# directed NUMBERS 2

## Into the negative zone

Space is often referred to as 'the final frontier', yet no one has been near the bottom of our oceans at their deepest parts, or knows what creatures might lurk there.

**S**ubmarines usually dive to around 500 metres below sea level (−500 m), and some have gone to −900 m. Submersibles have been withstanding the enormous pressures of the deep since the 1930s. A recent expedition went to −2232 m, where the explorers saw rounded mounds of lava that have welled up from inside the Earth and solidified in the near-freezing water, hills of black smoker chimneys, and an unknown creature, 'a pinkie sized white critter', which wriggled and appeared to wave at them. We still have a long way to dive, however. The deepest ocean trench is −11 022 m. Compare this with the height of Mount Everest: a mere +8848 m.

## outcomes

After completing this chapter you will be able to:

- add and subtract directed numbers
- multiply and divide directed numbers
- use directed numbers to represent real-life situations.

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# *prep*zone2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

**e Worksheet R2.1**

**1** Arrange the following in order from smallest to largest.

    **(a)** 3, 17, 5, 8, 0                  **(b)** 45, 4, 5, 9, 1

**e Worksheet R2.2**

**2** Copy and complete each of the following by writing a $<$ or $>$ sign between the given values.

    **(a)** 10 ____ 7    **(b)** 3 ____ 6    **(c)** 2 ____ 0    **(d)** 0 ____ 5

**e Worksheet R2.3**

**3** Evaluate:

    **(a)** $14 + 6$                  **(b)** $3 + 8 + 12$

    **(c)** $22 + 19$               **(d)** $509 + 33$

    **(e)** $9 + 22 + 35 + 87$       **(f)** $67\,734 + 30\,245 + 718$

**e Worksheet R2.4**

**4** Calculate:

    **(a)** $9 - 5$        **(b)** $18 - 9$        **(c)** $72 - 39$

    **(d)** $563 - 192$    **(e)** $1000 - 347$    **(f)** $45\,786 - 10\,344$

**e Worksheet R2.5**

**5** Evaluate:

    **(a)** $2 \times 8$    **(b)** $7 \times 9$    **(c)** $11 \times 11$    **(d)** $38 \times 6$

    **(e)** $45 \times 3$    **(f)** $59 \times 6$    **(g)** $368 \times 30$    **(h)** $705 \times 400$

**e Worksheet R2.6**

**6** Evaluate:

    **(a)** $4 \times 5 \times 2$               **(b)** $7 \times 3 \times 3$

**e Worksheet R2.7**

**7** Evaluate:

    **(a)** $18 \div 3$       **(b)** $616 \div 7$       **(c)** $\frac{54}{9}$

**e Worksheet R2.8**

**8** Calculate:

    **(a)** $25 - (5 + 8)$    **(b)** $(4 \times 9) + (35 \div 5)$    **(c)** $6 \times 8 + 2 \times 3$

    **(d)** $64 \div 8 - 4$      **(e)** $50 - 5 \times 2 + 3$    **(f)** $3 \times 8 \div 6 \times 3$

## KEY WORDS

| | | | |
|---|---|---|---|
| addition | division | negative | subtraction |
| directed number | multiplication | positive | |

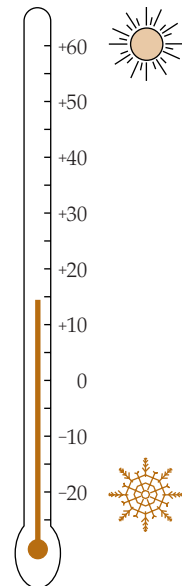Today's maximum temperature was 14°C after an overnight low of 2°C below zero.

Mt Kosciuszko, Australia's highest mountain, reaches 2230 m above sea level, whereas Lake Eyre is 16 m below sea level.

My bank account has gone into the red by $100!

Take the lift from the 7th floor of the building to the 2nd basement level.

+60
+50
+40
+30
+20
+10
0
−10
−20

All of these statements describe more than just the size of a number—they give a direction. Instead of using words, we may also use positive and negative numbers. Positive and negative numbers are called **directed numbers**.

We usually assign a + (**positive**) sign to things we consider to be gains or increases and a − (**negative**) sign to things we consider to be losses or decreases.

Stop being so negative about the cold!

## worked example 1

Write down a directed number suggested by each of the following.

**(a)** Winning $660 on Lotto

**(b)** Diving 5 m below the surface of the ocean

| Steps | Solutions |
|---|---|
| **(a)** 1. Does the case suggest a gain/increase or a loss/decrease? | **(a)** Gain/increase, so positive (+) |
| 2. Write the number with the appropriate sign. | +$660 |
| **(b)** 1. Does the case suggest a gain or a loss? | **(b)** Loss/decrease, so negative (–) |
| 2. Write the number with the appropriate sign. | –5 m |

**e** eTutorial

## exercise 2.1    Uses of directed numbers

**p** Preparation: Prep Zone Q2

### Core

**1** Write down a directed number suggested by each of the following.

**(a)**  A profit of $300
**(b)**  A wage increase of $2000
**(c)**  A loss of $50 000
**(d)**  A wage drop of $25
**(e)**  8 seconds before blast off
**(f)**  2 hours before take off
**(g)**  5 seconds after blast off
**(h)**  4 hours after take off
**(i)**  3 m above the surface
**(j)**  36 m above sea level
**(k)**  A deposit of $45
**(l)**  A lottery win of $1000
**(m)** A withdrawal of $20
**(n)**  A gambling loss of $90
**(o)**  A win by 6 goals
**(p)**  A win by 12 points
**(q)**  A loss by 10 goals
**(r)**  A 2 point loss
**(s)**  A rise of 5°C
**(t)**  A weight gain of 3.5 kg
**(u)**  A weight loss of 2.4 kg
**(v)**  Down 7 floors
**(w)** Up 28 floors
**(x)**  I owe the bank $40 000
**(y)**  I have $225 in the bank
**(z)**  The water level dropped by 9 m

**e** Hint

**2** Copy and complete:

**(a)**  If north is a positive direction, south is a _____ direction.

**(b)**  If right is a positive direction, _____ is a _____ direction.

**3** State the opposite of:

**(a)**  up 29 steps
**(b)**  west 300 km
**(c)**  6 km per hour over the speed limit
**(d)**  3 days late
**(e)**  80 m underground
**(f)**  7°C below zero
**(g)**  add 5
**(h)**  subtract 22
**(i)**  left 3 m
**(j)**  +4
**(k)**  –2
**(l)**  –11

**e** Hint
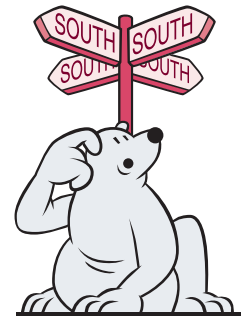
| (m) +16 | (n) +20 | (o) −400 |
|---------|---------|----------|
| (p) −350 | (q) −92 | (r) +87 |

### Extension

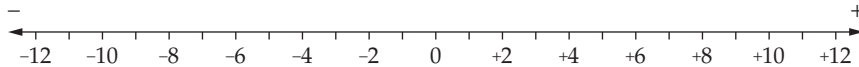**4** Give a real-life situation and its opposite and state the directed numbers suggested.

# 2.2 Comparing directed numbers

The number line is extremely useful when dealing with directed numbers. Drawing a horizontal number line (like the one shown) at the top of your page in your workbook can help you to add, subtract and compare values.

```
−                                                              +
  ←──┬────┬────┬────┬────┬────┬────┬────┬────┬────┬────┬────┬──→
    −12  −10   −8   −6   −4   −2    0   +2   +4   +6   +8  +10  +12
```

*Note:* Often the + sign is left off positive numbers. For example, 6 is really the same value as +6. From now on we will leave the + signs off the positive numbers on the number line, because there is no need for them.

We can use the number line to help us compare directed numbers, where numbers get larger as you move to the right.

Look at the positions of +5 and −10 on the number line shown. +5 is further to the right than −10. Mathematically speaking, we say that +5 is *greater than* −10, and write +5 > −10.

We could also say that −10 is *less than* +5, and write −10 < +5.

So the closer a number is to the positive end of the number line, the bigger its value is.

## worked example 2

Write the correct symbol (< or >) between each pair of values below.

**(a)** −9 _____ −4        **(b)** 0 _____ −10        **(c)** 7 _____ −48

### Steps

**(a)** −9 is further left on the number line than −4, so we write

**(b)** 0 is further right than −10, so we write

**(c)** Even though most number lines aren't drawn to −48, you can imagine −48 is very far to the left and that 7 is larger than −48, i.e.

### Solutions

**(a)**
   −9 < −4

**(b)** 0 > −10

**(c)**

   7 > −48
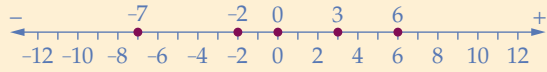
- The further to the right a number is on a horizontal number line, the greater its value.
- The further to the left a number is on a horizontal number line, the smaller its value.

Vertical number lines can also represent directed numbers, where the higher a number is on the line, the greater its value.

