 18 342 is divisible by 9.
- **(g)** 4 332 112 is divisible by 5.
- (i) 67 432 is divisible by 6.
- (b) 6872 is divisible by 6.(d) 552 is divisible by 4.
- (f) 5633902 is divisible by 3.
- (h) 56 432 is divisible by 2.
- (**j**) 3935 is divisible by 5.
- **10** Copy the following table and do the divisibility tests. Circle the number if the original number is divisible by it. The first one has been done for you.

100 000	2	3	4	5	6	9	10	
202 008	2	3	4	5	6	9	10	
12 121 212	2	3	4	5	6	9	10	
300 300 300	2	3	4	5	6	9	10	
7 500	2	3	4	5	6	9	10	
900 090	2	3	4	5	6	9	10	
123 456 789	2	3	4	5	6	9	10	
2564	2	3	4	5	6	9	10	
3 4 2 9	2	3	4	5	6	9	10	

Extension

- **11** Write three numbers greater than 11 000 that are divisible by both 4 and 5.
- **12** The following numbers are divisible by 11. 187, 220, 385, 1056, 2915, 9262, 10 593
 - (a) For each number add every second digit together. For example, for 10593, 1+5+3=9 and 0+9=9. Find the difference between each pair of numbers. What do you notice?

One of the

Hint

numbers involved

is 3.

- (b) Use your answers to part (a) to complete the following: To check if a number is divisible by 11 add every second digit, then find the _____ between the numbers you get. If the difference is ____ or __ then the original number is divisible by 11.
- **13** A number is divisible by 6 if it is divisible by 2 and 3. Follow the steps below to try to find the test for divisibility by 12.
 - (a) 2 and 3 multiply to give 6. What pairs of numbers multiply to give 12, apart from 12 and 1?
 - (b) 2 and 3 have no numbers that go into both of them. Which pairs found in part (a) have no numbers that go into both of them?
 - (c) Explain why you can't test divisibility by 12 by seeing if 2 and 6 go into the number.
 - (d) Copy and complete the following: To see if a number is divisible by 12 you need to do two divisibility tests: one to see if the number is divisible by _____ and one to see if it is ______ by ____.
- **14** Following the rules found for divisibility by 6 and by 12, find a test for divisibility by 18.

Working mathematically

problem solving

What number am I?

You may like to work in pairs to solve these.

- I have three digits.
 I am divisible by 5.
 I am odd.
 The product of my digits is 15.
 The sum of my digits is less than 10.
 I am less than 12 × 12.
 What am I?
 What if I am greater than 12 × 12?
- 2 I have three digits. My digits are all different. The sum of my digits is 12. My digits are all even. The sum of my units and tens digits equals my hundreds digit. I am divisible by 4. What am I?



Break it into a series of smaller steps.

<u>3.</u>3 Factors

A **factor** is a number that divides into another number exactly, with no remainder.

For example, 2 is a factor of 12 since 2'goes into' 12 (six times), with no remainder.

The largest factor of any number is the number itself. No number has any factors greater than the number itself.

One of the simplest ways to find all the factors of a number is to write all the pairs of numbers that multiply to give the original number.

Worked example 1 Find all factors of 12.	
Steps Find all pairs of numbers which multiply to give the original number. 	Solution $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
2. List the factors in ascending order (from smallest to largest).	The factors of 12 are 1, 2, 3, 4, 6, 12.

worked example 2

Find all the factors of 110.

Steps

- 1. Find all pairs of numbers which multiply to give the original number. Write the pairs underneath each other.
- 2. List the factors in ascending order (from smallest to largest).

Solution

- $\begin{array}{l} 1\times 110 = 110 \\ 2\times 55 = 110 \\ 5\times 22 = 110 \\ 10\times 11 = 110 \\ \text{No more to find as 10 leads on to 11.} \end{array}$
- The factors of 110 are 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes a factor will be multiplied by itself to give the original number. For example $7 \times 7 = 49$. We include 7 only once in the list of factors for 49. If we reach such a pair, this also tells us we have finished finding the pairs of numbers.

