RETURN TO MAIN MENU

1 whole NUMBERS

Faster than a speeding bullet

Calculation speed counts when it comes to producing realistic computer animations.

Ever-increasing computer calculation speed used for games such as *Tomb Raider* means that Lara Croft outperforms any character from the past. The first successful high-speed electronic digital computer, ENIAC (electronic numerical integrator and computer), was used from 1946 to 1955. It had 18 000 vacuum tubes, took up 170 square metres of floor space, and probably took all day to perform the same number of calculations as the programmers use to make Lara blink. Even modern day non-scientific calculators outperform ENIAC—and all of this is done using only two digits, 0 and 1! Recently, Japan's Earth Simulator supercomputer broke the speed record to perform 40 trillion calculations a second. What would Lara make of that?

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outcomes

After completing this chapter you will be able to:

- recognise and use other counting systems
- estimate answers to simple problems involving whole numbers
- apply order of operations.

prepzone 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your *eMaths Zone* CD or ask your teacher for the Replay Worksheet.

Worksheet R1.1

1 Write down the answer to these tables questions as quickly as possible.

(a) 7×8 **(b)** 9×9 **(c)** 4×6
(d) 11×12 **(e)** 4×7 **(f)** 8×9
(g) 12×8 **(h)** 9×7 **(i)** 12×9

Worksheet R1.2

2 Choose the correct answer.
What is the place value of the red number?

(a) 45 780
A eight **B** thousands
C tens **D** eighty

(b) 1 264 184
A ten thousands **B** two
C hundreds **D** hundred thousands

(c) 3490
A units **B** tens
C hundreds **D** zero

Worksheet R1.3

3 Set out these calculations in your normal way and work out the answers.

(a) $456 + 56$ **(b)** $16 + 2047$ **(c)** $90 + 1267 + 341$

Worksheet R1.4

4 Set out these calculations in your normal way and work out the answers.

(a) $298 - 123$ **(b)** $854 - 227$ **(c)** $1406 - 249$

Worksheet R1.5

5 Set out these calculations in your normal way and work out the answers.

(a) 45×21 **(b)** 134×52 **(c)** 234×95

Worksheet R1.6

6 Set out these calculations in your normal way and work out the answers.

(a) $844 \div 4$ **(b)** $3708 \div 9$ **(c)** $897 \div 7$

KEY WORDS

Babylonian system	decimal system	Hindu–Arabic system	order of operations
binary system	Egyptian system	magic square	Roman system
Chinese system	estimate	magic sum	rounding

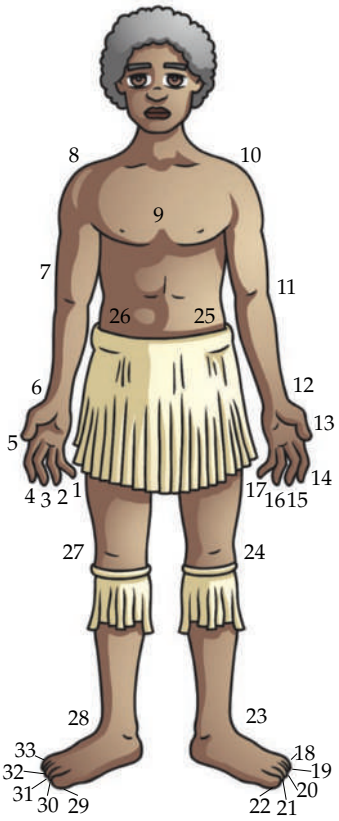
1.1 Number systems

The system of numbers we use is called the **Hindu–Arabic system**. We count using the base of ten.

Our number system started around 300–200 BC in India and was brought to Europe in the Middle Ages by the Arabs. It's now used all over the world, but many other number systems were once used.

The oldest 'counting machine' known is a 30 000-year-old wolf bone found in a country then known as Czechoslovakia, in 1937. It had 55 notches in two rows divided into groups of five. The age of the bone means that counting machines were invented very early in human history.

Many people used counting systems based on parts of their bodies. In the nineteenth century, some Torres Strait islanders counted by touching parts of their bodies.



Torres Strait system

The Egyptian system

The ancient Egyptians of around 3000 BC drew complex symbols, or hieroglyphics, for their numbers. The symbols could be placed from left to right or from right to left or from top to bottom.

one stick	two sticks								an arch
|	||	|||	||||	||| ||	||| |||	|||| |||	|||| ||||	||| ||| |||	∩
1	2	3	4	5	6	7	8	9	10

a coil of rope	a lotus flower	a bent reed	a tadpole	a genie
𓆼	𓆸	𓇋	𓆑	𓁐
100	1000	10 000	100 000	1 000 000

Examples:

23 is ∩∩|||

30 201 is 𓇋𓇋𓇋𓆼𓆼|

The Roman system

Around 300 BC, the Romans used letters as their symbols for numbers. The **Roman system** also made it easier to write numbers like 4 and 9. How?

I	II	III	IV	V	VI	VII	VIII	IX	X
1	2	3	4	5	6	7	8	9	10

L	C	D	M
50	100	500	1000

Examples:

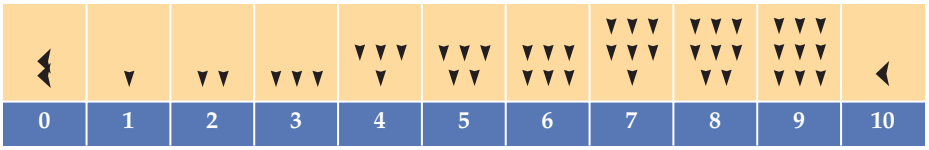
14 is XIV
76 is LXXVI
39 is XXXIX
2646 is MMDCXLVI

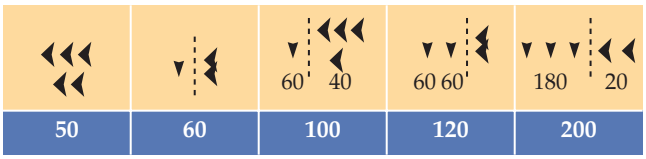
dangerzone

Look at the following:
4 is IV **40 is XL** **9 is IX** **90 is XC**
A letter *before* a higher value letter means we *take it away* from the higher value.
But 49 is not IL. Instead we have to write XLIX for 49, XL for 40 and IX for 9.
One of the rules in the Roman system is that you can only reduce the higher value by the next smaller value. Also, you are not allowed to reduce by 5, 50 or 500.

The Babylonian system

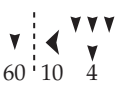
About 2100 BC, the ancient Babylonians developed a system which was unusual because it was not based on tens. See if you can work out what the Babylonian system is based on. All their numbers are wedge shaped because they wrote by pressing wedge-shaped sticks into damp clay.





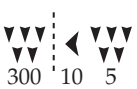
Examples:

32 is 

74 is 

130 is 

234 is 

315 is 

The modern Chinese number system

In China the Hindu–Arabic system has been adopted for many uses, but many Chinese people still use their character system. The Chinese system is also based on tens. The Chinese write from top to bottom and write two symbols for each number greater than 9. These two symbols show how many lots of ten, one hundred etc. are needed. The number 24 is written as two, ten and four. 132 is written as one, hundred, three, ten, two, but 14 is just written as ten, four, not one, ten, four. There is a Chinese character for zero but because a circle is often used that is what we will use here.

A simplified version of the characters is:

0	一	二	三	四	五	六	七	八	九
0	1	2	3	4	5	6	7	8	9

十	百	千	万
10	100	1000	10 000

Examples:

19 is 十九

270 is 二百七十

8005 is 八千五

exercise 1.1 Number systems

Preparation: Prep Zone Q2

Core

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1 Write out these numbers using the Egyptian number system.

Hint

(a) 12	**(b)** 14	**(c)** 600	**(d)** 500
(e) 24	**(f)** 31	**(g)** 172	**(h)** 253
(i) 41 020	**(j)** 21 301	**(k)** 1 210 411	**(l)** 1 360 002

eTester

2 Write out these numbers using the Roman number system.

Hint

(a) 13	**(b)** 12	**(c)** 20	**(d)** 30
(e) 19	**(f)** 29	**(g)** 2341	**(h)** 3132
(i) 629	**(j)** 439	**(k)** 3646	**(l)** 2466
(m) 1980	**(n)** 1959	**(o)** 1999	**(p)** 1694

3 Write out these numbers using the Babylonian number system.

Hint

(a) 20	**(b)** 40	**(c)** 45	**(d)** 56
(e) 63	**(f)** 62	**(g)** 75	**(h)** 84
(i) 102	**(j)** 91	**(k)** 144	**(l)** 162

4 Write out these numbers using the modern Chinese number system.

(a) 4 **(b)** 37 **(c)** 126 **(d)** 270

(e) 18 **(f)** 823 **(g)** 1053 **(h)** 6400

Hint

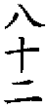
5 Write down what number system each number below is written in and what the number is in the Hindu–Arabic system.

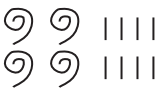
Worksheet C1.1

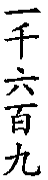
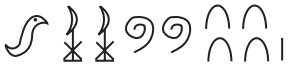
(a) XI **(b)**  **(c)** CXLIII **(d)** 

Hint

(e)  **(f)**  **(g)**  **(h)** CCXCII

(i)  **(j)**  **(k)**  **(l)** 

(m)  **(n)**  **(o)** DCIV **(p)** 

(q) MMDCLXIV **(r)**  **(s)** MMMCDXXVIII **(t)** 

6 Write each of these Hindu–Arabic numbers in:

(i) Egyptian numbers **(ii)** Roman numbers

(iii) Babylonian numbers **(iv)** Chinese numbers

(a) 15 **(b)** 65 **(c)** 92

(d) 300 **(e)** 199 **(f)** 236

Extension

7 (a) Which one of the three ancient number systems is still used?

(b) Where have you seen this number system used?

(c) What do the Babylonian, Chinese and Hindu–Arabic systems have that the other two don't have?

(d) Which of the five number systems usually takes the longest time to write down?

(e) Which usually takes the shortest time to write down?

(f) In which of the five number systems is the *position* of the numbers *not* important?

(g) Why do you think most number systems were based on fives or tens?