## exercise 2.2 Comparing directed numbers

*p* Preparation: Prep Zone Q1 and 2, Ex 2.1

*e* hi.com.au

### Core

1 Copy the following pairs of numbers, and write $<$ or $>$ between them to make a true statement.

*e* Worksheet C2.1

    **(a)** +7 ____ +9    **(b)** +8 ____ +2    **(c)** +2 ____ +11    **(d)** +4 ____ +9

*e* Hint

    **(e)** +6 ____ −3    **(f)** +3 ____ −12    **(g)** +1 ____ −3    **(h)** +6 ____ −4

*e* Worksheet C2.2

    **(i)** −3 ____ +5    **(j)** −7 ____ +5    **(k)** −6 ____ +1    **(l)** −2 ____ +6

    **(m)** −12 ____ −4    **(n)** −12 ____ −9    **(o)** −7 ____ −1    **(p)** −2 ____ −11

    **(q)** 0 ____ −10    **(r)** −9 ____ 0    **(s)** −5 ____ 0    **(t)** 0 ____ −2

    **(u)** −35 ____ +7    **(v)** −88 ____ +3    **(w)** −4 ____ +67    **(x)** −11 ____ +75

2 Arrange in order from smallest to largest (increasing order).

    **(a)** −3, −12, 6, −4, −1          **(b)** 1, −6, −4, 9, 2

*e* Hint

    **(c)** −11, 10, 9, 2, −10         **(d)** −8, −5, 7, 1, −6

*e* Worksheet C2.3

    **(e)** −1, 1, −4, −2, −11         **(f)** 2, −8, −3, −9, −5

    **(g)** −1, −4, −7, −8, −12        **(h)** −8, −6, −9, −3, −7

    **(i)** −56, 23, −79, 0, 6          **(j)** 76, −89, −7, 0, −12

    **(k)** 49, −33, −78, −90, 2       **(l)** −24, −93, 3, −14, −30

**3** Arrange in order from largest to smallest (decreasing order).

- **(a)** –2, –1, 2, 11, –3
- **(b)** –8, 7, 10, 1, –12
- **(c)** –5, –6, 3, 7, 8
- **(d)** 4, –4, –10, 9, –8
- **(e)** 1, –8, –4, –9, –6
- **(f)** –12, 0, –5, 7, –11
- **(g)** –2, –4, –9, –6, –7
- **(h)** –11, –5, –2, –12, –9
- **(i)** 77, –66, –82, 1, –18
- **(j)** –60, –55, –81, 6, 0
- **(k)** 0, –79, –54, 4, –9
- **(l)** 9, –48, 98, –99, –8

But we *are* lining up from smallest to largest!
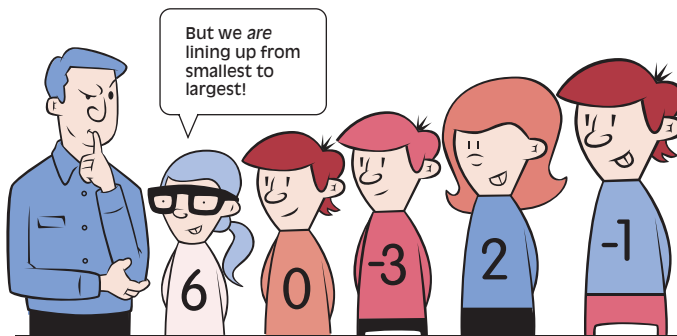


**4** Write TRUE or FALSE for each of the following.

- **(a)** $58 > -60$
- **(b)** $-44 < -50$
- **(c)** $-31 < -29$
- **(d)** $-29 > 30$
- **(e)** $-75 > 70$
- **(f)** $-92 > -11$
- **(g)** $19 > -100$
- **(h)** $-37 < -38$
- **(i)** $82 > -99$

**5** Write down the whole numbers between:

- **(a)** –4 and 3
- **(b)** –2 and 2
- **(c)** –7 and –3
- **(d)** –9 and –4
- **(e)** –5 and 0
- **(f)** –3 and 0
- **(g)** –45 and –50
- **(h)** –37 and –42
- **(i)** –120 and –115

**6** Write any three negative numbers greater than –8.

**7** Find the next three numbers in each pattern.

- **(a)** –18, –16, –14, ____, ____, ____
- **(b)** –20, –15, –10, ____, ____, ____
- **(c)** 9, 6, 3, ____, ____, ____
- **(d)** 8, 6, 4, ____, ____, ____
- **(e)** –6, –12, –18, ____, ____, ____
- **(f)** –7, –14, –21, ____, ____, ____
- **(g)** 30, 20, 10, ____, ____, ____
- **(h)** 60, 40, 20, ____, ____, ____
- **(i)** –13, –9, –5, ____, ____, ____
- **(j)** 12, 7, 2, ____, ____, ____

## Extension

**8** The minimum temperatures recorded at Uluru one week were:
Monday –3°C, Tuesday –7°C, Wednesday 0°C, Thursday –1°C,
Friday 2°C, Saturday –2°C and Sunday –4°C.

- **(a)** On which day was the lowest minimum temperature?
- **(b)** When was the highest minimum recorded?

**9** The balances of four traders' bank accounts show the following figures:

| | |
|---|---|
| Ben | $428 |
| Damien | –$260 |
| Sue | –$23 |
| Carol | $325 |

- **(a)** Who has the largest account?
- **(b)** Who owes their bank the most?

**10** In a combined operation, the airforce and navy have a helicopter and submarine at the levels shown.

Helicopter:       230 m above sea level

Submarine:      80 m below the surface of the ocean, directly under the helicopter

A parachutist from the helicopter has 76 m to fall before hitting the ocean.

**(a)** How far is the parachutist from the submarine?

**(b)** What distance from the helicopter is the parachutist?

**(c)** How far apart are the helicopter and submarine?



230 m

76 m

80 m
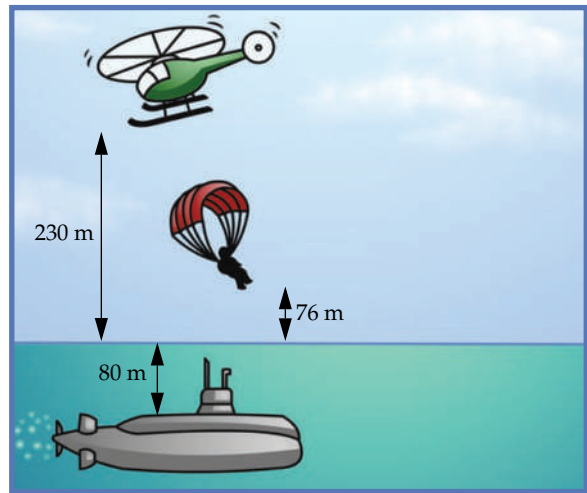
# 2.3 Adding directed numbers

The number line is also used to add directed numbers.

When adding two numbers using a number line, the following rules apply:

- The first number is the starting point.
- We think of the addition between the numbers as 'and'.
- The second number tells us whether to go right (+) or left (−), and how many steps to move.
- The finishing point is the answer.

## worked example 4

Use a number line to find the finishing point for each of the following trips.
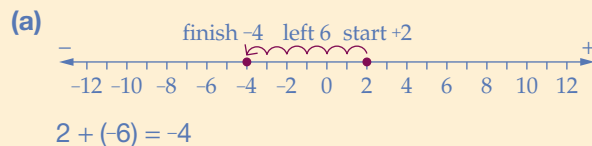
**(a)** $2 + (-6)$            **(b)** $-3 + (+7)$            **(c)** $-8 + (-2)$

### Steps

**(a)**        2 +   −   6

start at 2 and go left 6

Following these steps on the number line, we finish at −4, so we write

### Solutions

**(a)**



finish −4   left 6   start +2

−12 −10 −8 −6 −4 −2 0 2 4 6 8 10 12

$2 + (-6) = -4$

**(b)**      −3   +   +   7

start at −3 and go right 7

If we follow these steps on the number line, we finish at +4, so we write

**(b)**



start −3  right 7  finish +4

−12 −10 −8  −6  −4  −2  0   2   4   6   8  10  12

$-3 + (+7) = +4$

**(c)**      −8   +   −   2

start at −8 and go left 2

**(c)**



left 2
finish −10 start −8

−12 −10 −8  −6  −4  −2  0   2   4   6   8  10  12

The finishing point is −10, so
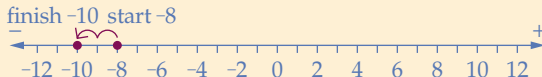
$-8 + (-2) = -10$

Note that we read these examples in the following way:

2 + (−6) is (*positive*) two plus *negative* six.

−3 + (+7) is *negative* three plus *positive* seven.

−8 + (−2) is *negative* eight plus *negative* two.

## exercise 2.3 Adding directed numbers

**p** Preparation: Prep Zone Q3 and 4, Ex 2.2

**e** hi.com.au

Draw a number line (from −12 to +12) on your page and use it to help answer the first two questions in this exercise.

### Core

**1** Use your number line to find the finishing point for each of the following trips.

**e** Worksheet C2.4

**e** Hint

| | | | |
|---|---|---|---|
| **(a)** +8 + (+3) | **(b)** +7 + (+1) | **(c)** +3 + (+8) | **(d)** +1 + (+7) |
| **(e)** +10 + 0 | **(f)** 0 + (+4) | **(g)** +5 + (−4) | **(h)** +3 + (−2) |
| **(i)** −7 + (−1) | **(j)** −6 + (−6) | **(k)** +6 + (−4) | **(l)** −2 + (−7) |
| **(m)** −3 + (−8) | **(n)** +5 + (−8) | **(o)** −5 + (−5) | **(p)** −4 + (−4) |
| **(q)** +12 + (−3) | **(r)** +8 + (−2) | **(s)** +2 + (−6) | **(t)** +4 + (−9) |
| **(u)** +7 + (−7) | **(v)** +5 + (−5) | **(w)** −10 + (+5) | **(x)** −12 + (+6) |

**2** Choose the correct answer.

**(a)** The finishing point for the trip +6 + (−12) may be found by following the instruction:

    **A** start at +6 and go right 12    **B** start at +6 and go left 12

    **C** start at −12 and go left 6    **D** start at +12 and go right 6

**(b)** The question −5 + (+3) may be answered by following the instruction:

    **A** start at +5 and go right 3    **B** start at −5 and go right 3

    **C** start at +5 and go left 3    **D** start at −5 and go left 3

**3** Evaluate:

  **(a)** +10 + (–17)    **(b)** +9 + (–20)    **(c)** –8 + (+14)    **(d)** –7 + (+19)

  **(e)** –6 + (–7)    **(f)** –11 + (–2)    **(g)** +8 + (–22)    **(h)** +10 + (–24)

  **(i)** –5 + (–10)    **(j)** –6 + (–8)    **(k)** –10 + (+20)    **(l)** –5 + (+30)

  **(m)** +6 + (–30)    **(n)** +8 + (–40)    **(o)** +74 + (–58)    **(p)** +67 + (–39)

  **(q)** –87 + (+56)    **(r)** –91 + (+45)    **(s)** –50 + (+100)    **(t)** –20 + (+40)

  **(u)** +30 + (–60)    **(v)** +50 + (–100)    **(w)** +47 + (–99)    **(x)** +62 + (–81)

**e** Hint

Some of these will be much too big for your number line, so you'll just have to imagine a bigger number line.