exercise 3.3 Enclors

Preparation: Ex 3.2

Core

1	Find	l all the factors	of ea	ach of the follow	wing	, numbers.		
	(a)	4	(b)	5	(c)	7	(d)	8
	(e)	10	(f)	9	(g)	13	(h)	11
	(i)	16	(j)	18	(k)	19	(1)	23
	(m)	20	(n)	24	(o)	32	(p)	36
	(q)	30	(r)	60	(s)	77	(t)	55
2	Cho	ose the correct	ans	wer.				
	(a)	Which of the	follov	wing is a factor	of 1	7?		
		A 7		B 14	(C 34	Ľ) 17
	(b)	Which of the	follov	wing is a factor	of 2	5?		
		A 3		B 5	(C 50	Ľ) 250
	(c)	Which of the	follov	wing is a factor	of 3	4?		
		A 4		B 12	(C 17	Ľ) 8
	(d)	Which of the	follo	wing is a factor	of 4	7?		
		A 1		B 9	(2 94	Ľ) 3
3	Cho	ose the correct	ans	wer.				
	(a)	Which of the	follo	wing is <i>not</i> a fac	ctor	of 15?		
		A 3		B 1	(2 15	Ľ) 30
	(b)	Which of the	follo	wing is <i>not</i> a fac	ctor	of 22?		
		A 22		B 4	(2 1	Ľ) 11
	(c)	Which of the	tollo	wing is <i>not</i> a fac	ctor	of 14?		
	(1)	A 4	c 11	B 2	. (2 14	L) 1
	(d)	Which of the	tollo	wing is <i>not</i> a fac	ctor (of 21?	г	11
	(a)	A 1 Which of the d	(_11	D /	,	- 3 - f 222	L	14
	(e)	11		\mathbf{R} 33		r r	г	1 3
	(f)	Which of the t	follo	\mathbf{D} UU wing is not a fact	tor v	~ 22	L	
	(1)	A 14		R 8	(~ 7	г) 6
	(σ)	Which of the	follo	wing is <i>not</i> a fac	rtor d	of 50?		• 0
	` &'	A 10	.5110	B 15	(2 50	Г	2
	(h)	Which of the	follo	wing is <i>not</i> a fac	ctor	of 63?	-	-
	()	A 63		B 21	(C 3	Ľ) 13

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Hint
Worksheet C3.3
Hint

😑 eTutorial



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e Questions	$\overline{\bigcirc}$

Working mathematically

investigation



The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BC to 195 BC. He taught at the University of Alexandria in Egypt. One of the things he was famous for was his'sieve'. See if you can copy what he did.

Copy the following table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- 1. Cross out the number 1.
- 2. Go to the next number, which is 2, and circle it. Then cross out all the multiples of 2

(i.e. cross out 4, 6, 8, ...).

- 3. Go to the next number that isn't crossed out. This should be 3. Circle it. Then cross out all the multiples of 3.
- Go to the next number that isn't crossed out. This should be 5. Circle it. Then cross out all the multiples of 5.
- 5. Repeat for the next number that isn't crossed out. Keep repeating until there is no'next number'.

Now see if you can answer the following questions.

- **1** Write the factors of each of the circled numbers.
- 2 Write the factors of any five of the crossed out numbers, except for 1.
- **3** What do you notice about the number of factors for each group of numbers, that is circled numbers and numbers crossed out?



3.4 Prime and composite numbers

From our work in previous exercises we know that numbers can have 1, 2, or more factors. We give special names to numbers depending on how many factors they have. A **prime number** has exactly two factors: itself and 1. A **composite number** has more than two factors. Between them prime numbers and composite numbers cover every whole number except for 1. We consider 1 to be a special number—it is neither prime nor composite.

A prime number has exactly two factors: itself and 1.

exercise 3.4 Prime and composite numbers



Core

- **1** (a) Find the factors of each of the numbers from 1 to 20.
 - **(b)** List the prime numbers from this list.
- 2 Why isn't 1 a prime number?
- 3 How many single-digit prime numbers are there?
- **4** List all the primes between 40 and 60.
- **5** How many even primes are there?
- 6 Explain why it is easy to tell that 4567278 is a composite number.
- **7 (a)** What is the next prime number after 60?
 - (b) What is the next composite number after 60?
 - (c) What are the two odd composite numbers less than 20?
 - (d) What is the next prime number after 30?
 - (e) What is the next composite number after 30?
 - (f) What is the next prime number after 40?
 - (g) How many prime numbers are there between 20 and 30?
- **8** Use one of the divisibility tests to show that these numbers are composites.

(a)	6785	(b)	34 450	(c)	7119
(d)	9909	(e)	977 824	(f)	4516803
(g)	2 987 625	(h)	87 912 404	(i)	2871027



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Extension

9 Write the following even numbers as the sum of two prime numbers.

(a)	4	(b)	6	(c)	10	(d)	12
(e)	18	(f)	20	(g)	22	(h)	100



11 What is the greatest difference between any two consecutive composite numbers?



Worksheet A3.1

Hint



3.5 Prime factors and factor trees

A **prime factor** of a number is a factor that is also prime. Every composite number can be expressed as a product of prime numbers.

A **factor tree** is a good way to find these prime factors.

The circled numbers are primes; we stop each branch when we get to a prime number. When every branch ends in a circled number we have found the prime factors of the number. So, $24 = 2 \times 2 \times 2 \times 3$. We usually write the factors in order from smallest to largest.

Note that there is often more than one way to construct the tree, as shown at right. However, the final product of the prime factors will always be the same.





maths in action

Prime numbers versus terrorism



Terrorism has cost many innocent lives so far this century, and mathematics has a role to play in stopping terrorist acts before they happen. Terrorists are able to commit these acts because they can plan and communicate in secret. Government intelligence agencies all over the world try to intercept terrorist computer emails and decode them.

At the time of the terrorist attacks in 2001, encryption or coding techniques existed that couldn't be cracked. These encryption techniques are based on prime numbers. Encryption is based on a key, which is a word or a number. These keys give the information needed to decode or decrypt the message. The keys currently most commonly used to encrypt emails are numbers that are the products of two prime numbers. Multiplying two primes is an effective key for encryption because it is easy to do in one direction (multiplication), but extremely difficult to do in the other direction (finding factors). To crack the code, you have to work out what the original two prime numbers are. When the numbers involved are very large, this is almost impossible to do because there are so many possibilities.