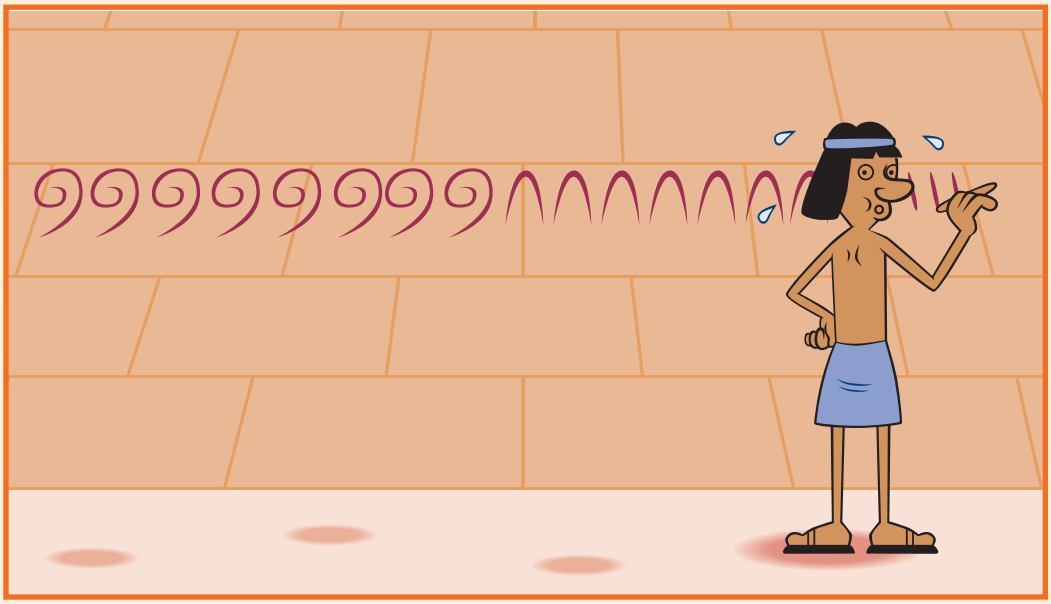
(h) Name something that you deal with every day which is based on sixties.

(i) Why do you think the Hindu–Arabic system is now the one used around the world?

8 Choose a number between 1500 and 2000 and write it using each of the modern Chinese, Egyptian, Babylonian and Roman number systems.

Worksheet A1.1

laughzone



Copy these Roman numerals and write them in Hindu–Arabic form, then arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

XIX	**E**	CM	**O**	CLI	**Y**
DII	**U**	CCXI	**A**	MDLX	**S**
CDXXI	**N**	MCDIX	**H**	CCCLXIV	**T**
DCXXVIII	**G**	MDCCXLII	**W**	MCMXXIV	**B**
CDXXVIII	**R**	MMDCXLV	**I**	MLXXIX	**D**

‘							’												
	364	1409	19	428	19		1560		628	900	364		364	900		1924	19		
	211		1924	19	364	364	19	428		1742	211	151		364	900				
	1742	428	2645	364	19		19	2645	628	1409	364								
	1409	502	421	1079	428	19	1079		211	421	1079								
	19	2645	628	1409	364	151	-		19	2645	628	1409	364	’					

1.2 Whole number problems

In primary school you would no doubt have spent many hours practising the four basic operations of mathematics—addition, subtraction, multiplication and division. In secondary school you are expected to be able to apply these basic skills to situations where the question is hidden in a little story. In questions like this you need to read through the whole question first and then decide which operation you will need to use to find the answer.

worked example 1

On 1 January the population of Summertown was 55 234. During the year, 1987 people died, 1245 babies were born, 4324 people left the town, and 3876 moved in.

(a) Find the total number of people who died or left the town.
(b) Find the total number of people who were born or moved into the town.
(c) Find the total number of people in the town at the end of the year.
(d) Clearly express the total change in population.

Steps	Solutions
(a) Add together the people who died and the people who left.	**(a)** 1987 + 4324 = 6311
(b) Add together the people who were born and the people who moved into the town.	**(b)** 1245 + 3876 = 5121
(c) 1. Subtract from the original population the total who left.	**(c)** 55 234 − 6 311 = 48 923
2. Add to this result the total of babies born and people who moved in.	48 923 + 5 121 = 54 044
(d) 1. The final population is lower than the original population so we will need to subtract the new from the old.	**(d)** 55 234 − 54 044 = 1 190
2. Write a short statement to answer the question.	The final population is 1190 less than the original.

exercise 1.2 Whole number problems

Preparation: Prep Zone Q1 and 3–6

Core

1 The highest mountain in the world, measured from sea level, is the Himalayan peak of Mount Everest. It is 8848 m above sea level. If we measure mountains which start under the ocean, the highest mountain in the world from base to tip is Mauna Kea on the island of Hawaii. Its total height is 10 203 m, of which 4205 m is above sea level.

(a) How much higher is Mauna Kea than Mount Everest?
(b) How much of Mauna Kea is below sea level?
(c) If we don't count the part of Mauna Kea which is under water, how much higher is Mount Everest?

Animation

2 The two longest rivers in the world are the Amazon (6448 km) and the Nile (6670 km). The longest river in Australia is the Darling (2739 km).

(a) How much longer is the Nile than the Amazon?
(b) How much longer is the Nile than the Darling?
(c) How much longer is the Amazon than the Darling?

Hint

3 The following table shows a number of inventors and when they made their famous inventions.

Invention	*Inventor*	*Year*
Calculator	Blaise Pascal (1623–1662)	1642
Thermometer	Gabriel Fahr enheit (1686–1736)	1714
Parachute	Louis Lenormand (1757–1839)	1783
Camera	Nicephor e Niepce (1765–1833)	1822
Telegraph	Samuel Morse (1791–1872)	1837
Telephone	Alexander Graham Bell (1847–1922)	1876
Car engine	Gottlieb Daimler (1834–1900) and Karl Benz (1844–1929)	1887
Radio	Guglielmo Mar coni (1874–1937)	1895
Television	John Logie Bair d (1888–1946)	1925
Laser	Theodor e Maiman (1927–)	1960

(a) How long after the telegraph was the telephone invented?
(b) How long after radio was television invented?
(c) How old was Gabriel Fahrenheit when he invented the thermometer?
(d) How long before the laser was the camera invented?
(e) Gottlieb Daimler and Karl Benz invented the car engine independently. How old was each man when he invented it?
(f) What age did Blaise Pascal live to?
(g) Who lived longer, Nicephore Niepce or Louis Lenormand, and by how much?

(h) Who out of all the inventors came up with their invention at the youngest age?

(i) Draw a timeline from 1600 to 2000 to represent the information in the table.

4 If a car uses 8 litres of petrol for every 100 km that it travels, how many litres would it use for a trip of:

(a) 900 km **(b)** 1200 km **(c)** 1500 km

(d) 1050 km **(e)** 725 km?

Animation

5 Harvey'Scoop'Roberts, a journalist with the *Monthly Farm News*, can type 50 words a minute. How long does it take him to type an article of 1800 words?

Hint

6 Kathy and her brother Herbert both have paper rounds. They have to start and finish at the shop. The routes of both their rounds and the lengths of each street section they cover are shown on the following diagram.

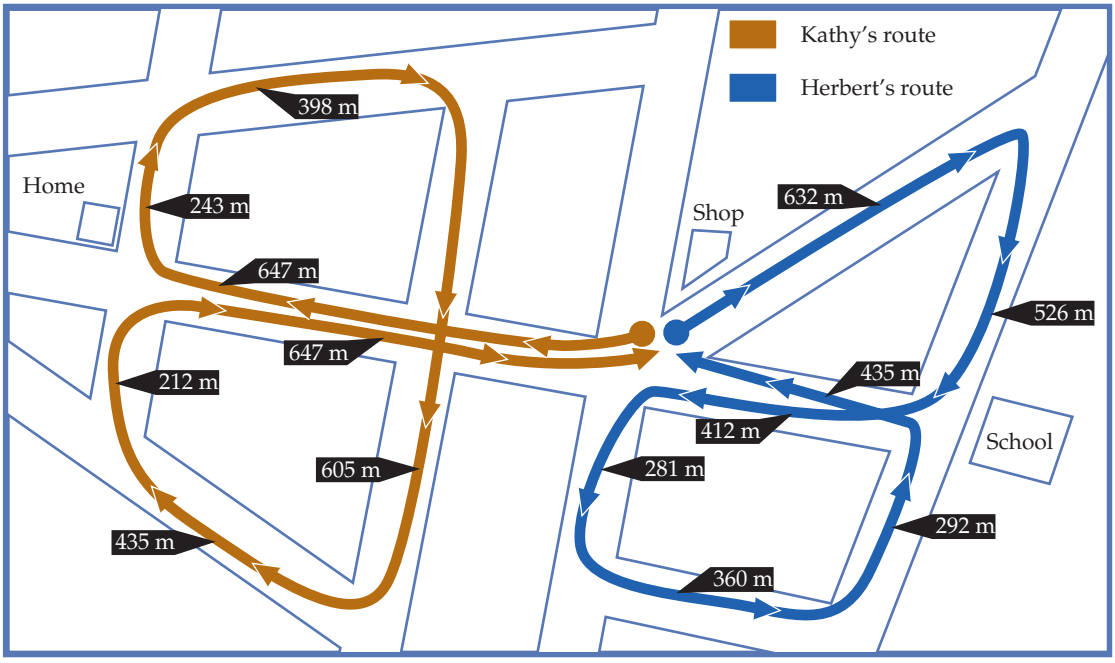
(a) How long is Kathy's paper round route?

(b) Whose paper round route is longer, Kathy's or Herbert's, and by how much?

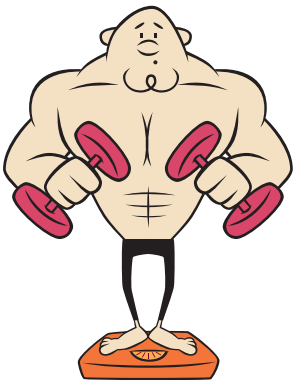
(c) How long would Kathy's paper round route be if Herbert was sick and she had to do his as well?

(d) If Kathy could go straight home after her paper round and not have to go back to the shop, approximately how long would her paper round be?

(e) If Kathy went straight to school after her paper round, approximately how far would she travel?



7 Arnie the body builder stands on a set of scales while he is holding two 3 kg dumb-bells. The scales show a weight of 102 kg. How much does Arnie weigh?