**4** Write down three different numbers that when added to 13 give a negative answer.

**5** Calculate:

  **(a)** +3 + (+4) + (+5)    **(b)** +5 + (+2) + (+4)    **(c)** +7 + (+1) + (+4)

  **(d)** +9 + (–7) + (–2)    **(e)** +6 + (–5) + (–7)    **(f)** +9 + (–12) + (–4)

  **(g)** –8 + (+6) + (+9)    **(h)** +7 + (–10) + (+1)    **(i)** +2 + (–8) + (+3)

  **(j)** –10 + (–2) + (+15)    **(k)** –11 + (–3) + (+20)    **(l)** –6 + (–3) + (–2)

  **(m)** –1 + (–4) + (–5)    **(n)** +10 + (–8) + (–7)    **(o)** +34 + (–12) + (–6)

  **(p)** +56 + (–11) + (–5)    **(q)** –29 + (+5) + (+6)    **(r)** –37 + (+7) + (+8)

**e** Hint

## Extension

**6** The temperature in the desert was –12°C. It rose by 20 degrees. What is the new temperature?

**e** Animation

**7** A lift is at the third basement floor (3 floors below ground level), and moves up 14 levels. On which floor does it end up?

**e** Hint

**8** A submarine 30 m below the surface of the ocean rises 16 m, then dives a further 20 m. What depth is it at now?

**9** A diver jumps from a platform 12 m high and dives 17 m to the bottom of a pool. How deep is the pool?

**e** Hint

**10** You have $5 in your pocket, but you owe a friend $12 while someone else owes you $20. If everyone pays up, how much will you have at the end?
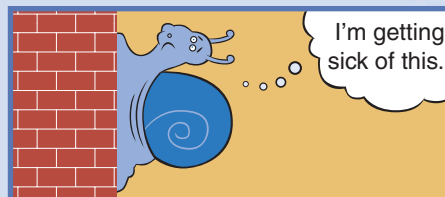
**e** Homework 2.1

**Working mathematically**

## problem solving

### The slippery snail

A snail is at the bottom of an empty trough with walls 40 cm high. If the snail manages to move 4 cm up a wall each day, but slips down 3 cm each night, how long does it take to reach the top of the trough?

I'm getting sick of this.

Solve a simpler version of the problem.

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The subtraction of two directed numbers may also be thought of as a number line journey with the following rules:

1.  The first number is the starting point.
2.  We read the subtraction between the two numbers as 'and do the opposite of'.
3.  The second number states right (+) or left (–) and how many steps.
4.  The finishing point is the answer.

### worked example 5

Use a number line to evaluate each of the following.

**(a)** $+8 - (+10)$          **(b)** $-4 - (+5)$          **(c)** $-2 - (-6)$

#### Steps                     Solutions

**(a)**      $+8$     –     +   10

start at $+8$ and do the opposite of right 10

i.e. start at $+8$       and go left 10    **(a)**



As you can see on the number line, the finishing point is $-2$, so we write      $+8 - (+10) = -2$

**(b)**      $-4$     –     +   5

start at $-4$ and do the opposite of right 5

i.e. start at $-4$       and go left 5    **(b)**



As you can see on the number line, the finishing point is $-9$, so we write      $-4 - (+5) = -9$

**(c)**      $-2$     –     –   6

start at $-2$ and do the opposite of left 6

i.e. start at $-2$       and go right 6    **(c)**



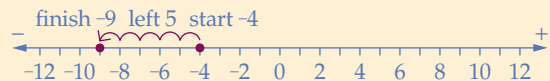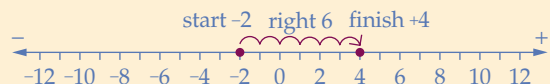As you can see on the number line, the finishing point is $+4$, so we write      $-2 - (-6) = +4$
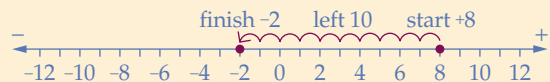
Note that we read these examples in the following way:
+8 – (+10) is *positive* eight take away *positive* ten.
–4 – (+5) is *negative* four take away *positive* five.
–2 – (–6) is *negative* two take away *negative* six.

> When subtracting directed numbers, the subtraction sign between the two numbers means 'and do the opposite of'.

## exercise 2.4  *Subtracting directed numbers*

**p** Preparation: Prep Zone Q3 and 4, Exs 2.2 and 2.3

Draw a number line (from –12 to 12) on your page and use it to help answer the first two questions in this exercise.

### Core

**1** Use your number line to evaluate the following.

| | | | | | | |
|---|---|---|---|---|---|---|
| **(a)** +7 – (+9) | **(b)** +5 – (+3) | **(c)** +8 – (+2) | **(d)** +12 – (+7) |
| **(e)** +11 – (+11) | **(f)** +6 – (+6) | **(g)** –3 – (+5) | **(h)** –5 – (+7) |
| **(i)** –9 – (+3) | **(j)** –8 – (+2) | **(k)** +10 – (–1) | **(l)** +4 – (–2) |
| **(m)** +5 – (–5) | **(n)** +6 – (–6) | **(o)** –7 – (–8) | **(p)** –3 – (–11) |
| **(q)** –12 – (–8) | **(r)** –9 – (–12) | **(s)** 0 – (–7) | **(t)** 0 – (–9) |
| **(u)** +8 – 0 | **(v)** –2 – 0 | **(w)** –9 – (+9) | **(x)** –4 – (+4) |

**2** Choose the correct answer.

**(a)** –15 – (+10) may be described by the number-line journey:
   **A** start at –15 and go right 10     **B** start at +10 and go left 15
   **C** start at +15 and go left 10     **D** start at –15 and go left 10

**(b)** The answer to +3 – (–20) may be found by following the instruction:
   **A** start at +3 and go right 20     **B** start at +3 and go left 20
   **C** start at –20 and go right 3     **D** start at –3 and go left 20

**3** Evaluate:

| | | | |
|---|---|---|---|
| **(a)** +10 – (+18) | **(b)** +6 – (+17) | **(c)** +8 – (+14) | **(d)** +11 – (+15) |
| **(e)** –9 – (–19) | **(f)** –4 – (–20) | **(g)** –3 – (–15) | **(h)** –5 – (–13) |
| **(i)** +16 – (+6) | **(j)** +19 – (+9) | **(k)** +30 – (+40) | **(l)** +20 – (+40) |
| **(m)** –24 – (+6) | **(n)** –36 – (+4) | **(o)** +2 – (+22) | **(p)** +7 – (+37) |
| **(q)** +11 – (–8) | **(r)** +8 – (–13) | **(s)** +67 – (–38) | **(t)** +78 – (–69) |
| **(u)** –45 – (+34) | **(v)** –66 – (+58) | **(w)** –36 – (–76) | **(x)** –94 – (–77) |

**4** Write down three directed numbers that when subtracted from –21 give a positive answer.

**5** Calculate:

**(a)** +6 – (+7) – (+4)     **(b)** +3 – (+5) – (+8)     **(c)** +8 – (+5) – (–6)
**(d)** +9 – (+4) – (–8)     **(e)** –7 – (+4) – (+3)     **(f)** –5 – (+6) – (+2)

**(g)** $+2 - (-1) - (-2)$      **(h)** $+1 - (-6) - (-2)$      **(i)** $-4 - (-7) - (+6)$

**(j)** $-7 - (-10) - (+5)$      **(k)** $+10 - (-2) - (+8)$      **(l)** $+8 - (-3) - (+7)$

**(m)** $+12 - (+16) - (+4)$      **(n)** $+14 - (+9) - (+10)$      **(o)** $-26 - (-14) - (-6)$

**(p)** $-22 - (-18) - (-7)$      **(q)** $+48 - (+32) - (+6)$      **(r)** $+51 - (+45) - (+7)$

**6** The temperature at sunset was 12°C, and it fell by 16°C during the night to reach a minimum. What was the minimum temperature?

**7** A shopper hops in a lift on the 7th floor and travels down 10 levels. Which level does he finish at?

### Extension

**8** Your company is $2000 in debt. You then receive notice that you owe the bank $1500. What is your financial state now?

**9** A farmer lowers a bucket from the top of a 2 m high well using a 10 m rope attached to the handle. If she lowers the whole length of rope, how far below ground does the bucket reach?

**10** A pelican flying 6 m above sea level suddenly spots a fish and dives 8.5 m straight down before catching the fish. How far below the surface was the fish?
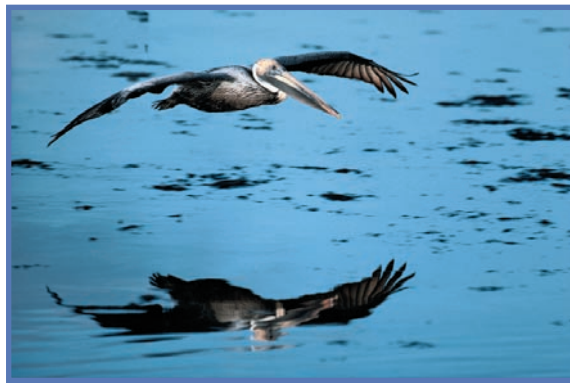
**11** A miner is sitting on a ledge 8 m below the top of a vertical mine shaft when he drops his torch. If the torch falls 23 m before hitting the bottom of the shaft, how deep is the shaft?

## speeding zone

Do these in your head as quickly as you can and write down the answers.