The largest prime ever found has over two million digits in it, and there are always new primes being found. Even the most powerful computers in the world would take centuries to find the two primes involved in encryption.

Governments are hoping that in the future quantum computers, which can process calculations simultaneously, will be able to find the two prime numbers quickly and enable them to crack the terrorists' codes.

Questions

1 Which two prime numbers were multiplied together to give each of these encryption keys?

(a) 77 (b) 38 (c) 65 (d) 202 (e) 143

2 Multiply the following prime numbers together to get encryption keys.

- (a) 3 and 17 (b) 7 and 31 (c) 47 and 73
- (d) 131 and 727 (e) 313 and 93 139
- **3 (a)** Write out the first five prime numbers.
 - **(b)** Write out the six smallest keys that you can create by multiplying two primes together.
 - (c) How many factors does each key in part (b) have?
 - (d) Do you think every key created by multiplying two primes together will have this number of factors? Why?
 - (e) Why couldn't we use any number (e.g. 120) as a key?

4 Here's a simplified version of how decryption using prime numbers works. Suppose your encrypted message is W L R GY S B E M R C R E B J CV and your key is 713.

You have to find the two prime numbers that multiply together to give you 713. The answer is 23 and 31. These digits tell you how much the letters in the original message have been shifted along the alphabet to give you the coded message.

You match these digits to the code as follows:

W	L	R	G	Y	S	В	Ε	Μ	R	K	С	R	Ε	В	J	С	V
2	3	3	1	2	3	3	1	2	3	3	1	2	3	3	1	2	3

The way this code works is that the number tells you how many letters to shift each letter along the alphabet. The 2 under the W means shift two letters forward through the alphabet from W, so the uncoded message, usually called the plaintext, has Y as its first letter. To get the second letter you find the letter 3 places on from L in the alphabet, which gives you O as the second letter of your plaintext.

- (a) Continue decrypting the code to get the full original plaintext message.
- (b) Describe what you did with coded letterY.
- **5** Decode this message given the key 943.

K X P G Q F O N D M N H K B E L N L N S Y K Y D

Note: You need to work out which order to put the two primes in. Only one will give you the message.

6 Encrypt your own message using this prime number system. Use only prime numbers under 100 and keep the message under 25 letters. Give your coded message and key to someone to decrypt.

Research

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Find out about the GIMPS (Great Internet Mersenne Prime Search) project and present a report on how prime numbers are used in email encryption. Discuss whether governments should ban encryption and make it possible for intelligence agencies to read everyone's emails. Should people have a right to privacy?

3.6 Other special numbers

There are many other special groups of numbers, far too many to cover in this book. We will discuss some of them here.

Palindromes

What do the fo	llowin	g words hav	ve in	n common?		
KAY	AK	MADAM		GLENELG	DID	MUM
These words an	e calle	d palindro	me	5.		
Sentences c	an be j	palindrome	s:			
Step o	on no pe	ets				
Numbers ca	n also	be palindro	ome	s. Like the pa	alindromic	words, they are
the same backy	wards a	is they are f	orw	vards. Here a	re some ni	umber
palindromes:		-				
-	22	676	8	12 321	6 004 006	5

Palindromes read the same backwards as forwards.

Fibonacci numbers

Leonardo Fibonacci was an Italian mathematician who lived from 1170 to 1240. He is most famous for describing the following number pattern:

1, 1, 2, 3, 5, 8, 13, 21, ...

What's the next number?

How is the pattern formed?

The number of petals on many flowers are **Fibonacci numbers**.

Triangular numbers

Look at the following patterns.



Each pattern of dots forms a triangular shape. We call the number of dots that make up these patterns **triangular numbers**. In the following exercise you will be asked to find some more triangular numbers and explore the patterns in them.



- **6** (a) Look at every third Fibonacci number. What sort of number is it?
 - (b) Look at every fourth number. What is it divisible by?
 - (c) Look at every fifth number. What is it divisible by?

- **7** It's possible to make different Fibonacci sequences by using different starting numbers. Copy and complete these sequences.
 - (a) 2, 2, 4, 6, 10, ____, ___, (b) 1, 2, 3, 5, 8, ___, ___, (c) 0, 4, 4, 8, ____, (d) 10, 1, 11, 12, ___, (d)
 - (c) 0, 4, 4, 8, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ___, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ___, ___, ___, ___, ___, ___, ___, ___, ___, ____, ____, ____, ____, ____,
 - (g) 6, ____, 8, ____, 18, ____
 - (i) _____, 5, ____, 11, ____, ___
- 8 Write two of your own Fibonacci sequences. Take out some of the numbers as in Question 7, and ask another person to work out the missing numbers in your sequences.

(f) 10, 10, 20, 30, ____, ___,

(h) 9, ____, 9, ____, ___,

(j) ____, 4, ____, 20, ____,

- **9** Draw diagrams for the fourth, fifth and sixth triangular numbers. Find the corresponding numbers.
- **10** Write out the triangular numbers in a row. What do you notice about the amount each number increases by?