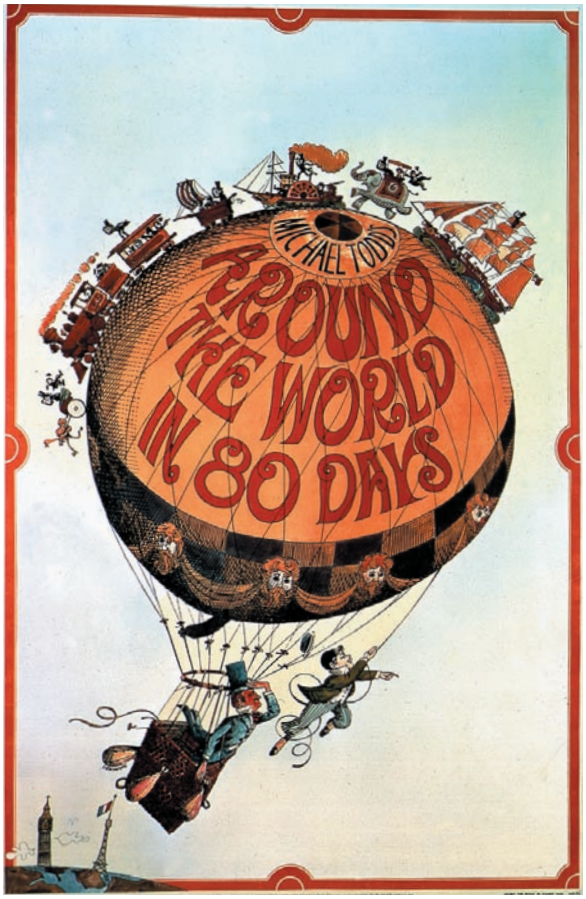
Extension

8 Stavros wants to buy some marmalade. One jar in the supermarket is 250 g and costs $1.25; another is 500 g and costs $2.25. Which one is better value?

Hint

9 Little Lucy is between 3 and 4 weeks old. Give three possible values for her age in minutes.

10 Jules Verne wrote about travelling around the world in 80 days. About how many weeks is that?



11 The length of a painting including the frame is 85 cm.
If the frame is 6 cm wide all the way around, what is the length of the unframed painting?

Hint

12 The Pizza Pit-Stop employs five people. The two cooks work 36 hours each per week for $12 an hour, and the three waiters work 30 hours each per week for $11 an hour. What does the Pizza Pit-Stop pay its five employees in total per week?

13 Wendy is training to be an Olympic swimmer. Every morning she swims 3600 m in a 50 m pool. How many laps is that?

14 For the numbers 108 and 9, find the:
(a) sum **(b)** difference **(c)** product **(d)** quotient

Hint

15 What is the sum of 42 and 76 added onto the product of 42 and 76?

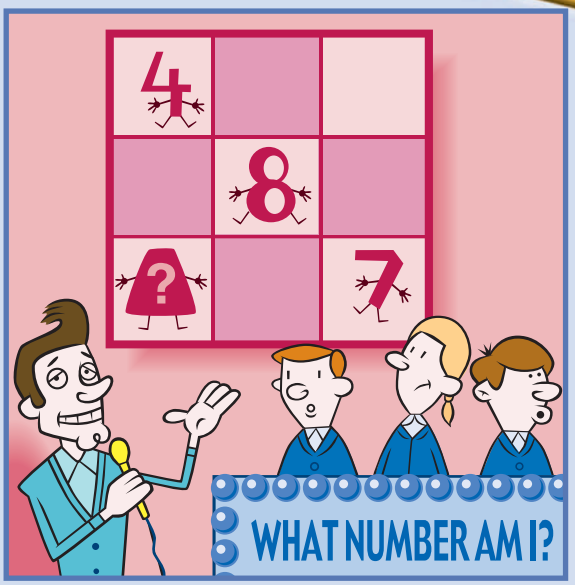
16 What is the result when the difference between 9864 and 8 is added onto the quotient of 9864 and 8?

Working mathematically

problem solving

What number am I?

I am a two-digit number greater than 50.
The product of my digits is not 12, but 12 goes into it exactly.
The sum of my digits is odd.
The sum of my digits is less than 13.
What am I?
What am I if I am less than 50?



Break it into smaller steps. First, find what the product of the digits could be, given that 12 goes into it.

1.3 Magic squares

Magic squares have fascinated people for over 4000 years. A magic square is a square of numbers in which every row, column and diagonal adds up to the same total. This total is called the **magic sum**. They were called 'magic' because people used to believe the squares had mystical powers.

One of the simplest magic squares is a 3×3 square using the digits 1 to 9, where all the totals are 15. The one shown here is called the Lo-Shu magic square and according to legend was brought by a turtle from the river Lo to the Chinese Emperor Lu around 2200 BC.

4	9	2
3	5	7
8	1	6

exercise 1.3 Magic squares

Preparation: Prep Zone Q3 and 4

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Worksheet C1.2
Hint

Core

1 Copy these magic squares into your book and find the missing numbers.

(a)

7		
	8	
11		9

(b)

14	19	12
18		

(c)

3		
2	4	6

(d)

		4
	10	
16		12

(e)

	1	6
	5	
	9	

(f)

2		
6	1	8

2 (a) Compare your answer to Question **1(a)** with the Lo-Shu magic square on page 14. Something was added to each of the numbers in the Lo-Shu square to get your answer. What was it?

(b) What can be done to the Lo-Shu magic square to get your answer to Question **1(b)**?

(c) What can be done to the Lo-Shu magic square to get Question **1(c)**?

(d) What can be done to the Lo-Shu magic square to get Question **1(d)**?

(e) What can be done to the Lo-Shu magic square to get Question **1(e)**?

(f) What can be done to the Lo-Shu magic square to get Question **1(f)**?

(g) If you add 8 to each of the numbers in the Lo-Shu square, what will be the magic sum of the new square?

(h) If you add 100 to each of the numbers in the Lo-Shu square, what will be the magic sum of the new square?

Extension

3 Magic squares were first investigated in Europe during the fifteenth century. The German artist Albrecht Dürer (1471–1528) made a woodcut called 'Melancholy' which includes a 4×4 magic square (shown on the next page).

(a) The year in which Dürer made the woodcut appears in the magic square. What is it? How old was he when he made 'Melancholy'?

(b) What is the magic sum of the Melancholy square?

(c) What do the four numbers that make up the 2×2 square in the top left-hand corner of the Melancholy square add up to?

(d) How many other 2×2 squares can you find inside the Melancholy square that add up to the magic sum?

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

(e) Make a 4 × 4 magic square of your own by adding 2 to each of the numbers in the Melancholy square. What is the magic sum?

(f) If you made a new magic square by adding 5 to each of the numbers in the Melancholy square, what would the magic sum be?

Hint

(g) Make a new magic square by swapping the first column of the Melancholy square with the fourth column. What is the magic sum of the new square?

(h) Work out the following 4 × 4 magic square and then see if you can work out what was done to the Melancholy square to get it.

	9		16
15		10	3
14			
	12		13

magic sum = 34

(i) Complete the following 4 × 4 magic square and then see how it was formed from your answer to part **(h)**.

8		9	
	10		
	11	15	6
5		12	

magic sum = 50

4 The great American thinker and inventor Benjamin Franklin created a number of magic squares. One of the most famous was an 8×8 square.

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

(a) What is the magic sum of Franklin's square?

(b) Look at the half-rows, that is, the first four numbers in each row and the last four numbers in each row. What do you notice?

(c) What do you notice about the half-columns?

(d) Look at the 4×4 squares in each of the four corners. Are they magic squares?

(e) Look at any 2×2 square inside Franklin's square and add up the four numbers. Do the same for another 2×2 square. What do you notice?

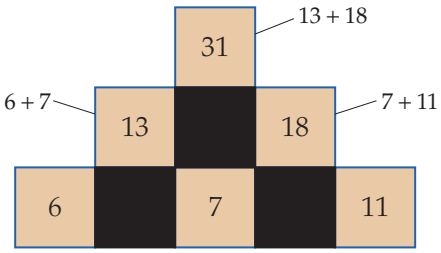
5 Construct two 3×3 magic squares using any from Question **1** as a base.

Homework 1.1
Worksheet A1.2
Worksheet E1.1

1.4 Number pyramids and cross-number totals

Number pyramids

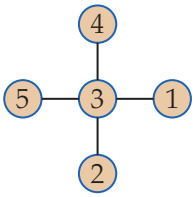
To get each number in the pyramid, add the two numbers below it.



Cross-number totals

This cross-number puzzle has been filled out using the numbers 1 to 5 exactly once so that the total along each line is nine.

When you are trying to solve a cross-number total, use pencil to write in the numbers so you can rub them out if necessary.



exercise 1.4 Number pyramids and cross-number totals

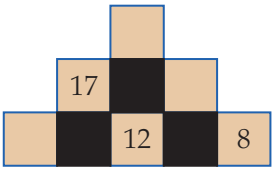
Preparation: Prep Zone Q3 and 4

Core

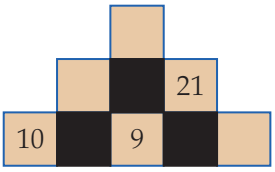
1 Copy the following number pyramids into your book and fill in the missing numbers.

Hint

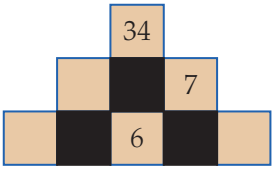
(a)



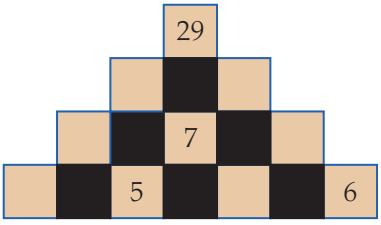
(b)



(c)



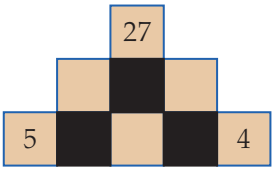
(d)



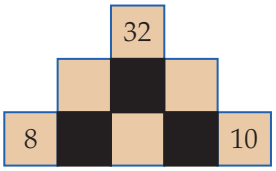
2 Copy the following number pyramids into your book and fill in the missing numbers.

Hint

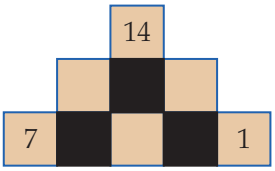
(a)



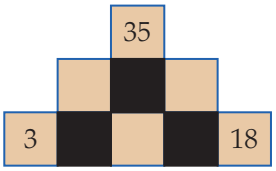
(b)



(c)



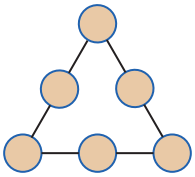
(d)



Can you find a pattern which helps you solve these?