🕐 Time target: 2 minutes

**1** $8 \times 90$      **2** $\frac{1}{2}$ of $7.50      **3** 25% of 2000

**4** $870 + 90$      **5** $1500 \div 5$      **6** $5.00 − $3.15

**7** $20 \times 8 + 20 \times 2$      **8** $1100 - 250$

**9** Find the cost of six stamps at 45 cents each.

**10** What time is it seven and a half hours after 11.41 a.m.?

You have seen that we can leave the + sign off when writing positive numbers. For example, +7 may be written as just 7. There are some other rules which help us simplify expressions involving directed numbers. You may have noticed some of these as you worked through earlier exercises.

−2 + (+4) = 2 is the same as −2 + 4 = 2

Adding a positive number is the same as just adding the number.

+5 + (−3) = 2 is the same as 5 − 3 = 2

Adding a negative number is the same as just subtracting the number.

7 − (+4) = 3 is the same as 7 − 4 = 3

Subtracting a positive number is the same as just subtracting the number.

6 − (−2) = 8 is the same as 6 + 2 = 8

Subtracting a negative number is the same as just adding the number.

So if the signs are the same you add; if they are different you subtract.

### In summary

+ + is the same as +          + − is the same as −
− + is the same as −          − − is the same as +

### dangerzone

To apply these rules the two signs **must** be next to each other. The rules do not apply otherwise.

### worked example 6

Simplify the following before calculating the answer in each case.

**(a)** 4 + (+9)          **(b)** −1 + (−3)          **(c)** 10 − (+1)          **(d)** −6 − (−5)

| Steps | Solutions |
|---|---|
| **(a)** 1. Simplify. (+ + = +) | **(a)** 4 + (+9) |
| | = 4 + 9 |
| 2. Calculate. | = 13 |
| **(b)** 1. Simplify. (+ − = −) | **(b)** −1 + (−3) |
| | = −1 − 3 |
| 2. Calculate. (Use a number line if you like.) | = −4 |

## *exercise 2.5* — *Simplifying addition and subtraction*

**p** Preparation: Prep Zone Q3 and 4, Exs 2.3 and 2.4

### *Core*

**1** Simplify each expression below by writing a single sign between the values, and calculate the answer in each case.

**e** Worksheet C2.6

**e** Hint

| | | | |
|---|---|---|---|
| **(a)** $-9 + (+7)$ | **(b)** $-1 + (+4)$ | **(c)** $+5 + (+2)$ | **(d)** $+5 + (+6)$ |
| **(e)** $+7 + (-9)$ | **(f)** $+12 + (-3)$ | **(g)** $+10 + (-4)$ | **(h)** $+4 + (-8)$ |
| **(i)** $-7 + (-3)$ | **(j)** $-9 + (-3)$ | **(k)** $-3 - (+7)$ | **(l)** $-7 - (+4)$ |
| **(m)** $+5 - (+3)$ | **(n)** $+8 - (+4)$ | **(o)** $+1 - (+12)$ | **(p)** $+5 - (+10)$ |
| **(q)** $+2 - (-10)$ | **(r)** $+1 - (-1)$ | **(s)** $-7 - (-6)$ | **(t)** $-3 - (-9)$ |
| **(u)** $-4 - (-9)$ | **(v)** $-2 - (-6)$ | **(w)** $+6 - (+27)$ | **(x)** $+8 - (+38)$ |

**e** Worksheet C2.7

**2** State TRUE or FALSE for the following.

| | |
|---|---|
| **(a)** $25 + (+5) = 25 - 5$ | **(b)** $30 + (-10) = 30 - 10$ |
| **(c)** $15 - (+20) = 15 - 20$ | **(d)** $40 - (-50) = 40 + 50$ |

**e** Worksheet C2.8

**3** Calculate:

**e** Hint

| | | | |
|---|---|---|---|
| **(a)** $2 + 6$ | **(b)** $6 + 4$ | **(c)** $-5 + 9$ | **(d)** $-3 + 6$ |
| **(e)** $-9 - 4$ | **(f)** $-7 - 5$ | **(g)** $3 - 10$ | **(h)** $6 - 12$ |
| **(i)** $-7 - 4$ | **(j)** $-1 - 9$ | **(k)** $-12 + 7$ | **(l)** $-10 + 2$ |
| **(m)** $8 - 5$ | **(n)** $12 - 4$ | **(o)** $6 - 18$ | **(p)** $9 - 20$ |
| **(q)** $-2 - 23$ | **(r)** $-5 - 31$ | **(s)** $4 - 35$ | **(t)** $12 - 47$ |
| **(u)** $-60 + 30$ | **(v)** $-52 + 21$ | **(w)** $-27 + 84$ | **(x)** $-28 + 93$ |

**4** Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

**5** Calculate:

**e** Hint

| | | |
|---|---|---|
| **(a)** $4 + 7 + 1$ | **(b)** $2 + 3 + 6$ | **(c)** $-4 + 6 + 5$ |
| **(d)** $-6 + 7 + 5$ | **(e)** $-12 + 3 + 2$ | **(f)** $-11 + 3 + 4$ |
| **(g)** $6 - 9 + 4$ | **(h)** $8 - 10 + 9$ | **(i)** $7 - 12 + 3$ |
| **(j)** $4 - 9 + 2$ | **(k)** $-8 + 5 - 3$ | **(l)** $-6 + 2 - 4$ |
| **(m)** $-4 + 12 - 4$ | **(n)** $-3 + 10 - 6$ | **(o)** $-5 - 1 - 6$ |
| **(p)** $-2 - 5 - 4$ | **(q)** $-7 - 7 - 5$ | **(r)** $-5 - 5 - 8$ |
| **(s)** $36 - 40 - 5$ | **(t)** $43 - 50 - 2$ | **(u)** $-20 + 12 + 16$ |
| **(v)** $-30 + 15 + 20$ | **(w)** $-43 + 22 - 8$ | **(x)** $-18 + 29 - 3$ |

## Extension

**6** Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions listed below.

| | |
|---|---|
| Deposit | $30 |
| Withdrawal | –$80 |
| Deposit | $200 |
| Withdrawal | –$60 |
| Withdrawal | –$500 |

What is your account balance at the end of the month?

**7** A lift operator starts on the ground floor, and then travels as follows:
UP 10 floors, DOWN 7 floors, DOWN 5 floors, UP 4 floors, DOWN 6 floors, UP 12 floors

**(a)** What was the highest floor the operator reached?

**(b)** What was the lowest floor he travelled to?

**(c)** Which floor did he finish on?

**8** Diana makes and sells soft toys. She keeps records of her profits (and losses) over a ten-week period. These records are shown in the table. Find Diana's overall profit (or loss) during this time.

| Week | Result | |
|---|---|---|
| 1 | Profit | $120 |
| 2 | Profit | $25 |
| 3 | Loss | $70 |
| 4 | Profit | $210 |
| 5 | Loss | $150 |
| 6 | Loss | $180 |
| 7 | Loss | $90 |
| 8 | Profit | $40 |
| 9 | Loss | $160 |
| 10 | Loss | $95 |

## Working mathematically

## investigation
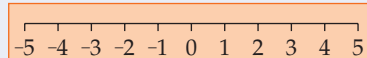
## Walking the plank

The following game will help you learn about the addition and subtraction of directed numbers.

### Set the scene

You and your partner have been captured by a bloodthirsty gang of pirates who have decided you are to walk the plank. However, the pirate captain decides to give you a chance to save yourself. The plank is marked as follows.

–5 –4 –3 –2 –1 0 1 2 3 4 5

You are to start at 0 and move according to the roll of two dice. The two dice are marked +, +, +, −, −, − and −3, −2, −1, +1, +2, +3. If you go *beyond* 5 you are saved, but *beyond* −5 you are in the water with the sharks!

   The pirate captain rolls the dice and calls out the result of the operations die first. A '+' means you face the safety of the boat, a '−' means you face the danger of the sharks. The captain then calls out the number on the other die. If it is positive you walk forward that number of steps, if it is negative you walk backwards the number of steps.
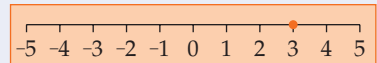
**1** You are in the position marked on the plank.
   The pirate captain rolls a + on the operations die and −2 on the number die. Copy the diagram and clearly mark your finishing position on the plank.
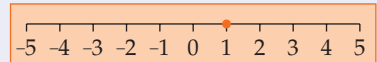


**2** You are at the position marked on the plank.
   Write down two different ways in which you could be saved on the next roll.



**3** You now find yourself on this longer plank at the position marked. This time if you go *beyond* 8 you are saved while *beyond* −8 sees you in with the sharks.
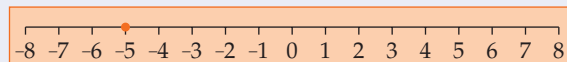


   You meet the sharks after two more rolls of the dice.
   Find as many different ways as possible that this might happen. Check with your partner to make sure you've covered all possibilities.

**4** The first four rolls of the dice have all shown − on the operations die. However, you are still at the starting point (0) on the longer plank from Question **3**.
   Write as many ways as you can to explain how this might have happened. Check with your partner to make sure you've covered all possibilities.

## problem solving

### Pattern sums

Copy and complete each of the following patterns. Arrange the numbers given, one in each circle, so that the sum of each line is equal to the stated value.

**1** Use -4, -3, -2, -1, 0, 1, 2; sum = -3     **2** Use -2, -1, 0, 1, 2, 3; sum = 0
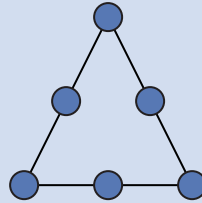
Try 'guess and check'.

## 2.6 Multiplication of directed numbers
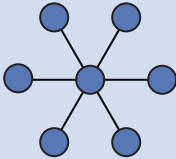
To work out how to multiply directed numbers we will think of multiplication as 'lots of'. Remember, $2 \times 5$ can be thought of as 2 lots of 5 (that is, $5 + 5$).
So, we can change multiplication questions into addition questions.

### worked example 7

Evaluate the following.