Extension

- **11 (a)** Add the first two triangular numbers together. Add the second and third numbers together, then the third and fourth numbers together. What do you notice about these numbers?
 - (b) Look back at your triangles and explain how pairs could be placed together to form squares. Relate this to what you found in part (a).

7 Square and cube numbers





The first square number, 1, makes a 1×1 square. The second square number, 4, makes a 2×2 square. The third square number, 9, makes a 3×3 square. The fourth square number, 16, makes a 4×4 square.

We say, for instance, that four squared is equal to sixteen. In mathematical symbols we write $4^2 = 16$.

C Hint

🔁 Hint





Calculators can be used to find squares.

For example $17^2 = 17 \times 17$ can be found by pressing



You can also use the x^2 key, if your calculator has one. To find

17², press x^2 1 Similarly, a number is called a **cube number** if we can arrange that number of dots in a cube pattern (including dots in the middle of the cube).

Look back at the 2×2 square. If you imagine putting another of the same square behind the first one you will get a $2 \times 2 \times 2$ cube and 8 dots will have been used. We say that two cubed is equal to eight.

In mathematical symbols we write $2^3 = 8$.

Now look at the 3×3 square. If you imagine putting two more of the same square behind the first one you will get a $3 \times 3 \times 3$ cube and 27 dots will have been used. Similarly $3 \times 3 \times 3 = 3^3 = 27$.

On the calculator you can press



Square and cube roots

Finding the **square root** or **cube root** is the opposite to squaring or cubing a number. For example, to find the square root of 16, you have to think of a number that when squared gives you 16.

The number is 4 since $4 \times 4 = 16$. This means the square root of 16 is 4. We write this as $\sqrt{16} = 4$.

To find the cube root of 27, you have to think of a number that when cubed gives 27.

 $3 \times 3 \times 3 = 27$. So the cube root of 27 is 3.

We write this as $\sqrt[3]{27} = 3$.

You can use $\sqrt{}$ and $\sqrt[3]{}$ keys on your calculator to find

square and cube roots.







eTutorial



ех	ercise 3.	7 <u>Sq</u>	uare ai	nd cube n	umbers
		(D) PI	reparation: Prep Z	one Q1, Ex 3.2	
Core					
1 (a)	Write the 5th, 6t	h and 7th squa	re numbers.	e Hint	Do not use a calculator
(b)	What is	1			unless the question tells you to.
	(i) the 12th	(ii) the 20th	(iii) the 100	th (iv) the 300th	
	square number?		(e Questions	Ma
2 (a)	Write the 4th, 5t	h and 6th cube	numbers.		(A) A.
(b)	What is				
	(i) the 9th	(ii) the 15th	(iii) the 100	th (iv) the 200th	1 Š
	cube number?		(e Questions	
3 (a)	Write down two	numbers betw	een 5 and 40 th	at are both even	
	and square.				
(b)	Write down two	numbers betw	een 30 and 90 t	hat are both odd	
	and square.		_		
4 (a)	Write in words h	now we would s	say these.	(\cdot, \cdot) 21 ²	
	(i) 5^2	(ii) 2^2	(iii) 14 ²	(iv) 31^2	Hint
(1.)	(v) 3^3	(v 1) 7 ³	(v 11) 19 ³	(v111) 27 ³	
(b)	(i) 42	numbers these a	are equal to.	(;;;) 02	
	(1) 4^2	$(11) 6^2$	-	(111) 9^2	HINT
	(IV) 13^{-1}	(v) 30	53 53	$(v) 50^{-}$	squared
	(v1) 7 (v) 40^3	(viii) 10 (vi) fit	, ty cubed	(xii) one hundre	ed squared
	(xiii) 5×10^2	(xiv) 4	$\times 10^2$	(xx) 3 + 10 ²	eu squareu
	(xvi) $10^3 \times 2$	(xvii) 1	$)^{3} - 7$	(xviii) $11 + 10^3$	
5 (a)	Evaluate	(,,		(
0 (u)	(i) $3^2 + 4^2$		(ii) $6^2 + 8^2$		
(b)	Rewrite both yo	ur answers fron	n part (a) as a n	umber squared.	
(c)	Look for a patte	rn in your two r	esults and ther	n copy and complete	e
	these.	5		17 1	
	(i) $9^2 + 12^2 = $	2 (ii) 12	$2^2 + 16^2 = $ 2	(iii) $30^2 + 40^2 =$	2
(d)	If $5^2 + 12^2 = 13^2$,	what does 10^2	+ 24 ² equal?		
6 Cop	py and complete.	Use your calcula	ator if necessar	y. 🦰	
(a)	(i) $3^2 \times 4^2$		(ii) (3 × 4	$(\mathbf{L})^2$	
	$= 9 \times 16$		$= 12^2$	Calcul	ator
	=		=		
(b)	(i) $5^2 \times 2^2$		(ii) $(5 \times 2)^{-1}$	$(2)^{2}$	
	=×	-	=2		
	=	1 . /	=		
(c)	What can you sa	iy about (<i>a</i> × <i>b</i>) ²	and $a^2 \times b^2$?		