3 Make up three number pyramids of your own and give them to a friend to work out.

4 Copy this cross-number total into your book three times.



Hint

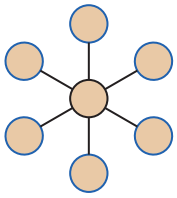
(a) Use the numbers 1 to 6 exactly once so that each line adds up to 9.

(b) Use the numbers 1 to 6 exactly once so that each line adds up to 10.

(c) Use the numbers 1 to 6 exactly once so that each line adds up to 11.

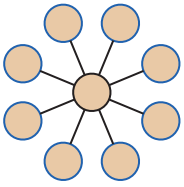
(d) Once you have found an answer in each case, how is it possible to find other answers fairly easily?

5 Copy these cross-number totals into your book and try to solve them.

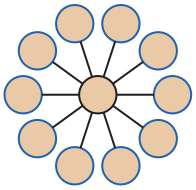


Hint

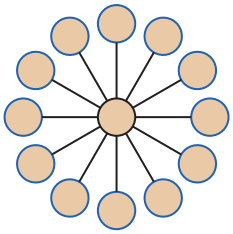
(a) Use the numbers 1 to 7 exactly once so that each line adds up to 12.



(b) Use the numbers 1 to 9 exactly once so that each line adds up to 15.



(c) Use the numbers 1 to 11 exactly once so that each line adds up to 18.



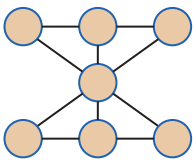
(d) Use the numbers 1 to 13 exactly once so that each line adds up to the same thing. To find what the cross-number puzzle has to add up to, look at the previous questions and see if you can find a pattern.

Extension

6 (a) Look at your answers to Question **5**. Can you find a pattern that will give you a quick way of finding which number goes in the middle? Which number would go in the middle of a cross-number puzzle of the numbers from 1 to 15? What about for 1 to 23?

(b) Can you find any other pattern to make it easier to solve this sort of cross-number total? What is it?

7 Copy this cross-number total into your book and use the numbers from 1 to 7 exactly once so that each line has the same total.



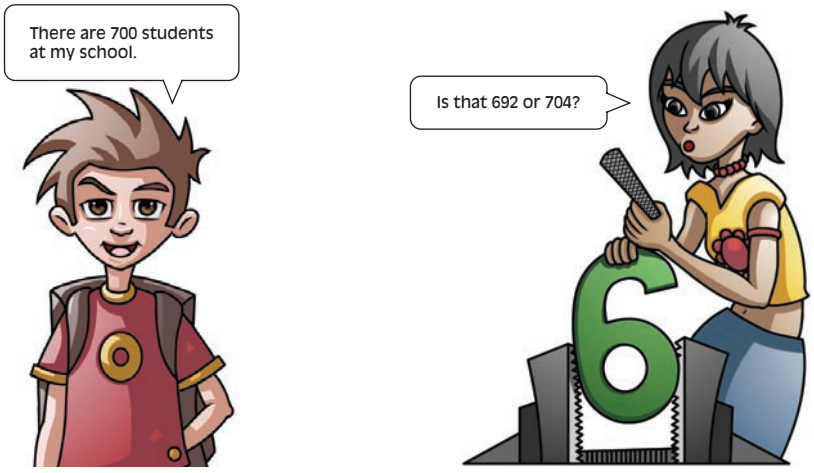
Worksheet E1.2

1.5 Estimating and rounding

Estimating

People make **estimates** of things all the time:

Rounding to the first digit



Are there *exactly* 700 students at Al's school? The figure is probably different from the actual number of students in the school due to **rounding**. The exact number of students in the school may be 692 or 704. In fact it could have been any number from 650 to 749.

Rounding to the first digit

When rounding, follow the rule 'if you're caught right in the middle, go up.' For example, 750 rounds to 800 (not 700) and 3500 rounds to 4000 (not 3000).

worked example 2

Round the following numbers to the first digit.

(a) 361 **(b)** 2050 **(c)** 8

Steps	Solutions
(a) Look at the second digit. Because it is 5 or greater, raise the first digit by one and replace the following digits with zeros.	**(a)** $361 \approx 400$
(b) Look at the second digit. Because it is less than 5, keep the first digit and replace the following digits with zeros.	**(b)** $2050 \approx 2000$
(c) Because there is only one digit this number is already rounded to the first digit.	**(c)** 8

The symbol $\approx$ means 'approximately equal to'.

Estimating multiplication

Often you don't need to know the exact answer to a multiplication problem. An estimate that you can do in your head or with very little work will often do.

worked example 3

Estimate 368×52.

Steps	Solution
1. Round off both numbers to the first digit.	400×50
2. Multiply the first digits.	$4 \times 5 = 20$
3. Count the number of zeros altogether in step 1.	three 0s (two in 400 and one in 50)
4. Write down the number from step 2 and put the number of zeros from step 3 after it.	20 000
5. So 368×52 is approximately 20 000. This is written as	$368 \times 52 \approx 20\,000$

Estimating division

Division problems can also be estimated using rounding.

worked example 4

Estimate 67 483 ÷ 421.

Steps	Solution
1. Round off both numbers to the first digit.	70 000 ÷ 400
2. Cancel off the same number of zeros on both sides.	70 0$\not{0}\not{0}$ ÷ 4$\not{0}\not{0}$ = 700 ÷ 4
3. Do the simple division.	$4\overline{)7^{3}0^{2}0}$ with quotient 1 7 5
4. So 67 483 ÷ 421 is approximately 175. This is written as	67 483 ÷ 421 ≈ 175

exercise 1.5 Estimating and rounding

Preparation: Prep Zone Q1, 2, 5 and 6

Core

1 Round these numbers to the first digit.

(a) 68	**(b)** 74	**(c)** 483	**(d)** 4846
(e) 3723	**(f)** 619	**(g)** 75 000	**(h)** 800 050
(i) 970	**(j)** 6	**(k)** 716 599	**(l)** 1 801 021
(m) 10	**(n)** 9643	**(o)** 9510	**(p)** 650

Hint

Worksheet C1.3

eTester

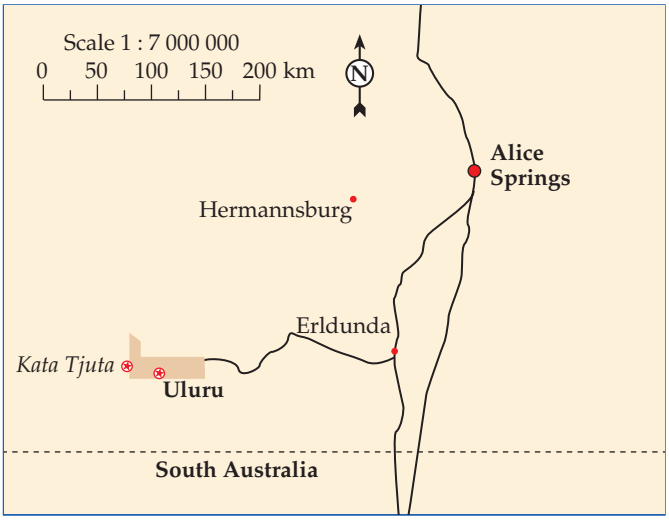
2 Which of the five estimates in each case do you think is the closest? Don't try to count them.

(a) Roughly how many people are there in the picture?

A 2000 **B** 30 000 **C** 7000 **D** 100 000

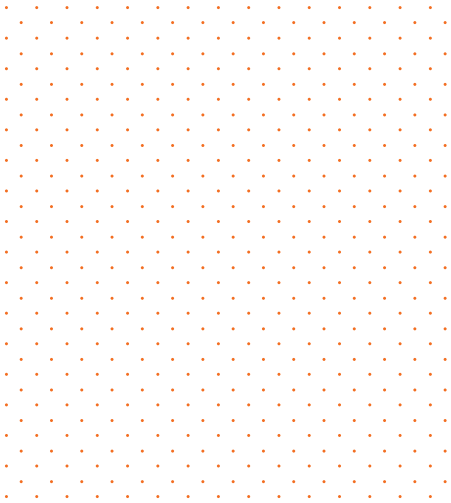
(b) Approximately how far is it directly from Uluru to Alice Springs?

A 63 km
B 789 km
C 350 km
D 40 km



(c) About how many dots are there in this square? Try to use a pattern.

A 7000
B 1000
C 1 000 000
D 500

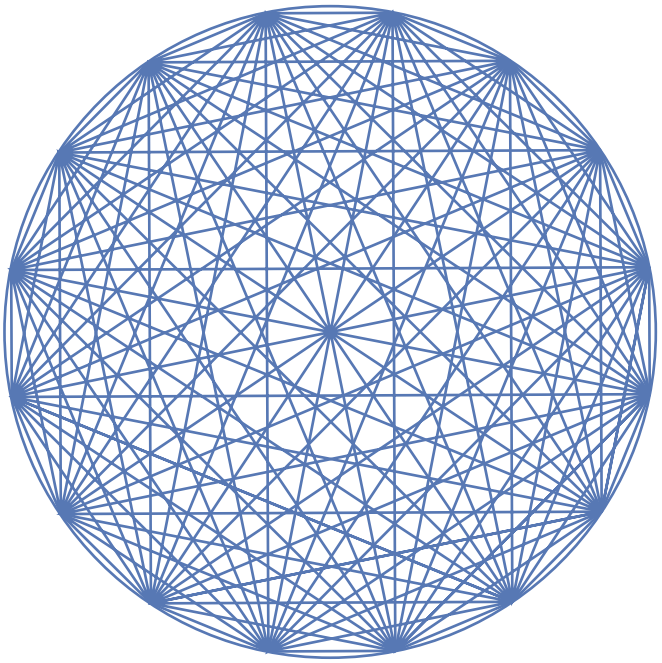


(d) Roughly how many stars are there in this picture?

A 50
B 300
C 3000
D 10 000

(e) **(i)** How many straight lines are there in this picture?

A 1000 **B** 200 **C** 7000 **D** 10 000



(ii) Try to work out a way of finding the exact number of lines without counting them all.