**(a)** $+2 \times +3$          **(b)** $+2 \times -3$          **(c)** $-2 \times +3$          **(d)** $-2 \times -3$

| Steps | Solutions |
|---|---|
| **(a)** 1. Write as a 'lots of' question. | **(a)** 2 lots of +3 |
| 2. Perform the addition. | $+3 + +3$ |
| | $= +6$ |
| **(b)** 1. Write as a 'lots of' question. | **(b)** 2 lots of -3 |
| 2. Perform the addition. | $-3 + -3$ |
| | $= -6$ |
| **(c)** 1. Write as a 'lots of' question. (The -ve sign at the start means 'the opposite of'.) | **(c)** 'the opposite of' 2 lots of +3 |

2. Do the arithmetic for 2 lots of +3. 

$$+3 + {}^+3$$
$$= {}^+6$$

3. Take into account 'the opposite of' 

$$-2 \times {}^+3 = {}^-6$$

**(d)** 1. Write as a 'lots of' question. (The –ve sign at the start means 'the opposite of'.) 

**(d)** 'the opposite of' 2 lots of –3

2. Do the arithmetic for 2 lots of –3. 

$$-3 + {}^-3$$
$$= {}^-6$$

3. Take into account 'the opposite of'. 

$$-2 \times {}^-3 = {}^+6$$

---

### Multiplying directed numbers

Multiplying two numbers with like signs gives a positive answer.
Multiplying two numbers with unlike signs gives a negative answer.

| | |
|---|---|
| $+ \times + = +$ | $+ \times - = -$ |
| $- \times + = -$ | $- \times - = +$ |

---

## worked example 8

Calculate:

**(a)** $-5 \times {}^-7$ **(b)** $-6 \times {}^+9$

| **Steps** | **Solutions** |
|---|---|
| **(a)** 1. Determine the sign of the answer. | **(a)** $- \times - = +$ |
| 2. Perform the multiplication. | $5 \times 7 = 35$ |
| 3. Write the question and answer together, putting in place the correct sign for the answer. Remember, if the final answer is + you can leave out the sign. | $-5 \times {}^-7 = {}^+35$ |
| **(b)** 1. Determine the sign of the answer. | **(b)** $- \times + = -$ |
| 2. Perform the multiplication. | $6 \times 9 = 54$ |
| 3. Write the question and answer together, putting in place the correct sign for the answer. | $-6 \times {}^+9 = {}^-54$ |

**e** eTutorial

## exercise 2.6 Multiplication of directed numbers

**e** hi.com.au

**p** Preparation: Prep Zone Q5 and 6, Exs 2.3 and 2.4

### Core

**1** Calculate the following by thinking of them as 'lots of' questions.

**(a)** $+3 \times {}^-4$ **(b)** $-2 \times {}^-5$ **(c)** $-2 \times {}^+5$ **(d)** $+2 \times {}^+7$

**(e)** $+3 \times {}^+4$ **(f)** $+3 \times {}^-6$ **(g)** $-3 \times {}^-5$ **(h)** $-3 \times {}^+4$

**e** Hint

**2** Another way to understand the multiplication rules is by using a pattern.
Copy and complete by following the pattern.

$3 \times 2 = 6$      $3 \times {-2} = \underline{\quad}$      ${-2} \times {-2} = \underline{\quad}$

$3 \times 1 = 3$      $2 \times {-2} = \underline{\quad}$      ${-2} \times {-1} = \underline{\quad}$

$3 \times 0 = 0$      $1 \times {-2} = \underline{\quad}$      ${-2} \times 0 = \underline{\quad}$

$3 \times {-1} = \underline{\quad}$      $0 \times {-2} = \underline{\quad}$      ${-2} \times 1 = \underline{\quad}$

$3 \times {-2} = \underline{\quad}$      ${-1} \times {-2} = \underline{\quad}$      ${-2} \times 2 = \underline{\quad}$

$3 \times {-3} = \underline{\quad}$      ${-2} \times {-2} = \underline{\quad}$      ${-2} \times 3 = \underline{\quad}$

**3** Calculate:

**(a)** $+6 \times +5$    **(b)** $+7 \times +3$    **(c)** $+11 \times +7$    **(d)** $+6 \times +9$    **e** Hint

**(e)** $+8 \times -5$    **(f)** $+12 \times -2$    **(g)** $+9 \times -3$    **(h)** $+5 \times -7$

**(i)** $-2 \times +4$    **(j)** $-2 \times +8$    **(k)** $-5 \times +5$    **(l)** $-4 \times +4$

**(m)** $-6 \times -5$    **(n)** $-2 \times -4$    **(o)** $-3 \times -2$    **(p)** $-5 \times -3$

**4** Evaluate:

**(a)** $-12 \times 5$    **(b)** $7 \times -20$    **(c)** $9 \times -10$    **(d)** $-11 \times -12$    **e** Hint

**(e)** $-40 \times -2$    **(f)** $-60 \times 3$    **(g)** $23 \times -5$    **(h)** $9 \times -52$

**(i)** $-56 \times -4$    **(j)** $-45 \times -8$    **(k)** $-76 \times 43$    **(l)** $91 \times -26$

**(m)** $-334 \times -3$    **(n)** $-653 \times 4$    **(o)** $828 \times -49$    **(p)** $-407 \times -66$    **e** Worksheet C2.10

**5** Evaluate the following:

**(a)** $-2 \times -4 \times 3$    **(b)** $-1 \times 3 \times 4$    **(c)** $6 \times -8 \times -1$    **e** Hint

**(d)** $3 \times -5 \times -1$    **(e)** $-1 \times -1 \times -6$    **(f)** $-2 \times 7 \times -2$

**(g)** $4 \times 5 \times -2$    **(h)** $-9 \times -1 \times -2$    **(i)** $8 \times -1 \times 3$

**(j)** $2 \times 5 \times -5$    **(k)** $5 \times -2 \times -2$    **(l)** $-8 \times -1 \times 4$

**(m)** $-3 \times -3 \times -3$    **(n)** $-2 \times -2 \times -2$    **(o)** $-10 \times -10 \times -10$

**(p)** $-4 \times -1 \times -2 \times -1$    **(q)** $3 \times -2 \times -1 \times -4$    **(r)** $-3 \times 2 \times -1 \times -2$

**(s)** $-4 \times -1 \times -2 \times 2 \times -3$    **(t)** $5 \times -2 \times 3 \times -1 \times -1$    **(u)** $-3 \times 2 \times 6 \times -1 \times -2$

**6** Copy and complete. (Many answers are possible.)

$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = -24$

## Extension

**7** Calculate:

**(a)** $-1 \times -1$        **(b)** $-1 \times -1 \times -1$

**(c)** $-1 \times -1 \times -1 \times -1$        **(d)** $-1 \times -1 \times -1 \times -1 \times -1$

**(e)** $-1 \times -1 \times -1 \times -1 \times -1 \times -1$        **(f)** $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

**(g)** Copy and complete:

In a multiplication chain involving directed numbers, the answer is positive if there is an $\underline{\quad}$ number of negative values and negative if there is an $\underline{\quad}$ number of negative values.

What would the answer be if:

**(h)** Eighty-six $-1$s were multiplied together?

**(i)** Ninety-nine $-1$s were multiplied together?

**8** Evaluate:

**(a)** $-2 \times -2 \times -2 \times -2$        **(b)** $-3 \times -3 \times -3$    **e** Hint

**(c)** $-4 \times -1 \times -4$        **(d)** $-2 \times -1 \times -2 \times -1$

**(e)** $-3 \times -2 \times -1 \times -1 \times -1 \times -2$        **(f)** $-4 \times -1 \times -2 \times -1 \times -2$

**(g)** $2 \times -3 \times -2 \times -1 \times 4$      **(h)** $-5 \times -2 \times -3 \times -1 \times -2 \times -3 \times -1$

**(i)** $-4 \times -1 \times 3 \times -2 \times 2 \times -1$      **(j)** $-5 \times 3 \times -2 \times 4$

**9** In a particular card game, each player's final score is obtained by counting up the number of cards of each suit, and allocating points as shown in the table.

Find the score of a player who finishes with:

| Card | Score |
|------|-------|
| Hearts | –5 points each |
| Diamonds | –3 points each |
| Clubs | +2 points each |
| Spades | +4 points each |

**(a)** 7 hearts      **(b)** 3 diamonds

**(c)** 5 hearts, 6 clubs      **(d)** 6 diamonds, 9 spades

**(e)** 4 hearts, 5 clubs, 3 spades      **(f)** 3 hearts, 3 diamonds, 8 clubs

**(g)** 6 hearts, 2 diamonds, 4 spades      **(h)** 4 hearts, 4 diamonds, 5 clubs, 3 spades

# 2.7 Division of directed numbers

Consider the multiplication $-2 \times 3 = -6$.

We can write this as:     $-6 \div 3 = -2$ or $-6 \div -2 = 3$

Similarly:                $-2 \times -3 = 6$ can be written as $6 \div -2 = -3$

We also know that $6 \div 2 = 3$.

We can see that the rules for division of directed numbers are the same as those for multiplication.