7	Wha	at number do y	you h	nave to square	to ge	t each of these	?		
	(a)	9	(b)	4	(c)	49	(d)	64	e Hint
	(e)	81	(f)	25	(g)	144	(h)	10 000	Q Worksheet C3.6
8	Wha	at number do y	you h	nave to cube to	o get e	each of these?			
	(a)	1	(b)	27	(c)	8	(d)	64	C Hint
	(e)	8000	(f)	1000	(g)	125	(h)	1000000	
9	Eval	luate:							
	(a)	$\sqrt{25}$	(b)	$\sqrt{49}$	(c)	$\sqrt{36}$	(d)	$\sqrt{64}$	e Hint
	(e)	$\sqrt{100}$	(f)	$\sqrt{121}$	(g)	$\sqrt{196}$	(h)	$\sqrt{169}$	
	(i)	$\sqrt{1}$	(j)	$\sqrt{0}$	(k)	$\sqrt{4900}$	(1)	$\sqrt{400}$	
	(m)	$\sqrt{1600}$	(n)	$\sqrt{2500}$	(o)	$\sqrt{360000}$	(p)	$\sqrt{810000}$	e Questions
10	Eval	luate:							
	(a)	3√8	(b)	3√64	(c)	3√1000	(d)	3√1	e Hint
	(e)	3√0	(f)	∛√125	(g)	3√8000	(h)	3√27 000	
11	Сор	y and complet	e.						
	(a)	(i) $\sqrt{9 \times 4}$:		(ii) $\sqrt{9} \times \sqrt{4}$			
		$= \sqrt{36}$				$= 3 \times 2$			
		=				=			
	(b)	(i) $\sqrt{100}$	× 16		(ii) $\sqrt{100} \times \sqrt{1}$	16		
		=				=×			
		=				=			

😜 Hint

😜 Hint

e

Worksheet A3.4

(c) What can you say about $\sqrt{a \times b}$ and $\sqrt{a} \times \sqrt{b}$?

Extension

- 12 (a) Can you find the square root of a negative number? Why or why not?(b) Can you find the cube root of a negative number? Why or why not?
- **13** Without using your calculator, find between which two consecutive whole numbers the following lie.

(a)	$\sqrt{10}$	(b) _~	/5 (c)	$\sqrt{20}$	(d)	$\sqrt{62}$
(e)	√99	(f) _~	/2 (g)	$\sqrt{70}$	(h)	$\sqrt{108}$

14 Without using your calculator, find between which two consecutive whole numbers the following lie.

(a)	3√10	(b)	3√52	(c)	3√1001	(d)	³√30
(e)	3√5	(f)	3√71	(g)	3√120	(h)	³√2

- **15** Check your answers to Questions **13** and **14** by finding the actual values using your calculator.
- **16** Is it more appropriate to use non-calculator strategies or a calculator to find the square root of each of the following numbers? Explain why.

(a) 9 (b) 24 (c) 10000 (d) 10

MATHS ZONE 7



3.8 Powers

Powers are a short way of writing large numbers. The small 2 in 5^2 is an example of a **power**. The other number is called the **base**.



The power tells you how many times the base will appear when you multiply the power out. For example:

 $\begin{array}{ll} 4^{3} = 4 \times 4 \times 4 & 7^{4} = 7 \times 7 \times 7 \times 7 \\ 5^{3} = 5 \times 5 \times 5 & 9^{5} = 9 \times 9 \times 9 \times 9 \times 9 \\ \end{array}$

For 4^3 we say 'four cubed' or 'four raised to the power of three' or just 'four to the power of three'.

For 5^3 we say 'five cubed' or 'five raised to the power of three' or just 'five to the power of three'.

For 7⁴ we say 'seven raised to the power of four' or just 'seven to the power of four'.

Another word for a power is an **index** (plural **indices**). When numbers are written using an index we say they are in **index form**.

 4^3 is in index form. $4 \times 4 \times 4$ is in **expanded form**.

worked example 3

Write out these numbers in expanded form and work out the answer. (a) 4^3 (b) 7^4

Steps

- (a) 1. Write 4^3 in expanded form.
 - 2. Calculate the first part of the product.
 - 3. Multiply this by the next part of the product.
 - 4. Write down the answer.
- **(b)** 1. Write 7^4 in expanded form.
 - 2. Calculate the first part of the product.
 - 3. Calculate the second part of the product.
 - 4. Multiply these two together.
 - 5. Write down the answer.