Animation

Worksheet C1.4

Hint

3 Use rounding to the first digit to find the estimates for these multiplications.

(a) 681×41	**(b)** 547×84	**(c)** 141×837
(d) 104×8946	**(e)** 650×23	**(f)** 62×819
(g) $38\,944 \times 771$	**(h)** 7340×250	**(i)** 950×3489
(j) 680×95	**(k)** 9×6511	**(l)** 8010×6
(m) $65\,000 \times 70$	**(n)** $56\,439 \times 9$	**(o)** $95 \times 75\,000$
(p) 950×9500	**(q)** 250×950	**(r)** $11 \times 62\,871$

4 In each case choose the best estimate from the alternatives given. Don't do the actual multiplication.

(a) 321×73

A 210 **B** 2100 **C** 2163 **D** 21 000

(b) 56×354

A 2400 **B** 1500 **C** 15 000 **D** 24 000

(c) 4570×429

A 2 000 000 **B** 1 600 000 **C** 160 000 **D** 200 000

(d) 6500×78

A 480 000 **B** 420 000 **C** 350 000 **D** 560 000

(e) 405×950

A 400 000 **B** 450 000 **C** 360 000 **D** 500 000

5 **(a)** Work out estimates for:

(i) 79×5003 **(ii)** 76×5488

(b) What do you notice about your results for part **(a)**?

(c) **(i)** Work out the exact values for the two multiplications in part **(a)**.

(ii) How far off are the estimates in each case?

(iii) Which one was the closer estimate? Why do you think this is?

6 **(a)** Work out estimates for:

(i) 673×417 **(ii)** 653×357

(b) **(i)** Work out the exact values for the two multiplications in part **(a)**.

(ii) How far off are the estimates in each case?

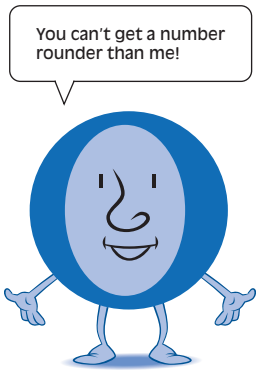
(iii) Which one was the closer estimate? Why do you think this is?

Hint

7 Use rounding to the first digit to find estimates for these quotients.

(a) $2940 \div 41$	**(b)** $3199 \div 62$	**(c)** $8742 \div 31$
(d) $1955 \div 78$	**(e)** $29\,110 \div 59$	**(f)** $52\,511 \div 37$
(g) $5218 \div 8$	**(h)** $7532 \div 4$	**(i)** $44\,895 \div 15$
(j) $75\,342 \div 80$	**(k)** $94\,101 \div 60$	**(l)** $10\,803 \div 95$
(m) $95\,000 \div 542$	**(n)** $36\,534 \div 35$	**(o)** $3\,082\,817 \div 19$
(p) $8\,300\,764 \div 950$	**(q)** $45\,278\,121 \div 9782$	**(r)** $62\,321\,084 \div 2341$

Hint



8 In each case choose the best estimate from the alternatives given. Don't do the actual division.

(a) $7865 \div 24$

A 900 **B** 1600 **C** 210 000 **D** 400

(b) $7546 \div 84$

A 100 **B** 1000 **C** 640 000 **D** 10

(c) $5500 \div 29$

A 350 **B** 200 **C** 700 **D** 50

(d) $99\,160 \div 527$

A 740 **B** 2000 **C** 330 **D** 220

(e) $126\,905 \div 9500$

A 100 **B** 10 **C** 10 000 **D** 300

Extension

9 **(a)** Estimate:

(i) $4529 \div 11$ **(ii)** $5391 \div 12$

(b) Does it make more sense to leave 11 and 12 as they are than to round them to 10? Why or why not?

Worksheet C1.5

10 Write down three pairs of numbers that when rounded to the first digit then multiplied together give:

(a) 3200 **(b)** 200 **(c)** 100 000 **(d)** 42 000

11 Copy the following table and complete it, using rounding to the first digit. Use your calculator to work out the exact answer after you've found the rounded answer.

Calculator

	Question	Rounded question	Rounded answer	Exact answer
(a)	58 + 789 – 301	60 + 800 – 300	560	546
(b)	923 + 67 – 466			
(c)	344 – 209 – 42 + 163			
(d)	67 × 342 × 77			
(e)	92 × 650 + 23 471			
(f)	749 × 96 – 35 987			

Hint

12 Without using your calculator, decide which of the alternative answers given is the best estimate. Check your answer by using your calculator.

(a) 853 + 67 041
A 6700 **B** 14 000 **C** 68 000 **D** 750 000

(b) 5634 + 9363
A 1400 **B** 567 **C** 140 000 **D** 15 000

(c) 45 + 884 + 10 057
A 1100 **B** 11 000 **C** 13 000 **D** 110 000

(d) 97 445 – 374
A 60 000 **B** 4000 **C** 97 000 **D** 63 000

(e) 349 × 43
A 1200 **B** 15 000 **C** 120 000 **D** 1 500 000

(f) 81 × 86
A 7000 **B** 70 **C** 700 000 **D** 700

(g) 170 × 1471
A 2500 **B** 25 000 **C** 2 500 000 **D** 250 000

(h) 43 736 ÷ 56
A 8000 **B** 780 **C** 67 **D** 80 000

eQuestions
Homework 1.2
Worksheet A1.3

1.6 Order of operations

Max, Minh, Al and Polly all worked out the following problem.

$$3 + 6 \div 3 + (8 - 3) - 1 \times 2$$

- Max's answer was 14.
- Minh's answer was 6.
- Al's answer was 8.
- Polly's answer was 18.

Why were there so many different answers?

Obviously we can't have different people getting different answers in mathematics. People have to agree on the rules about working things out. That's why mathematicians came up with some rules about the order in which to do the four operations.

These are called the **order of operation** rules.

The order of operation rules

1. Always do the parts in brackets first.
2. Do the multiplication and division next, in order from left to right.
3. Do the addition and subtraction next, in order from left to right.

Who had the correct answer to the problem at the start of this section: Max, Minh, Al or Polly?

worked example 5

Simplify $24 + 6 \div 2 - 1 \times 4$.

Steps	Solution
1. Do multiplication and division in the order in which they appear.	$24 + 6 \div 2 - 1 \times 4$ $= 24 + 3 - 1 \times 4$ $= 24 + 3 - 4$
2. Do addition and subtraction in the order in which they appear.	$= 27 - 4$ $= 23$

worked example 6

Simplify $12 - 9 + 8 \div (2 + 2) \times 3$.

Steps	Solution
1. Do the brackets first.	$12 - 9 + 8 \div (2 + 2) \times 3$ $= 12 - 9 + 8 \div 4 \times 3$
2. Do multiplication and division in the order in which they appear.	$= 12 - 9 + 2 \times 3$ $= 12 - 9 + 6$
3. Do addition and subtraction in the order in which they appear.	$= 3 + 6$ $= 9$

exercise 1.6 Order of operations

Preparation: Prep Zone Q1

Interactive

Hint

eTester

Core

1 Simplify.

(a) $6 \times 2 - 1$ **(b)** $8 \div 4 - 2$

(c) $7 + 6 \div 2$ **(d)** $1 + 8 \times 3$

(e) $15 - 8 \div 4$ **(f)** $8 - 5 \div 5$

(g) $8 + 3 \times 10$
(h) $25 - 2 \times 11$
(i) $6 \div 3 + 3 \times 5$
(j) $8 \times 5 - 4 \times 10$
(k) $9 - 6 \div 2 + 7$
(l) $8 - 24 \div 12 + 3$
(m) $8 \times 3 \div 4 \times 2$
(n) $4 \times 9 \div 6 \times 2$
(o) $20 + 12 - 17 + 3$
(p) $28 + 10 - 1 + 1$
(q) $9 \times (10 - 7) \div 3$
(r) $24 \div (7 + 5) \times 6$
(s) $88 \div 8 - 6 \times (5 - 4)$
(t) $12 \times 5 + 4 \times (10 - 4)$
(u) $18 - 7 \times 2 + 13 - 4 \div 2$
(v) $9 - 2 + 5 + 3 \times 4 \div 6$
(w) $28 - 7 \times 3 + (5 - 1) \div 2 + 3$
(x) $23 - 5 + (17 - 2) \times 3 + 5$

2 State TRUE or FALSE for the following.

(a) For $2 + 6 \times 4$ we would do $2 + 6$ first.
(b) For $9 - 4 \times 2$ we would do $9 - 4$ first.
(c) For $6 + 12 \div 3$ we would do $12 \div 3$ first.
(d) For $60 \div 5 \times 3$ we would do $60 \div 5$ first.
(e) For $8 + 40 \div (3 + 5) \times 10$ we would do $8 + 40$ first.
(f) For $24 + 6 \div 2 - 1 \times 4$ we would do $6 \div 2$ first.
(g) $4 + 12 \div 2$ simplifies to 8.
(h) $20 \div 5 - 1$ simplifies to 3.

3 (a) What would you do first and what would you do second in each of these questions?

(i) $(4 - 2) \times 6 \div (2 \times 2)$
(ii) $(24 + 15 \div 5) - 6 \times 3$
(iii) $[(7 + 9) \times 2] \div 4$
(iv) $\{[(21 - 17) \div 2] + 10\} - 1$

(b) Simplify each of the statements in part **(a)**.

4 Put brackets into these statements, where necessary, to make them true.

Hint

(a) $6 + 6 \times 3 = 36$
(b) $10 - 4 \times 5 = 30$
(c) $5 + 2 \times 3 + 7 = 25$
(d) $12 + 6 \div 7 - 4 = 14$
(e) $9 - 8 \times 6 + 4 = 10$
(f) $3 + 4 \times 5 - 10 = 25$
(g) $7 + 10 - 5 \div 2 = 6$
(h) $3 \times 4 - 2 \div 6 = 1$
(i) $6 \div 3 + 3 \times 5 = 5$
(j) $3 \times 6 \div 8 - 4 + 5 = 2$
(k) $12 + 4 \div 8 \times 3 - 6 = 0$
(l) $8 \div 2 + 2 \times 7 - 10 = 4$
(m) $3 \times 10 - 7 \div 9 + 12 = 13$
(n) $18 \div 3 \times 5 - 3 + 2 = 14$
(o) $7 + 3 \div 4 + 1 = 2$
(p) $5 - 3 \times 8 - 6 \div 2 = 2$

5 Replace each $*$ with one of the four operators $(+, -, \times, \div)$ to make the equation true.

Hint

(a) $2 + 21 * 3 = 9$
(b) $15 - 6 * 2 = 12$
(c) $5 * 3 - 8 = 7$
(d) $9 * 6 + 10 = 13$
(e) $14 - 8 * 6 = 0$
(f) $5 + 15 * 3 = 10$
(g) $7 * 5 * 6 = 29$
(h) $14 * 3 * 2 = 15$
(i) $(5 * 9) * 7 = 2$
(j) $(24 * 6) * 10 = 3$
(k) $8 * 5 * 2 - 6 = 12$
(l) $12 * 2 + 1 * 9 = 15$

6 Replace each ∗ with either $<$, $>$ or $=$ to make the statement true.

(a) $6 \times (4 \div 2) \times 3 \ * \ (6 \times 4) \div 2 \times 3$

(b) $(1 + 4) \times 20 \div 5 \ * \ 1 + (4 \times 20) \div 5$

(c) $8 + (5 - 3) \times 2 \ * \ 8 + 5 - (3 \times 2)$

(d) $100 + 10 \div 10 \ * \ (100 + 10) \div 10$

(e) $9 \times 2 + 0 \ * \ 9 \times (2 + 0)$

(f) $36 \div 6 \times (3 - 3) \ * \ 36 \div 6 \times 3 - 3$

Worksheet C1.6

Hint

Remember $<$ means 'is less than' and $>$ means 'is greater than'.