## Dividing directed numbers

$$- \div - = + \text{ and } + \div + = +$$
$$- \div + = - \text{ and } + \div - = -$$

## worked example 9

Calculate:

**(a)** $-56 \div -7$                            **(b)** $20 \div -4$

| Steps | Solutions |
|-------|-----------|
| **(a)** 1. Determine the sign of the answer. | **(a)** $- \div - = +$ |
| 2. Perform the division. | $56 \div 7 = 8$ |
| 3. Write the sign of the answer in front or, since the answer is +, leave off the sign and simply write | $-56 \div -7 = +8$ <br> $-56 \div -7 = 8$ |
| **(b)** 1. Determine the sign of the answer. | **(b)** $+ \div - = -$ |
| 2. Perform the division. | $20 \div 4 = 5$ |
| 3. Write the sign of the answer in front. | $20 \div -4 = -5$ |

**p** Preparation: Prep Zone Q7, Exs 2.3, 2.4 and 2.6

### Core

**e** hi.com.au

**1** Calculate:

(a) $-12 \div -3$    (b) $9 \div -3$    (c) $6 \div -2$    (d) $-18 \div -2$

(e) $-8 \div 4$    (f) $-30 \div 5$    (g) $20 \div -5$    (h) $21 \div -7$

(i) $-49 \div -7$    (j) $-48 \div -8$    (k) $-54 \div 6$    (l) $-32 \div 4$

(m) $35 \div -5$    (n) $90 \div -10$    (o) $-81 \div -9$    (p) $55 \div -11$

**e** Hint

**e** Worksheet C2.11

**2** Evaluate:

(a) $50 \div -2$    (b) $-30 \div 5$    (c) $-56 \div -8$    (d) $84 \div -12$

(e) $-27 \div 9$    (f) $-63 \div -9$    (g) $45 \div -5$    (h) $-16 \div 4$

(i) $-96 \div -12$    (j) $70 \div -7$    (k) $121 \div -11$    (l) $-64 \div 8$

(m) $-60 \div 6$    (n) $60 \div -12$    (o) $-100 \div -10$    (p) $-88 \div -8$

**e** Hint

**3** Use short division to calculate the following.

(a) $261 \div -3$    (b) $-684 \div 2$    (c) $-565 \div -5$

(d) $8050 \div -10$    (e) $-816 \div -4$    (f) $-561 \div -3$

(g) $-6864 \div 2$    (h) $-5634 \div -6$    (i) $-7263 \div -9$

(j) $7465 \div -5$    (k) $8036 \div -7$    (l) $-6248 \div 11$

**e** Hint

**4** Find the following:

(a) $\dfrac{48}{-3}$    (b) $\dfrac{-77}{11}$    (c) $\dfrac{-25}{5}$    (d) $\dfrac{120}{-8}$

(e) $\dfrac{52}{-2}$    (f) $\dfrac{-93}{-3}$    (g) $\dfrac{-84}{-4}$    (h) $\dfrac{-96}{4}$

(i) $\dfrac{-90}{-6}$    (j) $\dfrac{100}{-5}$    (k) $\dfrac{-360}{40}$    (l) $\dfrac{-240}{-80}$

(m) $\dfrac{120}{-24}$    (n) $\dfrac{-1000}{-20}$    (o) $\dfrac{-480}{2}$    (p) $\dfrac{280}{-4}$

**e** Hint

**5** Write three numbers that when divided by 8 give a negative whole number answer.

**6** A syndicate of 6 people loses $240 trying to win a lottery. How much is each person's share of the loss?

**7** The temperature falls 20°C in 5 hours one night in Copenhagen. What is the average temperature change each hour?

**e** Hint

### Extension

**8** A bathysphere begins at the ocean surface and is lowered 5 metres every minute. If the ocean is 95 metres deep in this area, calculate:

(a) the time taken for the bathysphere to reach the ocean floor.

(b) the time taken to rise to the surface if the bathysphere rises at a rate of 2 metres per minute.

**e** Animation

**9** A department store has 6 floors below ground level and 9 floors above ground. A tourist on the top level takes a lift to the bottom level, stopping every 3 floors to look around. How many stops will the tourist make?

**e** eQuestions

**e** Homework 2.3

# 2.8 Combined operations

The rules for order of operations also apply to directed numbers.

**1** Perform calculations in brackets first.

**2** Next, perform multiplications and divisions in order from left to right.

**3** Finally, perform additions and subtractions from left to right.

## worked example 10

Evaluate:

**(a)** $-7 \times 4 \div -2 \times -2$

**(b)** $-3 + (-2 \times -6) - 4$

### Steps

**(a)** 1. Write out the question.

  2. Since there are only multiplication and division involved, work from left to right.

**(b)** 1. Write out the question.

  2. Evaluate brackets first.

  3. Perform additions and subtractions from left to right.

### Solutions

**(a)** $-7 \times 4 \div -2 \times -2$

$= -28 \div -2 \times -2$

$= 14 \times -2$

$= -28$

**(b)** $-3 + (-2 \times -6) - 4$

$= -3 + (12) - 4$

$= 9 - 4$

$= 5$

## exercise 2.8    Combined operations

**ⓟ** Preparation: Prep Zone Q8, Exs 2.5, 2.6 and 2.7

### Core

**1** Evaluate:

**(a)** $10 - 11 + 6 - 7$

**(b)** $-7 - 3 + 5 - 4$

**(c)** $-4 + 8 - 3 - 5$    **ⓔ** Hint

**(d)** $5 - 12 - 3 + 8$

**(e)** $-4 + 7 - 6 + 8$

**(f)** $-10 + 7 - 6 + 18$

**(g)** $16 - 20 - 5 + 7$

**(h)** $-17 - 4 + 15 + 9$

**(i)** $-24 + 16 + 10 - 17$

**(j)** $45 \div -9 \times -2$

**(k)** $-81 \div 9 \times 3$

**(l)** $-32 \div -8 \times 5$

**(m)** $56 \div -7 \div -2$

**(n)** $-6 \times 7 \div 3$

**(o)** $-5 \times -9 \div 3$

**(p)** $-21 \div 7 \times 10 \div -5$

**(q)** $84 \div -4 \div -3 \times 2$

**(r)** $-48 \div -8 \times -3 \div 9$

**2** Evaluate:

**(a)** $4 + (6 \times -3) - 2$

**(b)** $(-3 - 5) \times -8$

**(c)** $-5 \times (-7 - 4)$    **ⓔ** Hint

**(d)** $-2 + (-64 \div -8)$

**(e)** $7 + (14 \div -2) - 3$

**(f)** $44 \div (-12 + 1)$

**(g)** $(-6 \times 5) + (-24 \div -6)$

**(h)** $(7 - 15) - (-7 + 3)$

**(i)** $(-8 + 2) \times (4 - 10)$

**(j)** $(12 \times -3) \div (-2 - 2)$

**(k)** $(-5 \times -9) - (7 - 10)$

**(l)** $(-45 \div 9) + (9 - 21)$

**(m)** $(-48 \div -4) \div (-1 + 3)$

**(n)** $(-7 + 19) - (-24 \div 12)$

**(o)** $(-12 + 16) \times (15 \div -5)$

**3** Copy and complete. (Different answers are possible.)

$-3 - ($ _____ $\div$ _____ $) = 5$

**4** Choose the correct answer.

(a) The expression $5 \times 6 \div -3 + 3 \times -1$ is equal to:

　　**A** –13　　　　**B** –5　　　　**C** –3　　　　**D** 7

(b) The expression $-4 - 2 \times -3 + 6$ is equal to:

　　**A** –12　　　　**B** –2　　　　**C** 8　　　　**D** 24

(c) The expression $-16 \div (2 - 6) + 8 \div (-2)$ is equal to:

　　**A** –18　　　　**B** 0　　　　**C** 2　　　　**D** 3

**5** Evaluate the following:

(a) $24 \div -3 + (-5) \times -2$　　　　(b) $18 + 2 \times -3 + 5 - 2$

(c) $-15 \div (2 - 5) - 6$　　　　(d) $30 \div -6 \times 5 + 6$

(e) $5 \times 8 \div -2 \times -5 + 4$　　　　(f) $-5 \times 8 \div 2 \times -2 - 6 \times -3$

(g) $-56 \div -7 + 4 \times -2$　　　　(h) $(7 - 10) \times 20 \div -5$

(i) $-6 \times 12 + 10 \div -2$　　　　(j) $28 + (-5 \times -7) \times -1$

(k) $72 \div (-2 + 14) - 11$　　　　(l) $13 - 2 \times -6 - 5 \times 4$

## *Extension*

**6** A company makes a profit of $3 million per month for 7 months, and then loses $9 million per month for 5 months. What was the overall result for the year?

**7** Jade places some water at a temperature of 24°C in an ice-cube tray and puts the tray in the freezer. The freezer operates at a temperature of –10°C, and is able to lower the water's temperature by 4°C per hour. What will be the temperature of the tray's contents after:

(a) 3 hours　　(b) 6 hours　　(c) 7 hours　　(d) 10 hours?

**8** Steve plays golf with a handicap of 14 (which means at the end of a game of 18 holes 14 is subtracted from his finishing score to determine his official score). Each hole of golf may be scored as follows (lowest score at the end of the game wins):

| Par | 0 |
| --- | --- |
| Birdie | –1 |
| Eagle | –2 |
| Albatross | –3 |
| Bogey | +1 |
| Double Bogey | +2 |
| Triple Bogey | +3 |
| Disaster | +10 |



I thought you said you wanted my handy cap.
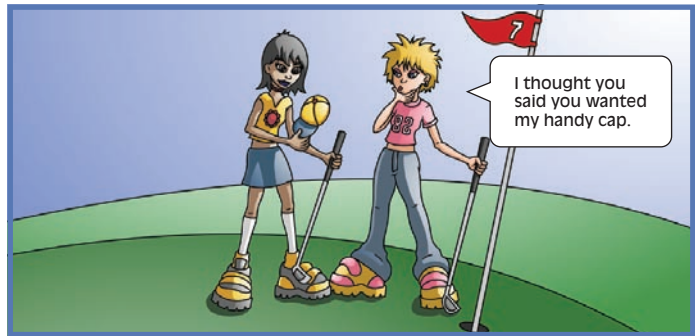
During one particular round of golf, Steve has 1 eagle, 4 birdies, 5 pars, 7 bogeys and 1 disaster.

(a) What was Steve's finishing score?