Solutions

- (a) $4^3 = 4 \times 4 \times 4$
 - $4 \times 4 = 16$
 - $16 \times 4 = 64$
 - $4^3 = 64$
- **(b)** $7^4 = 7 \times 7 \times 7 \times 7$
 - 7 × 7 = 49
 - $7 \times 7 = 49$
 - $49 \times 49 = 2401$
 - $7^4 = 2401$



Pol	vers o	n (cal	cular	7	0r5				
Many c	alculators have	a ke	y to he	lp you woi	ck	quickly v	with	powe	rs.	
It's usua	ally y^x or	x^{y}	or	a ^b or s	501	mething	simil	ar.	Calculato	
For	example, if you	wan	t to wo	rk out 12 ³	. v	ou press				
			2	_	, ,	ou proce				
Som	a other calcula	tors	includi	– ng graphi	20	calculato	rc 11	$e_0 2 n$	racass	
1		1015,		11g grapin	-2	calculate	л5, u	se a p	11:	
similar	to that used in	comp	outer sp	readsheet	p	rograms.	A ke	y like	this ^	
is used.	If we entered (6 ^ 3	this wo	ould mean	. 6	³ or six to	o the	powe	r of three.	e Tutorial
ех	ercise	3.8	3	Pow	-)	15				
				D Prepar	ati	on: Ex 3.6				
Core										
1 Wri	te each of thes	e in i	ndex fc	orm. (Don'	t v	vork out	the a	answe	r.)	
(a)	$8 \times 8 \times 8$			(b)	$4 \times 4 \times 4$	4×4	$\times 4 \times$	4	E Hint
(c)	$9 \times 9 \times 9 \times 9$			(d	l)	$7 \times 7 \times 2$	7			
(e)	$5 \times 5 \times 5 \times 5 \times 5$	< 5 ×	5	(f)	$9 \times 9 \times 9$	9×9	×9		
(g)	$12 \times 12 \times 12 \times$	< 12 >	< 12	(h)	16×16	×16	×16>	$\times 16 \times 16 \times 10$	$6 \times 16 \times 16$
(i)	$6 \times 6 \times 6 \times 6 \times 6$	< 6 ×	6×6×	6×6 (j))	11×11	×11	×11>	< 11 × 11 × 1	1 × 11
(k)	seventeen cul	oed		(1))	thirteen	to th	ne pov	wer of seven	
(m)	eight to the fo	ourth	power	(n	l)	nine to	the s	ix		
(o)	eleven to the	seve	n	(p)	nine to	the t	hree		
2 Wri	te these out in	expa	nded fo	orm. (Don	′t v	work out	the	answe	er.)	
(a)	4 ⁵	(b)	6 ⁵	(c)	5 ⁴		(d)	56	C Hint
(e)	2	(f)	3 ⁸	(g	;)	7 ²		(h)	83	
(i)	96	(j)	97	(k	:)	104		(1)	132	
(m)	54°	(n)	710	(0)	1115		(p)	5174	
3 Wri	te these out in	expa	nded fo	orm and th	nei	n work o	ut th	e ans	wer.	
(a)	2 ³	(b)	24 07	(c)	26		(d)	2°	e Hint
(e)	103	(f)	0' 105	(g	;) ``	00		(h)	1'	
(1)	10 ³	(\mathbf{j})	10 ³	(K	() \	6 [±]		(1)	8 [±]	
(m)	110	(n)	121	(0)	55		(p)	10°	
4 Sim	$14 \cdot 2^2$		(1.)	03 16			()	2^4	o 4	Uint
(a)	$1^{-} + 2^{-}$		(b)	$2^{\circ} - 1^{\circ}$ 10 ² + 2 ²	4	3	(C)	3 -	Z [≖] 16 , 23	
(a)	$2^{\circ} + 3^{-} - 6^{-}$		(e) (h)	$10^{-} + 5^{-} - 22 \times 24$	- 4		(I) (i)	26 ×	$1^{\circ} + 3^{\circ}$ 22	E Worksheet C3.7
(g) (i)	$2^{-} \times 2^{-}$ (8 + 5 ³) × 2 ⁴		(1) (1/)	$(6^4 - 7) \times$	1(n2	(1) (1)	3° × 1	$(3) \times 3^2$	
(j) (m)	$(0 + 3) \times 2$ 4×10^3		(K) (n)	$(0 - 7) \times 10^4$	10	0	(I) (D)	(+ - 9 × 1	05 × 0	
(n)	$9^2 \times 10^4$		(n) (n)	$6^2 \times 10^6$			(r)	$2^{2} \times$	10 ⁷	
(s)	$2 \times 10^4 + 7 \times 10^4$	10 ³ +	י יי 8 × 10 ²	$^{2} + 6 \times 10^{1}$	+	9	(1)	- ^		
(t)	$5 \times 10^6 + 3 \times 10^6$	10 ⁴ +	2×10^3	$3 + 9 \times 10^{2}$	' +	- 7 × 10 ¹ +	- 4			
(•)	2.7.10 1.07.	-0 1		10		10 1	-			

- **5** (a) Arrange these numbers in ascending order. 45, 54, 1200, 103, 46, 55
 - (b) Arrange these numbers in descending order. 100^2 , 10^5 , 1^{1000} , 0^{100} , 3^2 , 2^3
- **6** Evaluate:
 - (a) $\sqrt{2\times 2}$ **(b)** $\sqrt{10 \times 10}$ (c) $\sqrt{5\times5}$ (e) $\sqrt[3]{5 \times 5 \times 5}$ (f) $\sqrt[3]{3 \times 3 \times 3}$ (g) $\sqrt[3]{1 \times 1 \times 1}$

Extension

- **7 (a)** Copy and complete: $10^1 = 10 = 10$
 - $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1000$
 - (b) The number 10^{100} was called a googol by the mathematician Edward Kasner.
 - (i) How many times is 10 multiplied by itself to get a googol?
 - (ii) Look at the pattern in part (a). How many zeros would follow the 1 in a googool?