Extension

7 Copy and complete the following using each of the numbers 1, 3, 4, 7 exactly once for each question.

(a) $* \times * - * + * = 6$

(b) $(* - *) \div * + * = 2$

(c) $* \times (* - *) \div * = 9$

(d) $* + * \times * - * = 6$

(e) $(* + *) \times (* - *) = 20$

(f) $(* - *) \div (* - *) = 6$

(g) $(* + * - *) \times * = 42$

(h) $* \times (* + * - *) = 20$

(i) $* \times [(* + *) \div *] = 6$

(j) $[(* - *) \times *] + * = 8$

8 Make up five questions similar to those in Question **7**, and give them to a partner to work out. Then check to see if your partner is right.

Worksheet C1.7

Working mathematically

problem solving

The four 4s puzzle

Using the number 4 exactly four times, together with any of the four operators (+, –, ×, ÷) and brackets if you need them, see if you can make them equal the numbers 0 to 9. Copy down the following. The way to get 5 has been done for you. There is more than one way in many cases.

0 =

1 =

2 =

3 =

4 =

5 = $(4 \times 4 + 4) \div 4$

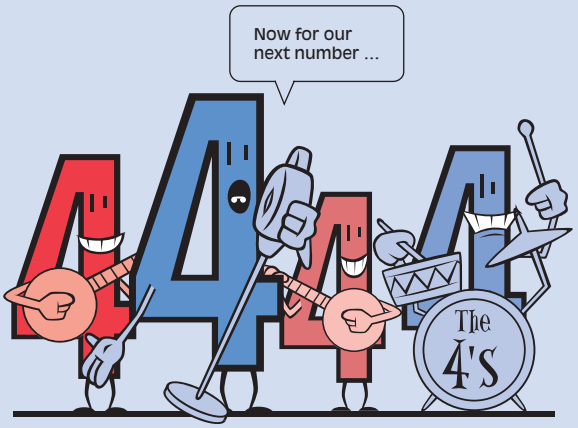
6 =

7 =

8 =

9 =

In a small group, see how many ways up to 100 you can find with just four 4s.



It's useful to remember some basics. For example, 4 ÷ 4 = 1 and 4 – 4 = 0.

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Working mathematically

investigation

The multiplication target game

What number do you have to multiply the arrow by to get a number within the target range?
Example: Arrow: 53; Target: 260 ⟷ 302

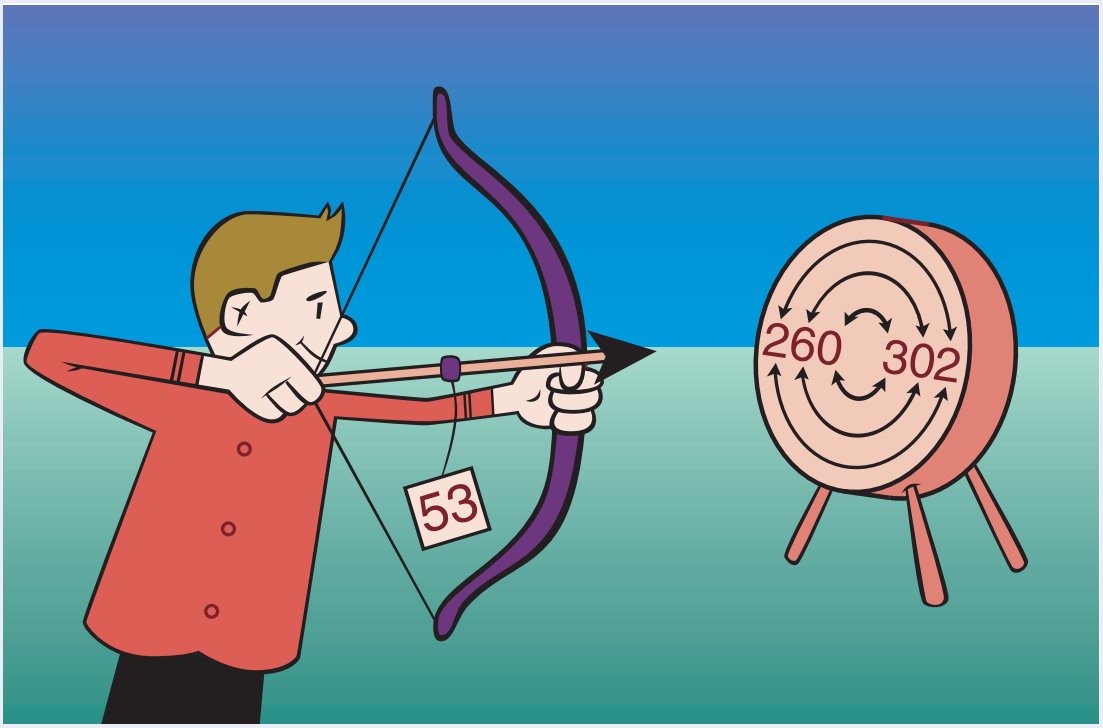
Try $53 \times 6 = 318$: outside the range.

Try $53 \times 5 = 265$: on target.

This has taken you two guesses to get on target. Use the following points system to keep score.

Number of guesses	One	Two	Three	Four	Five or more
Points	5	4	3	2	1

For the example above you would get four points.



Do these target questions and keep track of your total score. What is the best possible score?

(a) Arrow: 38; Target: 978 ⟷ 1000

(b) Arrow: 16; Target: 541 ⟷ 550

(c) Arrow: 29; Target: 276 ⟷ 293

(d) Arrow: 51; Target: 820 ⟷ 870

(e) Arrow: 47; Target: 4452 ⟷ 4489

(f) Arrow: 72; Target: 5100 ⟷ 5170

(g) Arrow: 824; Target: 17 300 ⟷ 18 100

(h) Arrow: 731; Target: 38 400 ⟷ 39 000

Worksheet T1.1

1.7 Mental and non-calculator maths strategies

Maths is made easier when we know some strategies to make computations simpler. In this section we will cover some of these strategies but there are many more. You may already know some.

Mental and other non-calculator strategies usually involve dealing with easier numbers than those obvious in the question, for example multiplying or adding numbers to form multiples of ten.

worked example 7

Use an appropriate mental strategy to help simplify each of the following.

(a) $2 \times 13 \times 5$ **(b)** $32 + 13 + 7$

Steps

(a) 1. Look for ways to form easy numbers.

2. Perform these calculations first. Complete the question.

(b) 1. Look for ways to form easy numbers.

2. Perform these calculations first. Complete the question.

Solutions

(a) $2 \times 13 \times 5$
$= (2 \times 5) \times 13$
$= 10 \times 13$
$= 130$

(b) $32 + 13 + 7$
$= 32 + (13 + 7)$
$= 32 + 20$
$= 52$

worked example 8

Use an appropriate strategy to help simplify each of the following.

(a) 9×15 **(b)** $162 \div 18$

Steps

(a) 1. When multiplying by 9 it is often easier to multiply by 10 first.

2. This gives one lot of 15 too many, so then subtract 15 from the answer.

(b) 1. When dividing by large numbers it can help to break the number up into factors.

Solutions

(a) $10 \times 15 = 150$

$150 - 15 = 135$
So $9 \times 15 = 135$

(b) $18 = 2 \times 3 \times 3$

2. Divide by each of these factors, one after the other.
   $162 \div 2 = 81$
   $81 \div 3 = 27$
   $27 \div 3 = 9$
3. Write the answer.
   $162 \div 18 = 9$

eTutorial

exercise 1.7 Mental and non-calculator maths strategies

Preparation: Ex 1.6

Core

1 Use an appropriate strategy to help simplify each of the following.

Hint

(a) $4 \times 6 \times 5$ **(b)** $15 \times 5 \times 2$ **(c)** $6 \times 7 \times 5$
(d) $2 \times 42 \times 5$ **(e)** $14 \times 5 \times 4$ **(f)** $8 \times 5 \times 3$
(g) $22 + 37 + 8$ **(h)** $165 + 6 + 14$ **(i)** $11 + 19 + 153$
(j) $37 + 128 + 63$ **(k)** $77 + 78 + 23$ **(l)** $89 + 116 + 11$

2 Use an appropriate strategy to help simplify each of the following.

Hint

(a) 9×17 **(b)** 49×6 **(c)** 31×4 **(d)** 19×8
(e) 11×14 **(f)** 61×7 **(g)** 13×19 **(h)** 8×19
(i) 99×7 **(j)** 101×18 **(k)** 91×7 **(l)** 21×16

3 Use an appropriate strategy to simplify each of the following.

(a) $210 \div 15$ **(b)** $192 \div 24$ **(c)** $112 \div 4$ **(d)** $96 \div 16$
(e) $750 \div 25$ **(f)** $252 \div 36$ **(g)** $196 \div 28$ **(h)** $448 \div 32$

4 (a) **(i)** Double eight, then double your answer. What do you get?
(ii) Complete this statement: Doubling a number twice is the same as multiplying the number by _____.

(b) Simplify the following by doubling twice.
(i) 13×4 **(ii)** 27×4 **(iii)** 32×4 **(iv)** 54×4

Extension

5 Try to come up with a strategy to help find the following. Write down your answer and explain what you did to find it. *Note:* There may be a range of suitable strategies to choose from.

(a) 15×8 **(b)** $187 - 93$ **(c)** $284 \div 4$

6 Below are some mistakes students made on a test and how they made them. Write what each student has done incorrectly and what the answer should be.