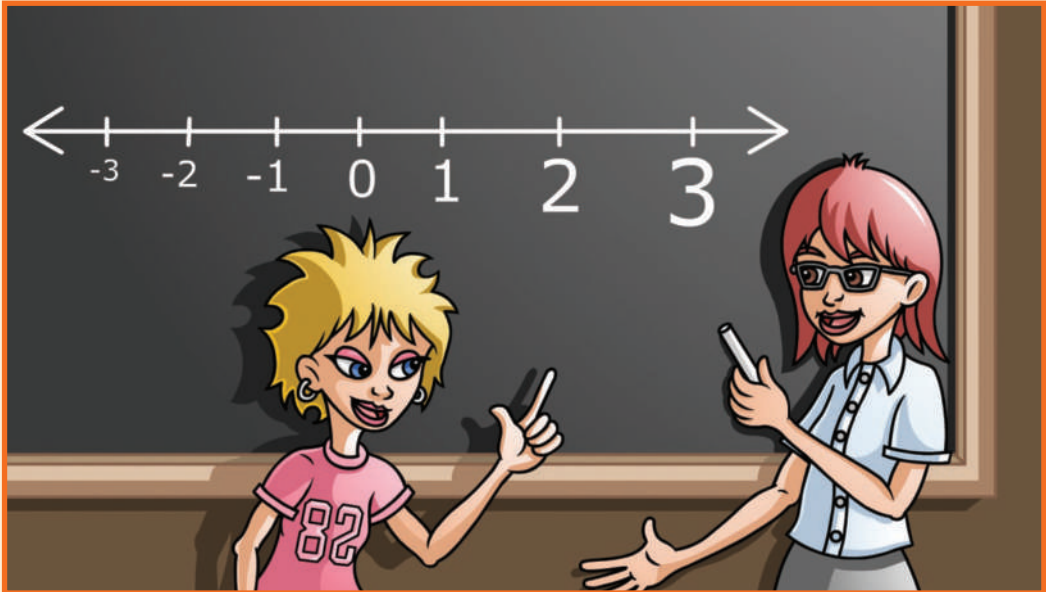(b) What was his official score, taking into account his handicap?

Answer the following, showing your working, then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

Evaluate:

| | | | |
|---|---|---|---|
| $8 \div -8 + 3$ | **F** | $2 \times -4 + 5$ | **E** |
| $-3 + 2 \times 4$ | **L** | $9 - 12 \div 2$ | **O** |
| $16 \div 4 \times -2$ | **P** | $36 \div -6 \times 3$ | **V** |
| $28 \div -4 + 6 \times -2$ | **T** | $-18 \div -6 - 4 \times 3$ | **A** |
| $-18 \times -2 + (-9 - 3)$ | **I** | $54 \div -9 - (7 - 8)$ | **U** |
| $(-23 + 17) \div (8 - 9)$ | **B** | $(45 \div -5) - (7 - 12)$ | **D** |
| $12 - 7 + 48 \div -8$ | **N** | $-44 \div -4 - 7 + 3$ | **C** |
| $-18 \div 6 \times -4 - 18$ | **S** | $3 \times -5 \times -2 - 40$ | **M** |
| $28 \div -4 - 8 + 2$ | **R** | | |

' 
| 24 |

| 2 | -3 | -3 | 5 |

| -8 | 3 | -6 | 24 | -19 | 24 | -18 | -3 |

| -9 | 6 | 3 | -5 | -19 |

| -4 | 24 | -13 | -3 | 7 | -19 | -3 | -4 |

| -1 | -5 | -10 | 6 | -3 | -13 | -6 |
'

# 2.9 Calculators and directed numbers

We can use our calculator to evaluate combined operation questions involving directed numbers.

In the following example the keystrokes given are for a calculator which has a special button to represent negative numbers. It looks like $(-)$ . On some other calculators there is a different button which performs the same task: $+/-$ .

Calculators have the order of operations built into them so we simply need to enter the numbers and operations as they are written.

## worked example 11

Evaluate the following using your calculator.
**(a)** $-120 \div 5 - 16 \times -2$
**(b)** $-8 \times (-34 + 50) \times 25 - 32$

### Steps

**(a)** Enter the numbers in order into your calculator and write down the answer.

| $(-)$ | 1 | 2 | 0 | $\div$ | 5 | $-$ |

| 1 | 6 | $\times$ | $(-)$ | 2 | $=$ |

**(b)** Enter the numbers in order into your calculator and write down the answer.

| $(-)$ | 8 | $\times$ | ( | $(-)$ | 3 | 4 |

| $+$ | 5 | 0 | ) | $\times$ | 2 | 5 |

| $-$ | 3 | 2 | $=$ |

### Solutions

**(a)** The solution is 8.

**(b)** The solution is $-3232$.

## exercise 2.9 Calculators and directed numbers

**p** Preparation Ex 2.8

### Core

**1** Evaluate the following using a calculator.

**(a)** $18 + (16 - 42) - 29$

**(b)** $-27 - (42 - 91) + 17$

**(c)** $16 \times -3 - 5 \times -2$

**(d)** $81 \div (9 - 18) + (16 - 25)$

**(e)** $(24 - 16) \div (16 - 20) + 17 \times -3$

**(f)** $-294 \div (-3 \times -2) - 16 \div -8$

**(g)** $-26 + 17 - 14 \times -2 - (-38 \div -2)$

**(h)** $4 \times -2 \times -3 + 200 \div -5 \div -8$

**e** Hint

**(i)** $(-33 \div 11 + 28 \div -4) \times -10$  **(j)** $(-26 \div -13 + 14) \div (40 + 6 \times -6)$

**(k)** $(55 \div -5) + 7 \times (-12 \div 6) + 16$  **(l)** $(15 \times -6 - 15) \times (18 - 21 + 5)$

**2** Evaluate the following using a calculator, writing your answers correct to two decimal places where appropriate.

**(a)** $26 - 17 \div -8 \times -4 + 3$  **(b)** $14 \times -26 + 18 - 13 \div -2$

**(c)** $115 \div -29 + 81 \div -16$  **(d)** $14 \times (-3 - 4 \times -3) \div (16 - 18)$

**(e)** $-22 \times 16 - 14 \times -21 \div 6$  **(f)** $81 \div -9 \times -2 + (14 - 17)$

**(g)** $-15 - 31 \div 16 + 4 \times -2 + 1$  **(h)** $33 \div -11 + 17 \times -6 \div -2$

**(i)** $-182 \times -16 - 14 \div 16 \div -2 \times -3$  **(j)** $-39 \div -13 + 14 \times -7 \div -2$

**(k)** $-17 + 52 \times -2 \div 3 - 21 \div 4$  **(l)** $-98 \times 6 \div 15 - 35 \times 6 \div 17$

**3** Choose the correct answer by working these out using your calculator.

**(a)** $45.78 - 52.5$ correct to two decimal places:

    **A** 67.20  **B** 98.28  **C** -6.72  **D** -13.28

**(b)** $\sqrt{7} - \sqrt{12}$ correct to two decimal places:

    **A** -0.818  **B** -0.81  **C** 0.82  **D** -0.82

**(c)** $17^2 - 18^2$:

    **A** -35  **B** -2  **C** -1  **D** 35

**(d)** $6^5 - 5^6$:

    **A** -7849  **B** 1  **C** -1  **D** 7849

**4** Use your own values for $a$ and $b$ and your calculator to verify that the following equations are true.

**(a)** $a - (-b) = a + b$  **(b)** $a + (-b) = a - b$

**(c)** $a - (+b) = a - b$  **(d)** $a + (+b) = a + b$

## *Extension*

**5** Use your calculator to work these out by using the memory (if necessary) and the $\boxed{(-)}$ or $\boxed{+/-}$ key. Give your answer correct to three decimal places.

**(a)** $(78.9 \times 3.4) - (54.8 \times 7.77)$  **(b)** $(54.9 \times 5.6) - (6.2 \times 103.7)$

**(c)** $(45.66 \div 7.94) - (95.5 \div 3.6)$  **(d)** $(60.19 \div 3.22) - (87.43 \div 3.28)$

**(e)** $4(\sqrt{5})^7 - (17.7 \times 9.2)$  **(f)** $3(\sqrt{3})^5 - (46.2 \times 8.1)$

**(g)** $2(\sqrt{10})^2 - (53.66)^2$  **(h)** $7(\sqrt{5})^3 - (76.2)^2$

**(i)** $\sqrt{3} - \sqrt{2} - \sqrt{5}$  **(j)** $\sqrt{7} - \sqrt{3} - \sqrt{6}$

**(k)** $(4.6)^3 - (6.7)^5 + (4.8)^2$  **(l)** $(2.3)^4 - (5.9)^5 + (1.5)^2$

**(m)** $(\sqrt{6})^3 - (10.5)^4 - (1.005)^2$  **(n)** $(12.3)^2 - (\sqrt{11})^5 - (3.833)^3$

**(o)** $-\sqrt{7} + \sqrt{6} - \left(\dfrac{5}{7}\right)^3$  **(p)** $-(6.7)^3 - \sqrt{17} + \left(\dfrac{3}{11}\right)^6$

**(q)** $(48.5 + 7.3) \times (-16.7 + 3.22)$  **(r)** $(56 - 72.8) \times (-12.3 + 17.1)$

**(s)** $(-14.7 + 5.8) \times (4.3 - 8.25)$  **(t)** $(4.667 - 15.7) \times (9.6 - 11.98)$

**(u)** $(3.8 - 5.7)^2 - (\sqrt{8})$  **(v)** $(19.7 - 24.12)^3 - \sqrt{11}$

## The ultimate cool



How cold can you go? The coldest temperature ever recorded on Earth was –89°C at the Russian research station Vostok in Antarctica. By comparison, the lowest temperature in the northern hemisphere, recorded in Oimekon, Siberia, was a mere –78°C.

The average Antarctic temperatures for different seasons are shown in the following table.

| Season | Inland temperature (°C) | Coastal temperature (°C) |
|---|---|---|
| Autumn | –57 | –2 to –14 |
| Winter | –59 | –8 to –20 |
| Spring | –49 | –3 to –11 |
| Summer | –32 | +1 to –2 |

At these sorts of inland temperatures, it becomes painful to breathe because the moisture on the hairs of your nose freezes instantly. But is this as cold as it can get? The coldest place in our solar system is probably Triton, a moon of the planet Neptune, where the temperature reaches –235°C. Parts of deep space, at –270°C, have the lowest temperature of anything in nature.

So, is this the ultimate cool? Scientists have long been interested in finding the coldest temperature possible. In early 2001, scientists at the Australian National University produced the coldest known substance for the first time in Australia. It was a super-cold

cloud of atoms reaching very close to –273°C. To create the cloud, the temperature of one million atoms was lowered using laser beams.