 $10^4 =$

 $10^5 =$

 $10^{6} =$

- (c) If you raise the number ten to the power of a googol, you get a number called a googolplex.
 - (i) How many times is 10 multiplied by itself to get a googolplex?
 - (ii) How many zeros would follow the 1 in a googoolplex? How much time do you think you save by writing a googolplex in index form?

8 Simplify these with your calculator, using the shortest method.

(a) 6⁵ **(b)** 8⁵ (c) 73 (d) 9³ **(e)** 13⁶ 15^{3} (g) **(h)** 25³ (f) 28^{3} (i) 32³ 2^{7} **(k)** 3⁶ (j) (1) 45 (n) $21^4 - 4481$ (m) $16^3 - 4096$ (o) $15^4 - 5625$ (p) $36^3 \times 53$ (g) $14^4 \times 14$ (r) $19^5 \times 21$ (s) $2^{12} + 2^{18}$ (t) $3^{10} + 12^4$ (u) $4^9 + 3^{11}$

9 (a) Use your calculator to answer TRUE or FALSE to each of the following statements.

- (i) 4^6 is bigger than 6^4 . (ii) 2^{10} is bigger than 10^2 .
- (iv) 19^2 is bigger than 2^{19} . (iii) 3^9 is bigger than 9^3 .
- (b) Look at your answers for part (a) and *without* using your calculator answer TRUE or FALSE to these statements.
 - (i) 9^8 is bigger than 8^9 . (ii) 2^{100} is bigger than 100^2 .
- **10** Use your calculator to find the following through trial and error.
 - (a) A number which when raised to the power of three gives: (i) 343 (ii) 1728 (iii) 2744 (iv) 39304
 - (b) A number which when raised to the power of four gives 4096.
 - (c) A number which when raised to the power of five gives 161051.
- **11** Find a number that when raised to the power of four gives a number between 150 000 and 300 000.

means 'from largest to smallest' (h) $\sqrt[3]{10 \times 10 \times 10}$

Descending order

(d) $\sqrt{8\times8}$

Ascending order means 'from

smallest to largest







Worksheet C3.9

MATHS ZONE 7



languagezone

Summary

Copy and complete the following summary of this chapter using the words and phrases from the list. A word or phrase may be used more than once.

- **1** A _____ of 6 is 18.
- **2** A number that is not _____ by any numbers other than 1 and itself is called a _____.
- **3** A number with more then two factors is called a _____
- **4** 1, 9 and 25 are all examples of ______. 1, 8 and 27 are all examples of ______.
- **5** The _____ of 8 is 2.
- 6 The long way of writing a number in index form is in _____
- **7** A _____ is a number that looks the same when the order of the numbers is reversed.

Questions

- **1** Write a non-mathematical meaning for the word 'factor'.
- **2** Write in words how we would say $\sqrt[3]{64} = 4$.
- **3** Label the parts. $\longrightarrow 5^3 \checkmark$ _____.
- **4** Write $6 \times 6 \times 6 \times 6$ in a shorter form. What do we call this form?
- **5** Write each of the following using symbols.
 - (a) five cubed
 - **(b)** seven to the power of four
 - (c) nine squared
- **6** Try to make at least 10 words, of three letters or more, from the letters of 'composite'.
- **7** The following words from this chapter are missing their vowels (a, e, i, o, u). Copy and complete the words.

```
(a) m_lt_pl_
```

- **(b)** c_lc_l_t_r
- (c) $d_v_s_b_l_t y$

Key words

base composite number cube numbers cube root divisibility tests divisible expanded form factor factor tree Fibonacci index index form indices multiple palindrome power prime factor prime number square numbers square root triangular numbers

Worksheet L3.1 Worksheet L3.2



FAQs

I keep getting the wrong answers for power questions. What could I be doing wrong?

A common mistake is thinking that something like 3^2 means 3×2 . The best way to avoid this is to read it correctly. Don't say to yourself 3^2 is 'three twos'. Say it is 'three to the power of two'.

I get factors and multiples confused. Is there an easy way to remember the difference? A factor is always that number and numbers smaller than it. Multiples are always that number and numbers larger than it. Tell yourself that because they are called 'multiples' we need to *multiply* that number to get its multiples.



How do I know when I have found all the factors of a particular number?

Follow a pattern. Start with 1 and its pair, then 2 and its pair, and so on. When you get to a factor that you have already found, then you have found all the factors. For example, to find the factors of 15, first you will find 1 and its pair 15, then 3 and its pair 5. The next factor is 5 but you have already found this so all factors are found. The factors of 15 are 1, 3, 5 and 15.