(a) $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So then I added 21.'
(b) $35 \times 3 = 140$. Sam: 'I doubled 35 then doubled my answer to get 140.'
(c) $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'
(d) $27 \times 12 = 214$. Joseph: 'I got 20 lots of 10 and added 7 lots of 2.'

7 Use strategies to find the following.

(a)	$149 + 790$	**(b)**	$2067 - 358$	**(c)**	$374 + 572$	**(d)**	$5802 \div 2$
(e)	$2784 - 892$	**(f)**	$298 + 649$	**(g)**	$1246 \div 2$	**(h)**	128×3
(i)	134×11	**(j)**	$876 \div 2$	**(k)**	360×5	**(l)**	$1270 - 648$

eQuestions

1.8 Extension: Number bases other than 10

Our usual counting system is the **decimal system**. It represents numbers written to base 10. There are some number systems that use other bases. One example is the **binary system** (base 2). This system is commonly used in computers and electronics. In electronics there are only two signals that can be sent: an electrical current (on) or no electrical current (off).

The decimal system has ten digits (0 to 9) that can be used in each place. In the binary system each place can only be filled by one of the two digits 0 or 1.

Look at this table, showing place values for the number 1101 in the decimal system.

Hundred-thousands 100 000	*Ten-thousands 10 000*	*Thousands 1000*	*Hundreds 100*	*Tens 10*	*Units 1*
		1	1	0	1

The number 1101 means 1 lot of 1000, 1 lot of 100, no lots of 10 and 1 unit.

You can see that each place value is worth ten times more than the place to its right.

Now compare place values for 1101 in the binary system:

32	*16*	*8*	*4*	*2*	*1*
		1	1	0	1

You can see that each place value is worth two times more than the place to its right. The number 1101 in the binary system is written as $(1101)_2$.

So $(1101)_2$ as a decimal number is

1 lot of 8, 1 lot of 4, no lots of 2 and 1 unit

$= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = 13$ in base 10.

worked example 9

Write $(11001)_2$ as a number in decimal form.

Steps

1. Multiply each digit by its place value.

Solution

$(11001)_2$

$= 1 \times 16 + 1 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1$

2. Calculate the answer.

$= 16 + 8 + 1$
$= 25$

worked example 10

Write 75 as a number in binary form.

Steps	Solution
1. Find the largest binary place value number that is less than 75. Binary place value numbers are 1, 2, 4, 8, 16, 32, 64, 128, … Find the remainder.	$75 - 64 = 11$
2. Find the largest binary place value number that is less than the remainder, then find the remainder. Repeat until you get to a remainder of 1 or 0.	$11 - 8 = 3$ $3 - 2 = 1$
3. Write the decimal number as the sum of binary place value numbers.	$75 = 64 + 8 + 2 + 1$ $= 1 \times 64 + 0 \times 32 + 0 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$
4. As in the decimal system, we don't start a number with a zero. So write 1 for the first binary place used then put a 1 or 0 for each place following, down to the units.	$75 = (1001011)_2$

exercise 1.8 Extension: Number bases other than 10

Preparation: Prep Zone Q2–4, Ex 1.1

1 Write these binary numbers in decimal form.

(a) $(100)_2$ **(b)** $(110)_2$ **(c)** $(1001)_2$ **(d)** $(10)_2$
(e) $(101)_2$ **(f)** $(1101)_2$ **(g)** $(111)_2$ **(h)** $(11)_2$
(i) $(1100)_2$ **(j)** $(1011)_2$ **(k)** $(1111)_2$ **(l)** $(10001)_2$
(m) $(11010)_2$ **(n)** $(110011)_2$ **(o)** $(101111)_2$ **(p)** $(111111)_2$

Hint

2 Write these decimal numbers in binary form.

(a) 8 **(b)** 15 **(c)** 27 **(d)** 36
(e) 41 **(f)** 53 **(g)** 65 **(h)** 90

Hint

3 Write every decimal number from 1 to 16 in binary form.

4 Choose the correct answer.
The decimal number 125 written in binary form is:
A $(1111011)_2$ **B** $(1111101)_2$ **C** $(1111100)_2$ **D** $(1110011)_2$

5 Each of these numbers is one less than a binary place value:
1, 3, 7, 15, 31, 63.

(a) Write each of these numbers in binary form.

(b) What do all of these numbers have in common when written in binary form?

(c) Starting with 9, list the first six decimal numbers that are one less than a place value number in the decimal system.

6 Another number system is called the octal system. In this system you are only allowed to use the numbers 0 to 7, and each place value is 8 times that of the previous place.

(a) Copy and complete this list of numbers, which shows the place value in the octal system: 4096, ___, 64, 8, ___

(b) Copy and complete the following table, which converts numbers from decimal form to octal form.

		Octal system place value				
	Decimal number	*512*	*64*	*8*	*1*	*Octal number*
(i)	29	$0 \times 512 = 0$	$0 \times 64 = 0$	$3 \times 8 = 24$	$5 \times 1 = 5$	$(35)_8$
(ii)	10	$0 \times 512 = 0$	$0 \times 64 = 0$	$\_\_ \times 8 = \_\_$	$\_\_ \times 1 = \_\_$	$(\_\_)_8$
(iii)	___	$0 \times 512 = 0$	$\_\_ \times 64 = 64$	$\_\_ \times 8 = \_\_$	$\_\_ \times 1 = \_\_$	$(121)_8$
(iv)	___	$1 \times 512 = \_\_$	$0 \times 64 = 0$	$\_\_ \times 8 = \_\_$	$\_\_ \times 1 = \_\_$	$(1003)_8$
(v)	97	$0 \times 512 = 0$	$\_\_ \times 64 = \_\_$	$\_\_ \times 8 = \_\_$	$1 \times 1 = \_\_$	$(\_\_)_8$
(vi)	130	$0 \times 512 = 0$	$\_\_ \times 64 = \_\_$	$\_\_ \times 8 = \_\_$	$\_\_ \times 1 = \_\_$	$(\_\_)_8$
(vii)	600	$\_\_ \times 512 = \_\_$	$1 \times 64 = 64$	$\_\_ \times 8 = \_\_$	$0 \times 1 = 0$	$(\_\_)_8$

Hint

7 Choose three decimal numbers between 300 and 400 and write them using the octal system.

Homework 1.3

speedingzone

Do these in your head as quickly as you can and write down the answers.

Time target: 2 minutes

1 17×6 **2** $112 - 38$ **3** $1\frac{4}{5} - \frac{3}{5}$

4 $0.2 + 1.4$ **5** $3600 \div 900$ **6** $\$2.60 + \$9.50$

7 What time is it 5 hours and 40 minutes after 9.17 a.m.?

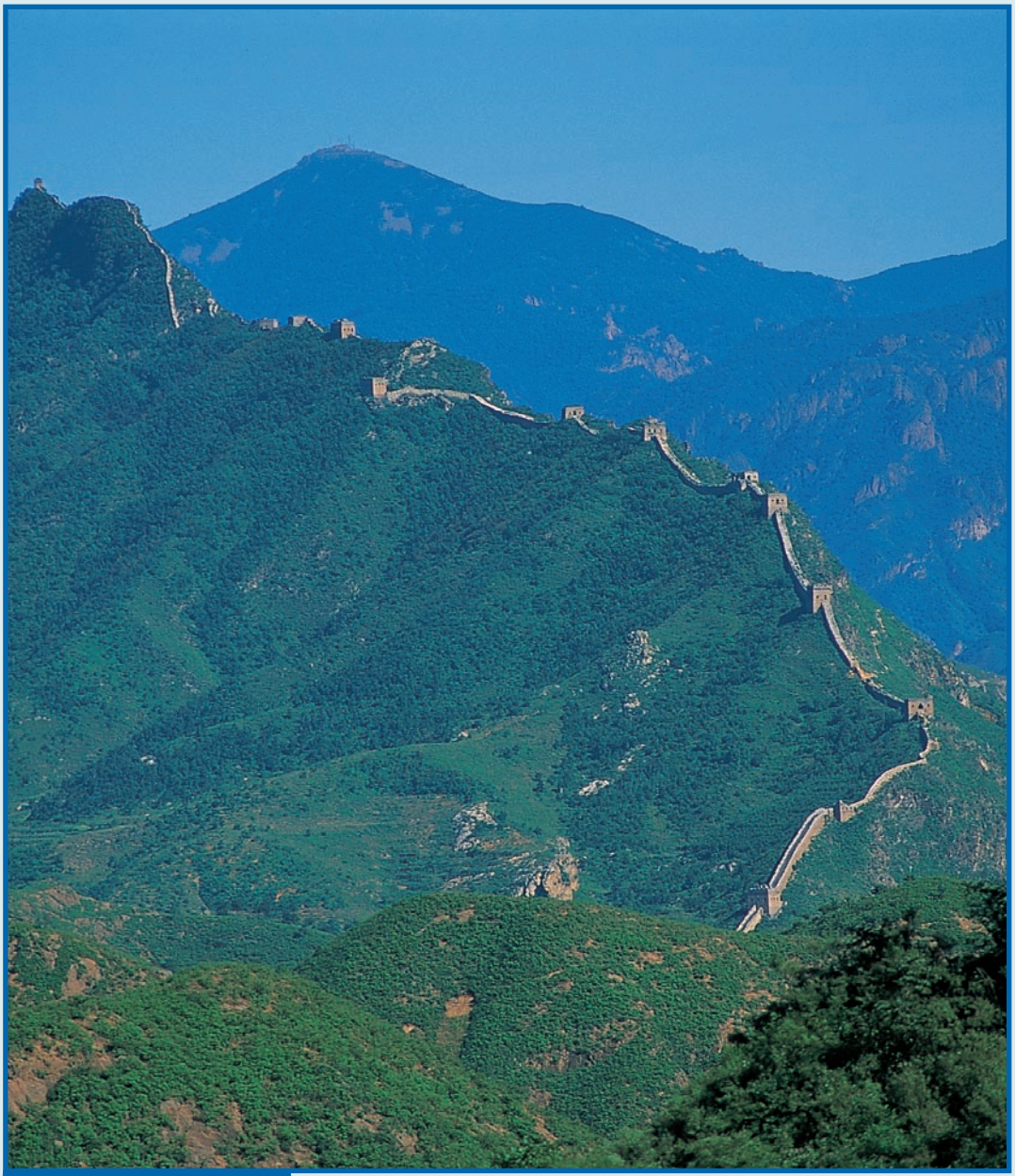
8 What is one-third of $7.50?