The coldest temperature possible is –273°C, which is also called 'absolute zero' in the Kelvin temperature scale. This is where atoms reach their lowest possible level of movement and everything freezes solid—even air. The Australian scientists didn't quite get there. They were about 100 billionths of a degree above absolute zero.
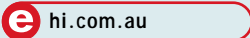
## Questions

**1** How much warmer is:
   **(a)** the coldest temperature ever recorded in the northern hemisphere compared to the coldest temperature recorded in Antarctica
   **(b)** the average winter inland temperature in Antarctica compared to Triton?

**2** How many degrees difference is there between the maximum and minimum coastal temperatures in Antarctica in:
   **(a)** autumn  **(b)** winter  **(c)** spring  **(d)** summer?

**3** The United Nations Intergovernmental Panel on Climate Change has predicted that because of the rising levels of carbon dioxide and other greenhouse gases in the atmosphere, the global average surface temperature will rise between 1°C and 6°C by the turn of the century. If temperatures in Antarctica show the same change as the global average, what would be the range of average inland temperatures in Antarctica for each of the four seasons by the end of the century according to the predictions?

**4** Look at the following table of temperatures.

| Temperature (°C) | Temperature fact |
| --- | --- |
| +5700 | Temperature at the Sun's surface |
| +1065 | Gold melts |
| +100 | Water boils |
| +57 | Highest recorded temperature on Earth |
| +37 | Human body temperature |
| 0 | Water freezes |
| –37 | Car antifreeze freezes |
| –89 | Lowest recorded temperature on Earth |
| –273 | Absolute zero |

What is the difference between:
   **(a)** human body temperature and the temperature when car antifreeze freezes
   **(b)** the temperature of the Sun's surface and absolute zero
   **(c)** the temperatures at which gold melts and water freezes
   **(d)** the highest and lowest temperatures recorded on Earth?

## Research

**e** hi.com.au

Present a report on the Kelvin temperature scale and describe how to convert from degrees Kelvin to degrees Celsius. Explain why –273°C is called absolute zero.

# languagezone

## Summary

Copy and complete the following summary of this chapter using the words and phrase from the list. A word or phrase may be used more than once.

**1** Two negative numbers multiply to give a _____ number.

**2** A _____ number divided by a positive number gives a negative number.

**3** For multiplication and _____ of two directed numbers, if the signs are the same then the answer will be _____.

**4** _____ _____ represent not only a quantity, but also a direction.

**5** The addition of two negative numbers will give a _____ answer.

## Key words

**addition**
**directed numbers**
**division**
**multiplication**
**negative**
**positive**
**subtraction**

## Questions

**1** Write the opposite of each of the following.
   **(a)** depositing $90 into a bank account
   **(b)** walking south 5 km

**2** Write down a non-mathematical meaning of the word 'positive'.

**3** Write two real-life examples where numbers, but not negative numbers, are used.

**4** Write two examples of real situations where negative numbers are commonly used.

**5** Write out how we would say $-9 + (-8) - (+4)$.

**6** Make at least 12 words of three or more letters from the letters in the grid. All words must include the middle letter. Letters cannot be used more than once. There is one nine-letter word.

| D | O | I |
|---|---|---|
| C | E | I |
| T | R | N |

**7** Write the verbs for 'addition', 'subtraction', 'multiplication' and 'division'.

e Worksheet L2.1

e Worksheet L2.2

## FAQs

*Why does −3 − 4 equal −7? Don't two negatives give a positive?*
Two negatives give a positive when they are multiplied, divided or the signs
are next to each other. Subtracting a positive number from a negative number
will always give a negative answer because by taking away a positive number
we are moving to an even more negative position on the number line.
(Remember that −3 − 4 is the same as −3 − (+4).)

*I keep getting confused about which sign goes in front of the answer to a question
like −27 + 60. How can I remember what to do?*
A quick way to remember this is that in this kind of question when you have
two numbers with different signs, the sign of the answer will be the same as
the sign of the 'bigger' number (ignoring its sign). Because 60 is bigger than
27 the answer will be positive. So −27 + 60 = 33.

### Core

**1** Write a directed number suggested by the following.   **2.1**
   **(a)** 14 degrees below zero       **(b)** Altitude: 200 metres

**2** State the opposite of:   **2.1**
   **(a)** north 5 km       **(b)** +27

**3** Write < or > between the following pairs of numbers to make a true   **2.2**
statement.
   **(a)** −52 ____ 25       **(b)** 19 ____ −20

**4** Arrange the following numbers in order from smallest to largest.   **2.2**
   **(a)** −7, 12, 0, −9, 7       **(b)** 4, −4000, 40, 400

**5** Evaluate. (Do not use a calculator.)   **2.3**
   **(a)** +7 + (−10)    **(b)** +9 + (−6)    **(c)** −3 + (+18)
   **(d)** −15 + (+5)    **(e)** −11 + (−4)    **(f)** +16 + (+2)
   **(g)** +5 + (−3) + (−3)    **(h)** −8 + (−5) + (+6)

**6** Evaluate. (Do not use a calculator.)   **2.4**
   **(a)** +1 − (+5)   **(b)** +16 − (+8)   **(c)** +9 − (+3)   **(d)** +7 − (+12)
   **(e)** −4 − (−4)   **(f)** −14 − (+18)   **(g)** −16 − (+21)   **(h)** −12 − (−52)

**7** Evaluate. (Do not use a calculator.)   **2.5**
   **(a)** 9 − 11   **(b)** −3 + 10   **(c)** −10 + 4   **(d)** 8 − 12
   **(e)** −4 + 41   **(f)** −14 + 28   **(g)** −5 − 8 + 2   **(h)** 4 − 9 + 2

**8** Find the following products. (Do not use a calculator.)

(a) $-14 \times 2$     (b) $4 \times -5$     (c) $-6 \times -6$     (d) $-7 \times -12$

(e) $15 \times -9$     (f) $-22 \times 8$     (g) $-60 \times 30$     (h) $-28 \times -200$

**9** Evaluate. (Do not use a calculator.)

(a) $-36 \div 3$     (b) $55 \div -11$     (c) $-28 \div -4$     (d) $-27 \div 3$

(e) $66 \div -11$     (f) $-91 \div -7$     (g) $\dfrac{72}{-9}$     (h) $\dfrac{-45}{-5}$

**10** Evaluate the following expressions. (Do not use a calculator.)

(a) $45 \div 9 \times -2 - 4$             (b) $-44 \div -2 \times 2$

(c) $12 \times -6 + 2 - 10$            (d) $46 + (-6 \times 7) + 20 \div -5$

(e) $-9 \times -5 - 3 \times 4 + 2$        (f) $-8 + (-18) \div -3 - 4 \times -4$

**11** Choose the correct answer.

(a) The difference between half the sum of -2 and 8 and twice the sum of 5 and -6 is:

    **A** 2          **B** 1          **C** 5          **D** 6

(b) The sum of the product of -2 and -6 and the difference between 5 and -3 is:

    **A** 8          **B** 12          **C** 14          **D** 20

## Extension

**12** Michelle made deposits of $210, $25, $45.50 and $66.75 into her bank account during one month and withdrawals of $35.75, $56.90, $214 and $102.50 during the same period. By how much did her balance *increase* during this period?

**13** In the game called Count 'Em Up, red tokens are worth 5 points, black tokens are worth -3 points and white tokens are worth -1 point. Calculate the total point score at the end of a round for each of the following players.

Felicity: 2 red, 3 black and 1 white

Georgia: 3 red, 4 black and 2 white

Rosalie: 3 red, 5 black and 4 white

Doreen: 2 red, 5 black and 2 white

**14** The minimum overnight temperatures for one week at Katoomba were -2°C, -3°C, 1°C, 2°C, -3°C, -2°C, 0°C. Find the mean minimum overnight temperature for the week. (The mean is found by adding all values then dividing the result by the number of values.)

**15** Three friends invest a total of $270 in a lottery draw. If they collect prizes worth a total of:
- **(a)** $60, find the loss for each friend
- **(b)** $300, find the profit for each friend
- **(c)** $28 500, find the profit for each friend.

# ◄◄ REPLAY

**1** Set out these calculations in your normal way and work out the answers.
- **(a)** 23 + 68
- **(b)** 156 + 3047
- **(c)** 12 + 674 + 3825

**2** Rearrange in order from smallest to largest:
990, 909, 1001, 1101, 999, 1009, 1010

**3** Calculate:
- **(a)** $200 \times 37$
- **(b)** $798 \times 5000$
- **(c)** $25 \times 1200$

**4** Copy and complete each of the following to make true statements by writing < or > between the given numbers.
- **(a)** 201 ___ 1999
- **(b)** 2.6 ___ 2.0
- **(c)** 0.09 ___ 0.9

**5** Find the missing number that makes each of the following true.
- **(a)** $17 = 28 -$ ___
- **(b)** $6 \times 9 =$ ___ $- 6$
- **(c)** $7 +$ ___ $= 60 \div 3$

**6** List the numbers you get if you count by sevens, starting at 50 and finishing at 99.

**7** Copy and complete the following by finding the pattern.
- **(a)** 12, ___, 20, 24, ___, ___, 36
- **(b)** 1, ___, 4, 8, ___, ___, 64

**8** List all numbers divisible by both 8 and 6 that are less than 100.

**9** Write the following Hindu–Arabic numbers as Roman numerals.
- **(a)** 12
- **(b)** 79
- **(c)** 145
- **(d)** 683

**10** Cathryn works for 12 hours a week earning $9 per hour. How much does Cathryn receive for a week's work?

**11** Round these numbers to the first digit.
- **(a)** 27
- **(b)** 102
- **(c)** 982
- **(d)** 42 467

**12** Simplify:
- **(a)** $3 + 6 \times 4$
- **(b)** $48 \div 6 - 2 \times 4$
- **(c)** $4 \times (27 - 18) \div 12$