Core

1	Find	l the first	three mu	ltiples	of:							3.1
	(a)	7	(b)	10		(c)	12		(d)	52	
2	Cop the	y the foll original r	owing tal number is	ole and divisit	l do ti ole by	he div 7 it.	visit	oility te	ests. Cii	rcle th	e number	if 3.2
		5301	2	3	4	5	6	9	10			
		10 000	2	3	4	5	6	9	10			
		333 333	2	3	4	5	6	9	10			
	43	521 820	2	3	4	5	6	9	10			
	10	021 020	_	U	-	U	Ŭ	Í	10			
3	Find	l all the f	actors of:									3.3
	(a)	35		(b)	31				(c)	44		
	(d)	48		(e)	51				(f)	100		
4	Stat	e whethe	er each of	the fol	lowii	ng nu	mbe	ers is a	prime	num	ber or a	3.4
	com	posite, a	nd explai	n why.		Ũ			•			
	(a)	5		(b)	16				(c)	1		
	(d)	77		(e)	17				(f)	276 3	350	

5	Wri	te out the factor tr	ees for th	ese numbe	rs, and then e	express each	3.5	
	nun	nber as a product o	of its prim	ne factors.				
	(a)	24 (b)	30	(c)	88	(d) 200		
6	Cop	y and complete th	iese Fibor	nacci seque	nces.		3.6	
	(a)	6, 3, 9,, 21, _	/	_/				
	(b)	5,, 11,	, 28,					
7	Eval	luate:					3.7	
	(a)	12 ²	(b)	7 ²	(c)	20 ²		
	(d)	$\sqrt{64}$	(e)	$\sqrt{900}$	(f)	$\sqrt{225}$		
8	Eval	luate:						
-	(a)	2 ³	(b)	10 ³	(c)	5 ³	3.7	
	(d)	3/0	(e)	³ /27	(f)	$\frac{3}{64}$		
9	Wri	te each of these ni	imbers in	indev form	<i>،</i>	· • • -		
,	(a)	$7 \times 7 \times 7 \times 7 \times 7 \times 7$		(b)	ten cubed		3.8	
	(c)	five squared		(b)	twelve to the	e nower eight		
10	Wri	te these numbers	in evnand	led form an	d then work	out the answer		
10	(a)	5 ³	(b)	24		$(3^2 - 2^3) \times 16^2$	3.8	
	(a)	0		0	(C)	$(5 2) \times 10$		
Ex	te l	nsion						
11	(a)	Copy and comple	ete the fol	llowing.			3.7	
		(i) $2^2 = 1^2 + 3$		(ii)) $3^2 = 2^2 + _$			
		(iii) $4^2 = 3^2 + _$		(iv)) $5^2 = 4^2 + _$			
	(b)	Describe the rela	tionship b	oetween a s	quare numbe	er and the square		
		number before it						
	(c)	Using the relation	nship you	ı establishe	d in part (b) ,	copy and		
		complete the foll	owing.					
		(i) $12^2 = 11^2 + $ _		(ii	$20^2 = 19^2 + 10^2$			
12	Is 2	¹⁰ larger or smaller	than 100	00? Write th	ne difference.		3.8	
13	Put	these numbers in	ascending	g order.			3.8	
	24, ,	$\sqrt{121}, 10^2, 3^3, 4 \times$	$\sqrt{81}$					

<_ R	EPLAY		
Calculate:			e Worksheet R3.6
(a) 1200 – 567	(b) 2971 – 730	(c) 8903 – 6784	
2 Calculate:			e Worksheet R3.7
(a) 675 ÷ 9	(b) 3164 ÷ 7	(c) 5052 ÷ 6	
3 (a) How many second	ls are there in three minute	es?	e Worksheet R3.8
(b) How many minute	es are there in 10 hours?		
4 Frank takes \$50 to purc	hase his mum's birthday p	present. He buys her a	e Worksheet R3.9
book for \$22 and a shir	t tor \$19.50. How much ch	ange does he have	
	flat aidaa) daga a sub a l	-2	Warkshoot D2 10
(a) How many faces (i	lat sides) does a cube have	e: ure base baye?	WUIKSHEEL NO. 10
(c) How many faces d	oes a pyramid with a trian	igular base have?	
6 Copy and complete the	following magic squares.		
(a)	(b)	(a)	1.3
(a)	10	3	
6	9 11	11	
8 18 4	8	19 5	
7 Round these numbers of	off to the first digit.		1.5
(a) 28 (b)	136 (c) 968	(d) 12	
8 Simplify:		/ N	1.6
(a) $6 \times 4 \div 2 \times 6$	(b) $5 + 6 \times 7$	(c) $18 + 12 - 7 + 6$	
9 Write the following nu	nbers in order from smalle	est to largest.	2.2
(a) -4, 10, 0, -6, 3	(b) -89, 78, -100, 29, -1	(c) 0, -5, 6, 7, -4	
U Simplify: $(a) = 00 = 110$	(b) 17 50	(a) $15 - 20 + 12$	2.5
(a) $90 - 110$	(b) $-17 - 52$	(c) $15 - 20 + 12$	
Calculate:	(b) $E_{\rm V}(10)$	(a) $2 \times (2) \times 7$	2.6
(a) -7×0	$(0) -3 \times (-12)$	(c) $3 \times (-3) \times 7$	
2 Calculate:			2.7
(a) $\frac{72}{-8}$	(b) -100 ÷ (-10)	(c) −120 ÷ 60	
			e Assignment 3

MATHS ZONE 7

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