9 How many odd numbers are there between 20 and 36?

10 Find the missing number: 816, 408, ___, 102.

maths in action

Calculating the Great Wall



The Great Wall of China

Could the Great Wall of China have been built without the help of some sort of calculating device? Nowadays, computers and calculators are used in building design, but how did people in ancient times do their sums when really large and complicated numbers were involved?

Most of the early number systems were awkward to use for anything but simple recording. 'Carrying the one' and other easy techniques we use on paper just didn't work for the number systems of the ancient Greeks, Romans and Chinese. They needed other methods.

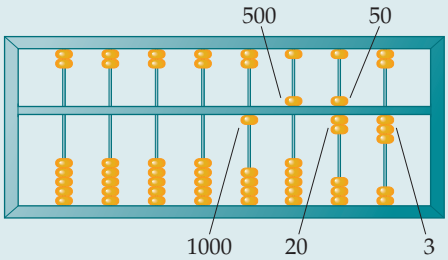
The first aid was the sand abacus where figures were drawn with a pointed object onto a flat surface covered in sand and erased with a finger. The sand abacus evolved into the line abacus, where counters were moved across lines drawn on a table.

A skilled user of an abacus can perform complicated arithmetic very quickly, which is why the abacus is still used in some countries today.

The bead abacus was developed in the late Middle Ages in China, where it was called the suan-pan. These involve two sets of beads moving on parallel strings. The first set contains five beads and allows counting from 1 to 5, and the second set has only two beads representing 5 or 10. Beads are 'counted' by moving them from the outside towards the middle beam. The far right column represents the 'ones', the next column the 'tens', the next, the 'hundreds' and so on.

The abacus at right shows the number 1573.

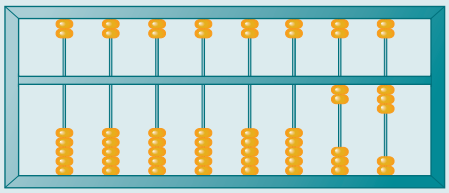
1000 + 20 + 3 + 500 + 50 = 1573



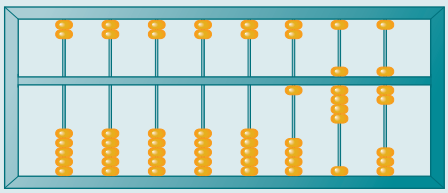
Questions

1 What do the following abacuses show?

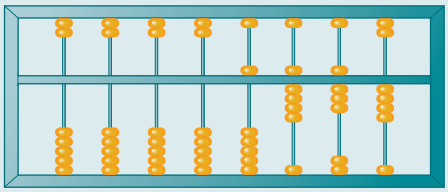
(a)



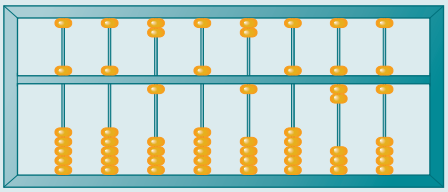
(b)



(c)



(d)



2 Draw abacuses that show the following numbers.

(a) 341 **(b)** 64 **(c)** 79 **(d)** 843 **(e)** 6492 **(f)** 76 894

Research

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Create an annotated poster that reflects the history of mechanical calculators right up until the twentieth century. Include Napier's bones, Schickard's machine, the Pascaline and the Difference Engine.

languagezone

Summary

Copy and complete the following summary of this chapter using the words and phrases from the list. A word or phrase may be used more than once.

1 The number system we use is called the _____ _____.

2 The _____ _____ of numbers is still used, often on watch faces.

3 The sum of one line in a _____ _____ is called the _____ _____.

4 One method we can use to give an _____ for the value of a calculation is by _____ the numbers to the first digit.

5 If there are no brackets involved, then according to the _____ _____ _____ we must do division before addition.

Key words

Babylonian system
binary system
Chinese system
decimal system
Egyptian system
estimate
Hindu–Arabic system
magic square
magic sum
order of operations
Roman system
rounding

Questions

1 Put the following operations in the order they should be done according to order of operations: multiplication, subtraction, brackets, addition, division.

2 James wrote 950 as LM using Roman numerals. Explain why this is not correct and write 950 correctly using Roman numerals.

3 When we use estimation we can call the answer an 'estimate'. This is the noun form. Write a sentence using 'estimate' in verb form. Notice how we say (pronounce) it differently.

4 You have come across the following words in this chapter: 'product', 'odd', 'prime' and 'power'. Each of these words also has a non-mathematical meaning. Use each word in a short sentence to show its non-mathematical meaning.

5 Describe some benefits of the Hindu–Arabic number system.

6 Make at least 10 words of four letters or more from the letters in 'estimate'.

7 Arrange each of the five counting systems in the list above in order depending on how many letters they have in their title, starting from the one with the fewest letters.

Worksheet L1.1

Worksheet L1.2

chapter 1 REVIEW

FAQs

Do I only use order of operations if the question asks me to?
No, you must always use order of operations. It is a mathematical rule.

Do all calculators do order of operations automatically?
Most new calculators do, but some older ones don't. It is best to check if your calculator does by putting in an easy question that would get a different answer if order of operations wasn't followed; for example, $1 + 2 \times 4$ should give an answer of 9 if the order of operations is followed.

Core

1 Look back at the symbols on pages 3–5. 1.1
Write out these numbers in:

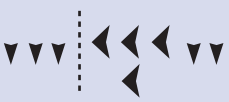
(i) the Egyptian number system
(ii) the Roman number system
(iii) the Babylonian number system
(iv) the modern Chinese number system.

(a) 54 **(b)** 146 **(c)** 238 **(d)** 309

2 Look back at the symbols on pages 3–5. 1.1
Write out these numbers in the Hindu–Arabic number system.

(a) CCCXL **(b)** MMCDLXXIII

(c)  **(d)** 

(e)  **(f)**  **(g)** 九十八 **(h)** 一千六百四

3 Round off these numbers to the first digit. 1.5

(a) 528 **(b)** 189 **(c)** 2500 **(d)** 3088

4 Use rounding to the first digit to estimate these products. 1.5

(a) 3741×22 **(b)** 265×341 **(c)** 986×35

5 Use rounding to the first digit to estimate these quotients. 1.5

(a) $25\,736 \div 49$ **(b)** $96\,001 \div 17$ **(c)** $25\,000 \div 621$

6 Use rounding to the first digit to estimate these, and then use your calculator to work out how far off your estimate was from the exact answer. 1.5

(a) $73 - 29 + 5628$ **(b)** $17 \times 35 \times 241$ **(c)** $28 \times 89 - 2455$

7 Find: 1.6

(a) $9 \div (2 + 1) - 2$ **(b)** $(3 \times 8) \div 4 + 7$

(c) $12 - 6 \times 2 + 11$ **(d)** $7 + 12 \div 4 - 1 \times 2$

(e) $(13 - 5 \times 2) + (20 \div 10)$ **(f)** $[5 \times (9 + 1)] - 3$

8 Choose the correct answer. 1.6

In the calculation of $2 \times [30 \div (4 - 1)] + 6$ the first operation to do is:

A + **B** − **C** × **D** ÷

9 Use an appropriate strategy to help simplify each of the following. 1.7

(a) $5 \times 18 \times 2$ **(b)** 21×35 **(c)** 15×8

Extension

10 Copy and complete these magic squares. 1.3

(a)

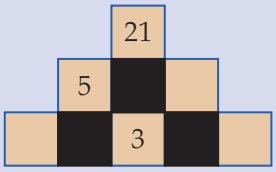
13		
8		
9		7

(b)

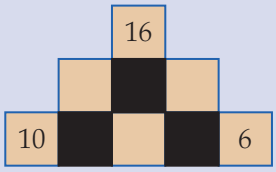
		16	3
	12	13	
		9	14
18	5	4	

11 Copy and complete these number pyramids. 1.4

(a)



(b)



12 Put brackets into these statements, where necessary, to make them true. 1.6

(a) $4 \times 2 + 3 \div 5 - 1 = 3$ **(b)** $5 + 1 \div 6 + 4 + 2 = 7$

13 Replace each * with one of the four operators (+, −, ×, ÷) to make the statement true. 1.6

(a) $9 * 7 * 3 = 30$ **(b)** $16 * 4 \times 2 * 2 = 12$

14 Write these binary numbers in decimal form. 1.8
(a) $(110)_2$ **(b)** $(1001)_2$ **(c)** $(11111)_2$ **(d)** $(101101)_2$

15 Write these decimal numbers in binary form. 1.8
(a) 11 **(b)** 29 **(c)** 34 **(d)** 71

REPLAY

1 Set out these calculations in your normal way and work out the answers. Worksheet R1.7
(a) 138 – 97 **(b)** 1902 – 845 **(c)** 5485 – 1099

2 List the numbers you get if you count by nines, starting at 30 and ending at 75. Worksheet R1.8

3 Copy and complete each of the following by writing < or > between the given numbers. Worksheet R1.9
(a) 1001 __ 982 **(b)** 3.9 __ 3.38 **(c)** 0.03 __ 0.19

4 Copy and complete the following by finding the pattern. Worksheet R1.10
(a) 9, 15, 22, 30, __, __, __ **(b)** 1, 3, 7, 15, __, __, __

5 Calculate: Worksheet R1.11
(a) 8000 ÷ 200 **(b)** 1200 ÷ 4 **(c)** 45 000 ÷ 90

6 Find the missing number that makes each of the following true. Worksheet R1.12
(a) __ + 7 = 24 **(b)** 4 × __ = 15 – 3 **(c)** 7 × 0 = 15 – __

7 Perform the following divisions. Worksheet R1.13
(a) 768 ÷ 3 **(b)** 1404 ÷ 9 **(c)** 7865 ÷ 5

8 List all numbers that 8 goes into that are greater than 70 and less than 150. Worksheet R1.14

9 Simplify: Worksheet R1.15
(a) 2 + 5 × 9 **(b)** 18 ÷ 6 – 3 **(c)** 8 × (15 – 5)

10 Which whole number are the following decimals closest to? Worksheet R1.16
(a) 1.9 **(b)** 5.089 **(c)** 37.6001

11 Calculate: Worksheet R1.17
(a) $\frac{7}{9} - \frac{2}{9}$ **(b)** $\frac{3}{4} + \frac{1}{4}$ **(c)** $1\frac{5}{6} - 1$

12 If Celen purchases 3 tops at $15.50 each, how much change will she receive from a $50 note? Worksheet R1.18

Assignment 1